

# Local Reasoning about the Presence of Bugs: Incorrectness Separation Logic (ISL)

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<sup>4</sup> University College London

# State of the Art: **Correctness**

- ❖ Lots of work on **local reasoning** for proving **correctness**
  - ➔ Prove the **absence of bugs**
  - ➔ **Over-approximate** reasoning

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  - ➔ **Scalability** to large teams and codebases

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  - ➔ Using **correctness**-based compositional analysis

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→ E.g. symbolic model checking, symbolic execution for testing

→ *UI*

***Incorrectness Logic*** (O'Hearn)

Formal Foundations

for

Bug Catching

❖ Base

❖ Exce

→ e.

→ Us

# Incorrectness Logic (IL)

Hoare triples

$\{p\} C \{q\}$

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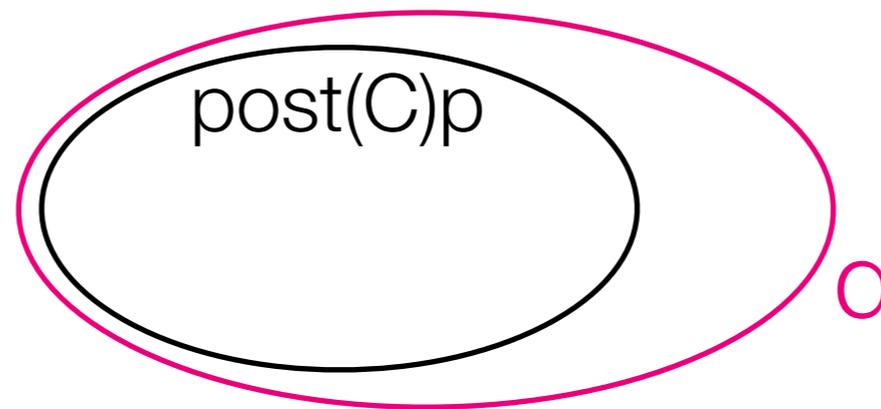
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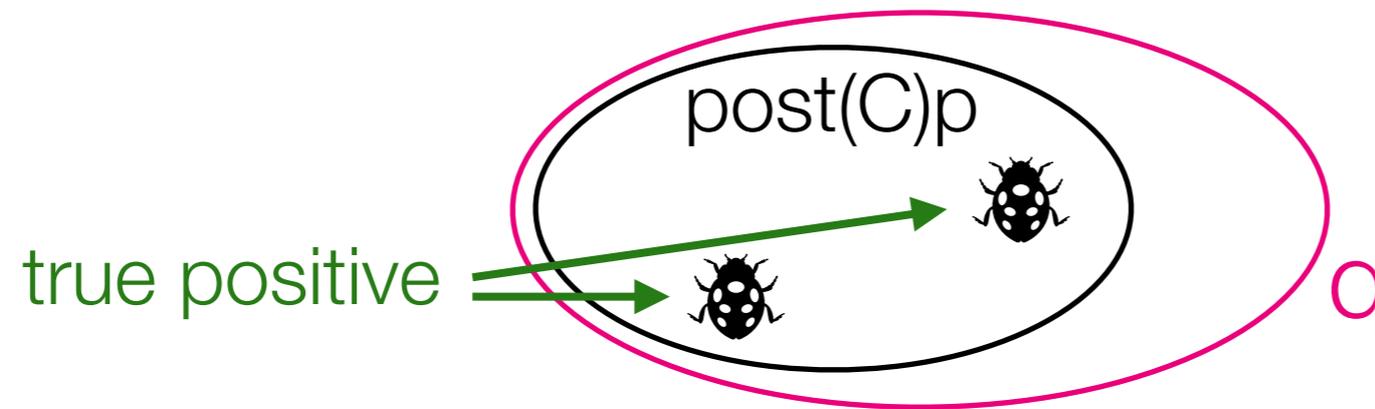


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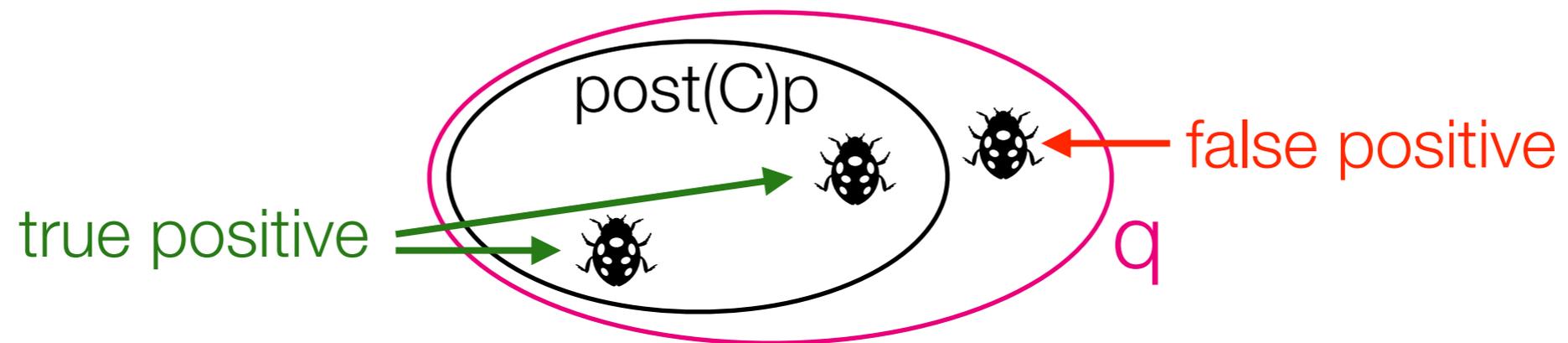


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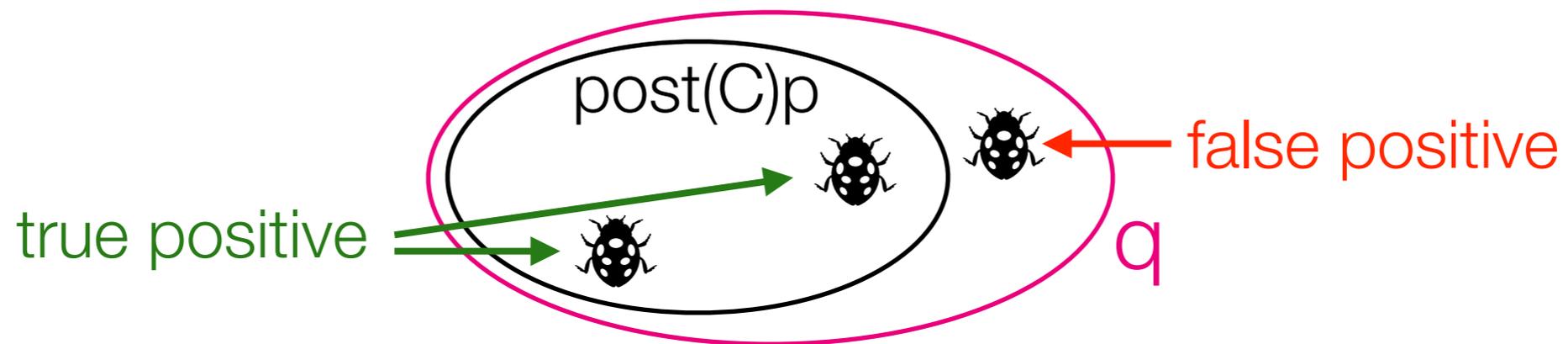


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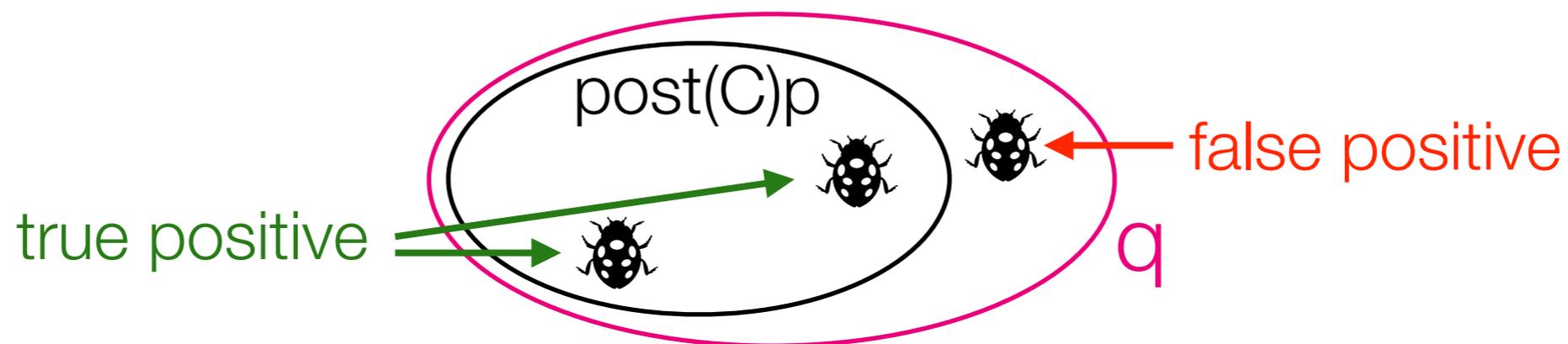
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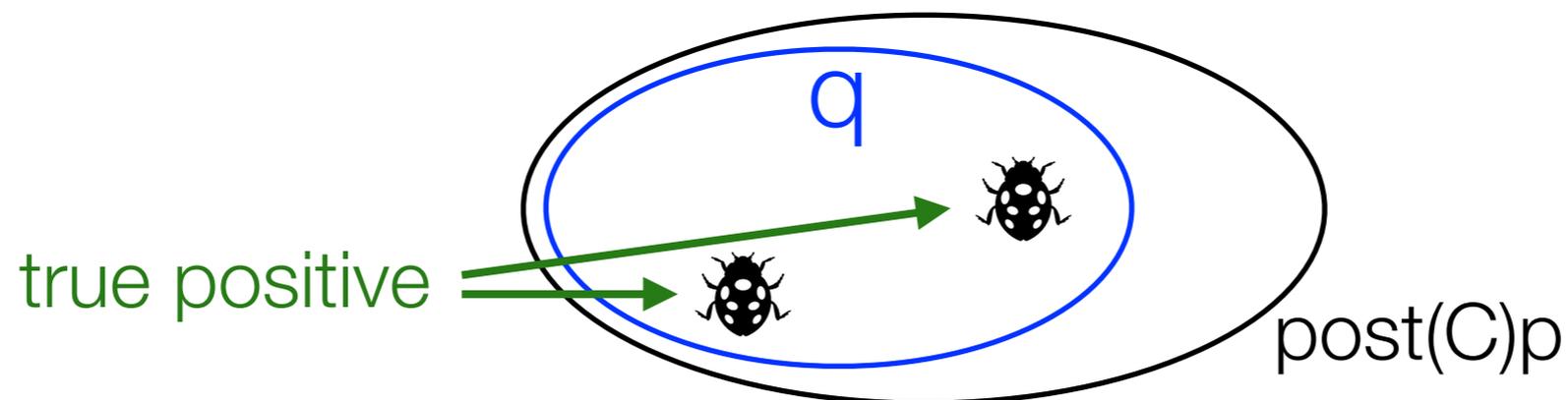
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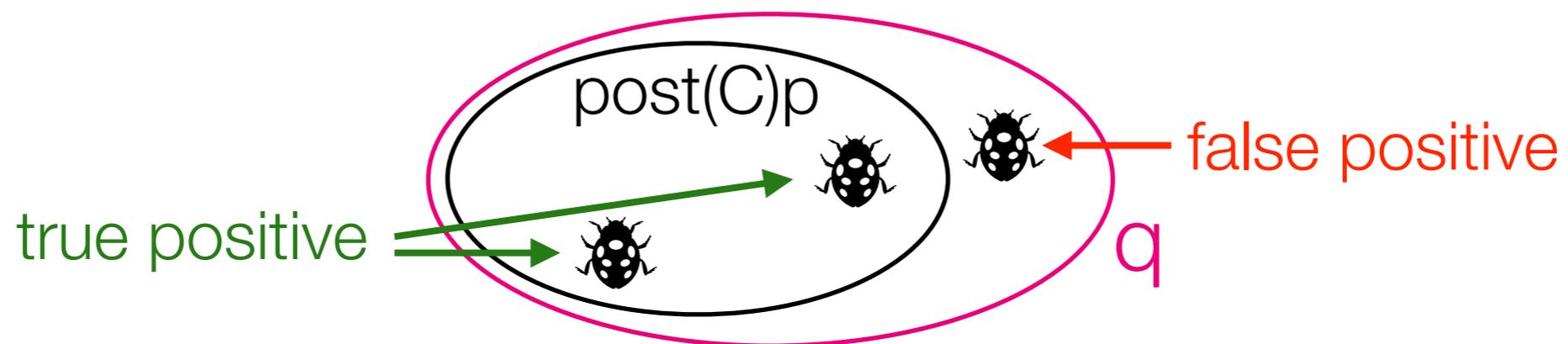


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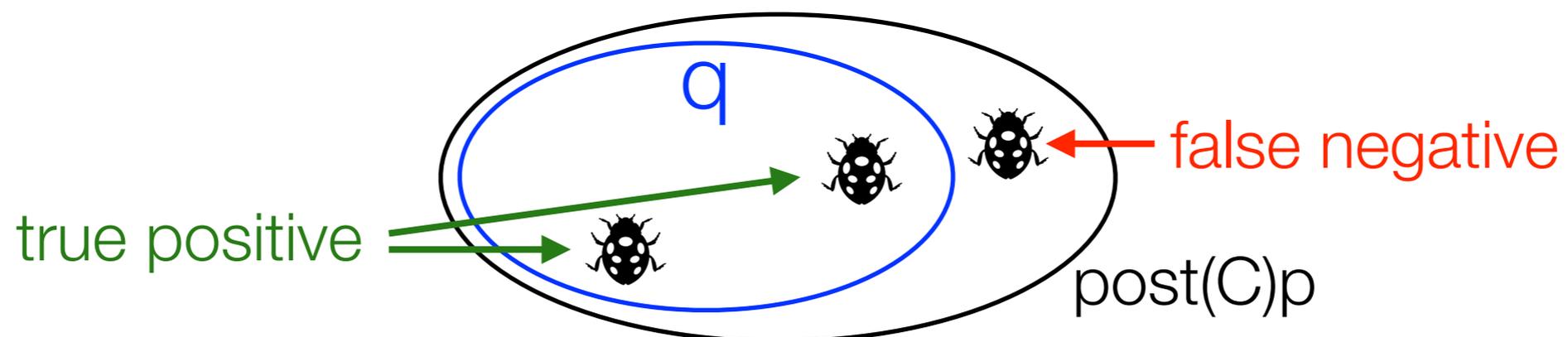
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$$[p] C [\varepsilon: q] \quad \textit{iff} \quad \text{post}(C, \varepsilon)p \supseteq q$$

Equivalent Definition (reachability)

$$[p] C [\varepsilon: q] \quad \textit{iff} \quad \forall s \in q. \exists s' \in p. (s', s) \in [C]_{\varepsilon}$$

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- + ***Under-approximate*** analogue of Hoare Logic
- + Formal foundation for ***bug catching***
- Global reasoning: ***non-compositional*** (as in original Hoare Logic)
- Cannot target ***memory safety bugs*** (e.g. use-after-free)

# Incorrectness Logic: Summary

+ *Under-approximate* analogue of Hoare Logic

+ Formal foundation for *bug catching*

- Glob

- Can

**Our Solution**

***Incorrectness Separation Logic***

(Logic)

# Contributions

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**This talk**



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$n!/2$  conjuncts !

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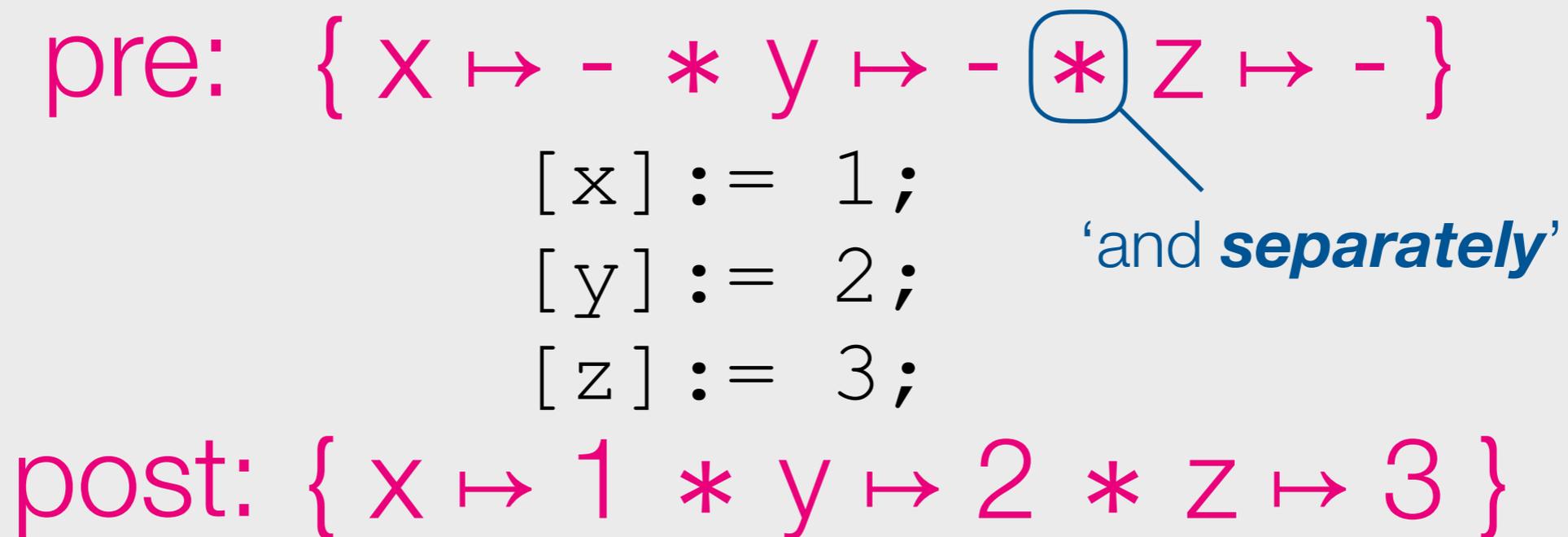
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$\forall x, v, v'. x \mapsto v * x \mapsto v' \Rightarrow \text{false}$

# The Essence of Separation Logic (SL)

## ***Frame Rule***

$$\frac{\{p\} \text{ C } \{q\}}{\{p * r\} \text{ C } \{q * r\}}$$

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**FREE**  $\{x \mapsto -\} \text{free}(x) \{\text{emp}\}$

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IL

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SL

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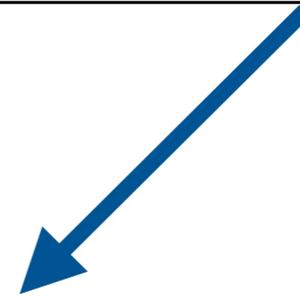
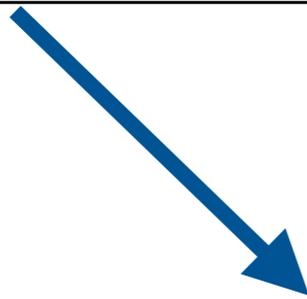
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$$x \mapsto v * x \mapsto v' \Leftrightarrow \text{false}$$

$$x \mapsto v * \text{emp} \Leftrightarrow x \mapsto v$$

$$[x \mapsto v] \text{ free}(x) \text{ [ok: emp]}$$

$$[p] \text{ C } [\varepsilon: q] \quad \textit{iff} \quad \forall s \in q. \exists s' \in p. (s', s) \in [C]_{\varepsilon}$$

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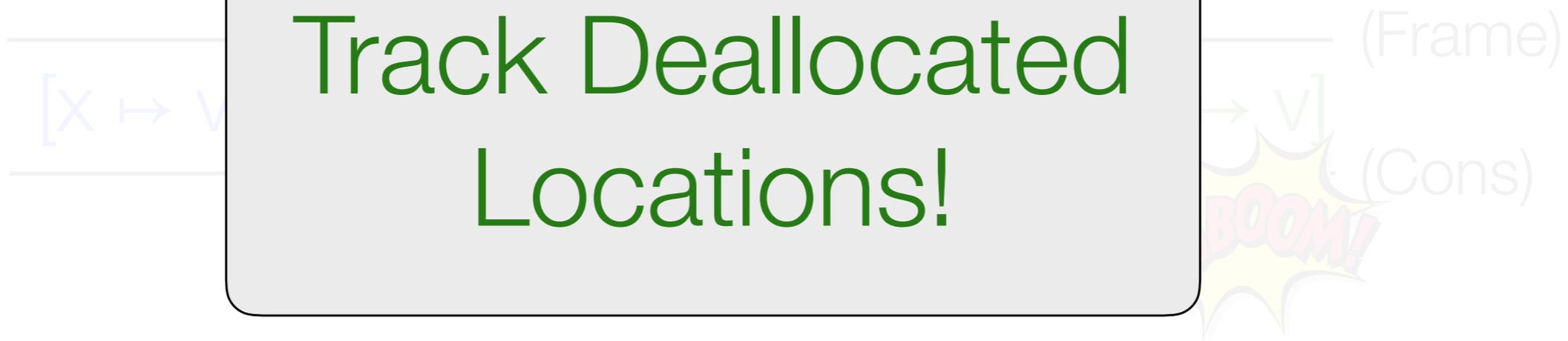

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**Solution:**  
Track Deallocated Locations!



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Thank You for Listening!

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**Incorrectness** logic: global reasoning for **bug catching**  
+  
**Separation** logic: correctness-based **local** reasoning

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Formal foundation for **local & compositional bug catching**