Concurrent Tree Update

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Abstract Tree Module
(High level)

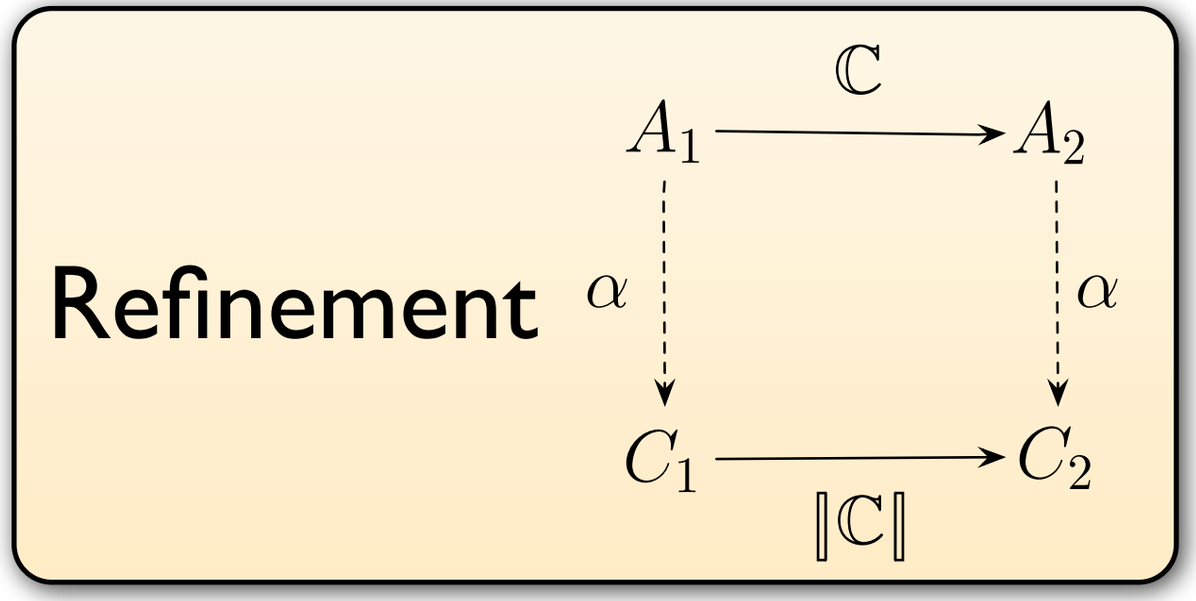
- Abstract data
  - e.g. DOM Trees
- High level commands

Concrete Tree Module
(Low level)

- Concrete data
  - e.g Firefox's DOM
- Implementation

Abstract Tree Module (High level)

- Abstract data
  - e.g. DOM Trees
- High level commands

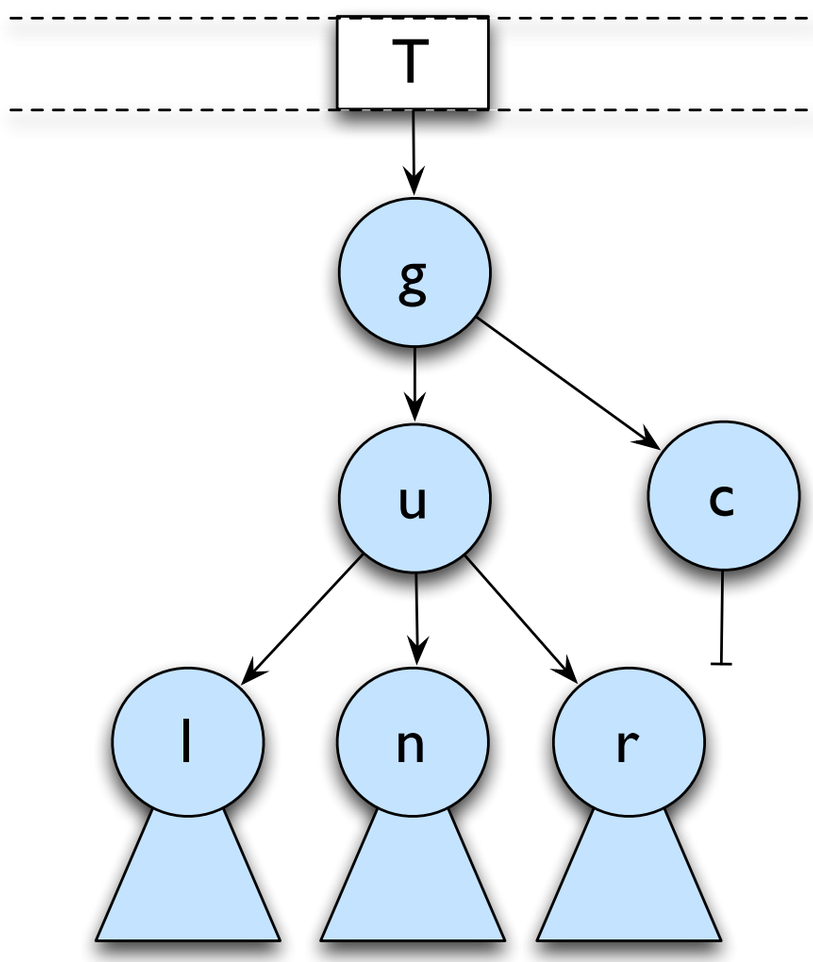


Concrete Tree Module (Low level)

- Concrete data
  - e.g Firefox's DOM
- Implementation

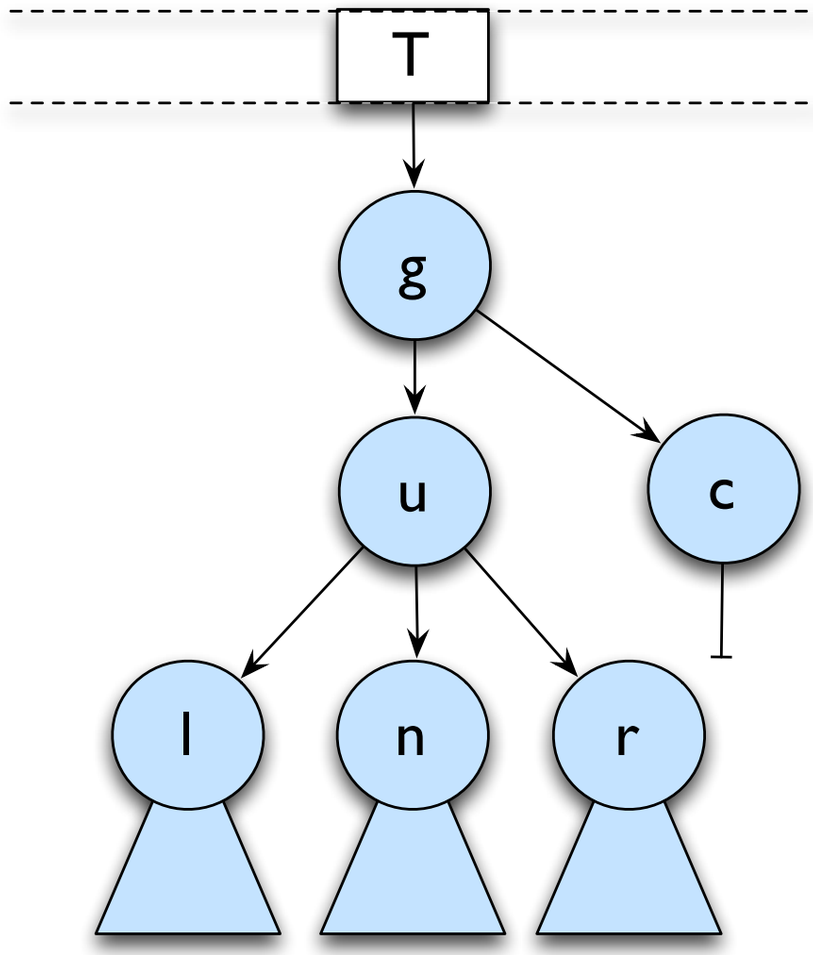
High Level Trees

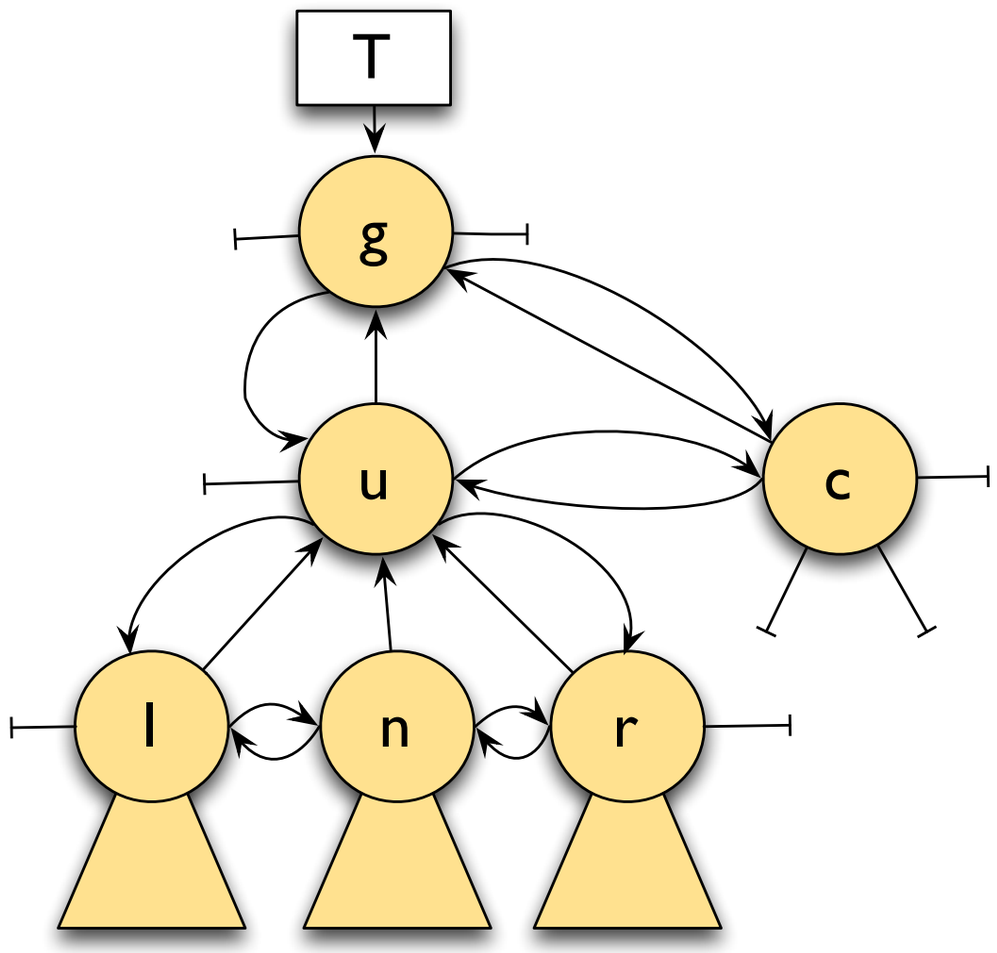
Abstract Tree Representation



High Level Trees

Abstract Tree Representation





High Level Trees

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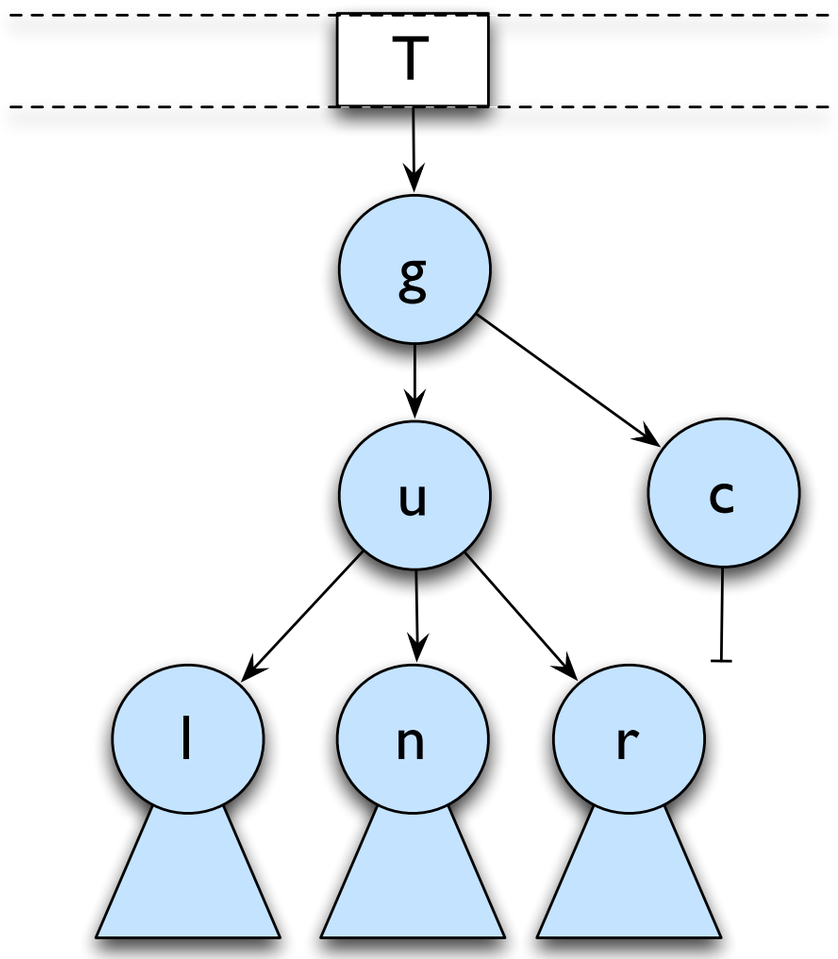
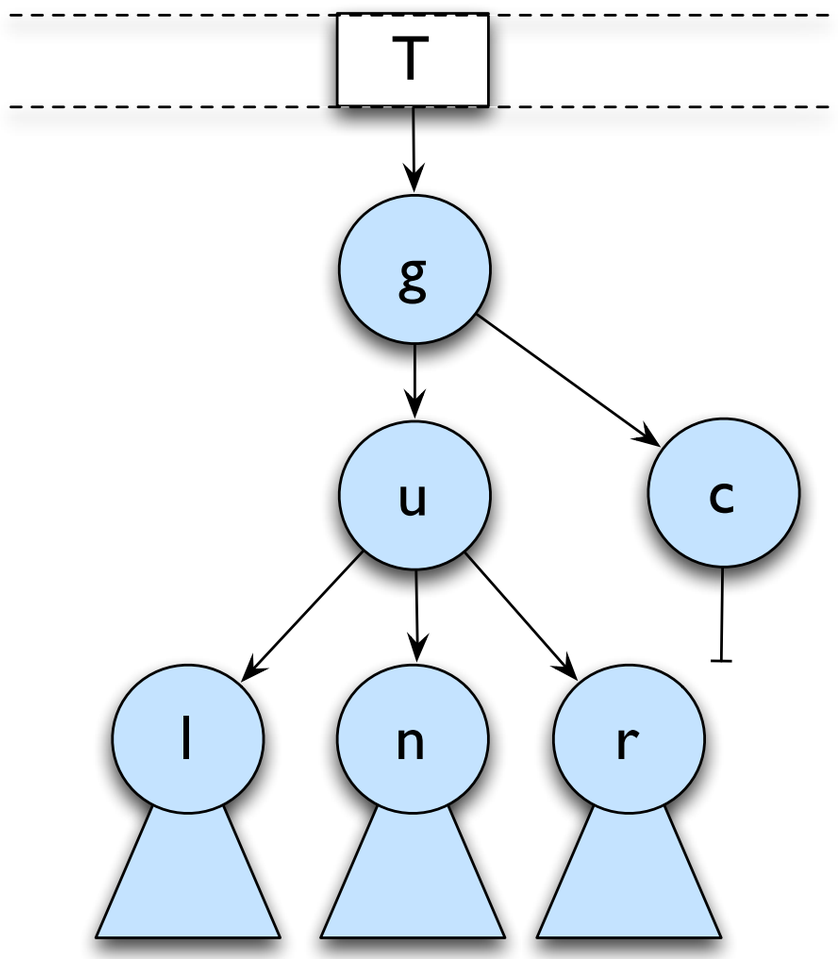


Tree Manipulation Commands

```
getLeft(n)
getRight(n)
getUp(n)
getFirst(n)
getLast(n)
newNodeAfter(n)
deleteTree(n)
appendChild(n)
```

High Level Trees

Abstract Tree Representation



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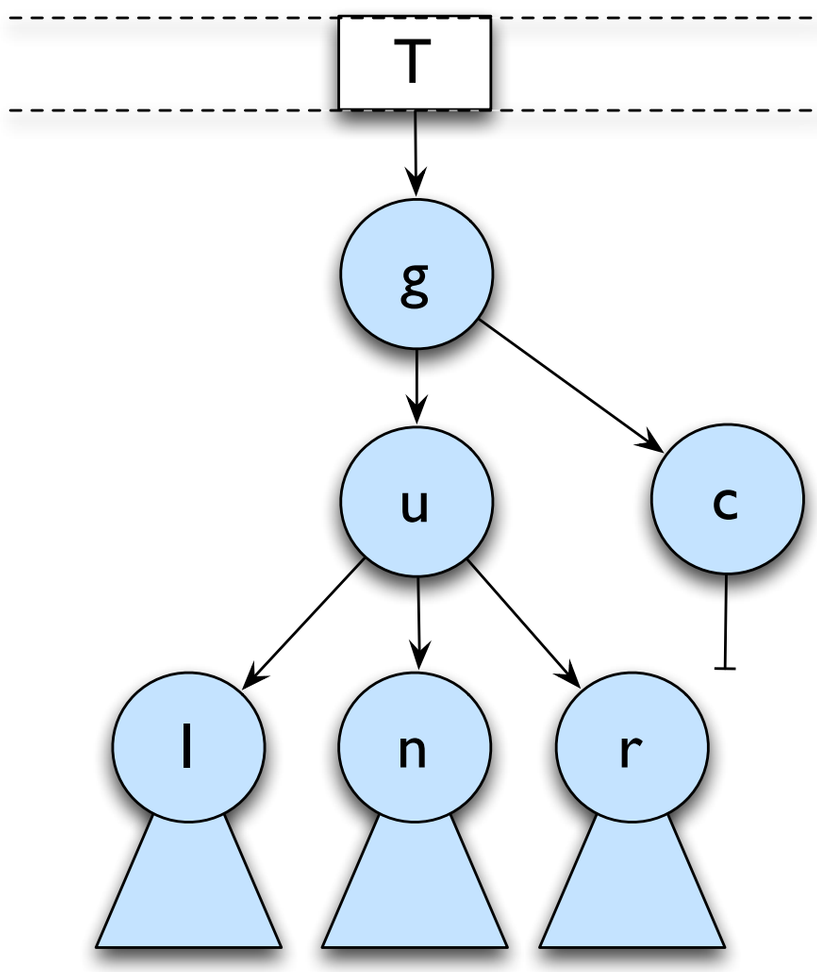
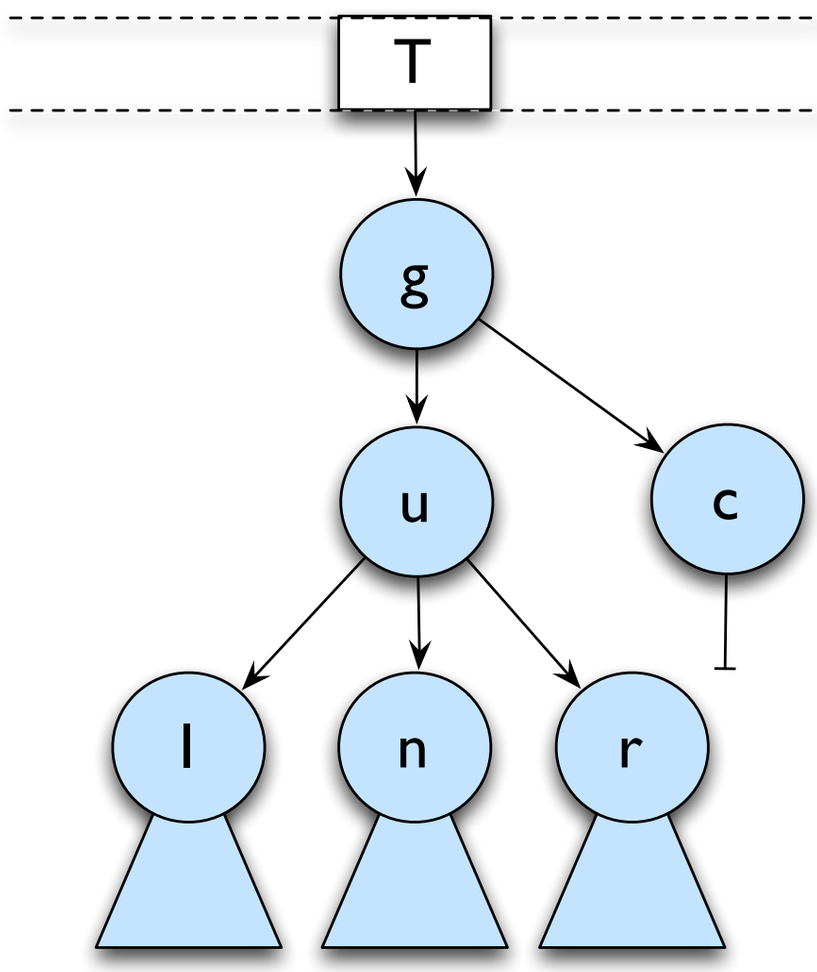
Unique identifiers to locate data

High Level Trees

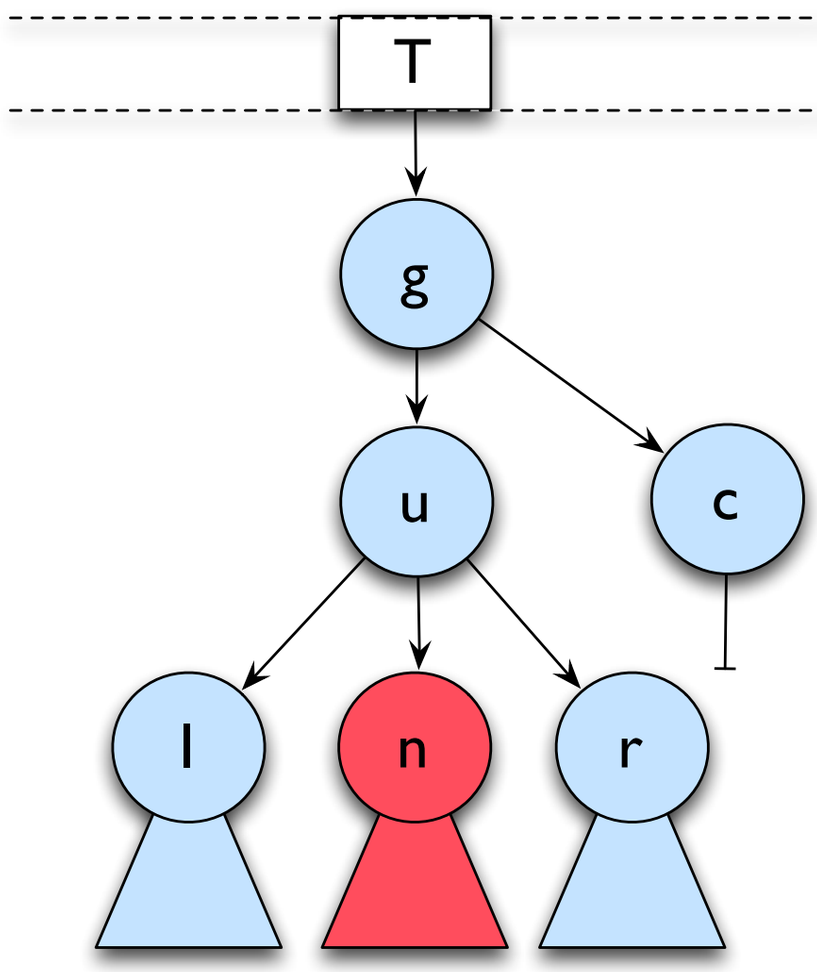


High Level Trees




```
deleteTree(n)
```

High Level Trees




```
deleteTree(n)
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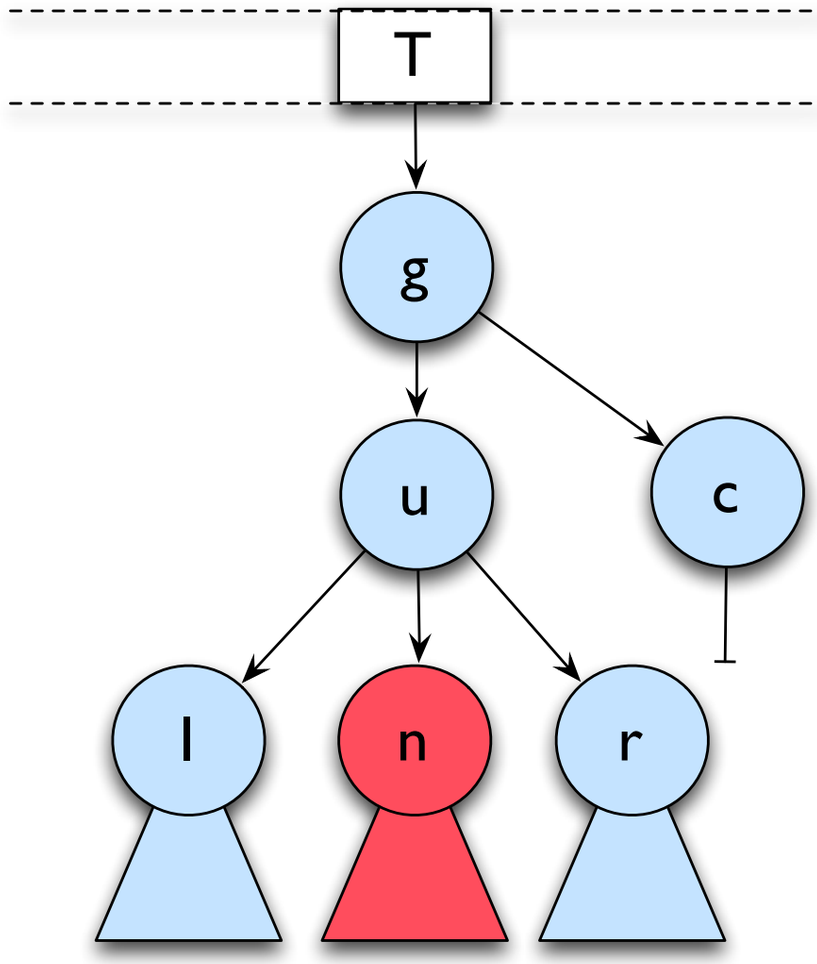
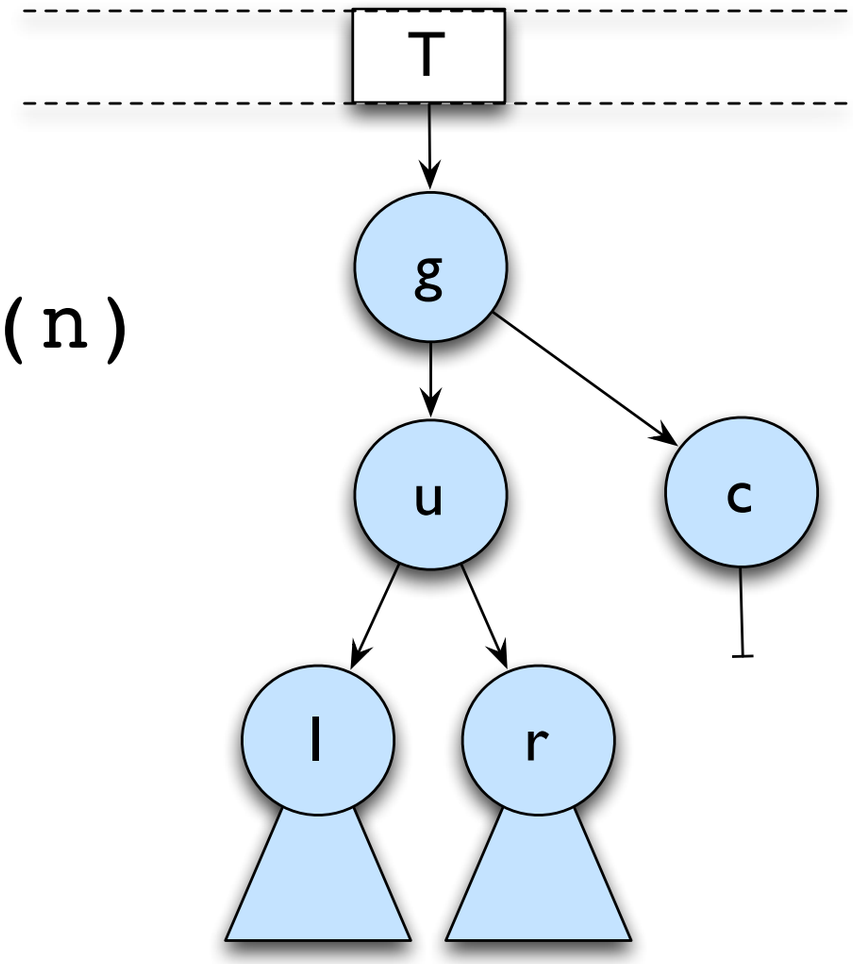
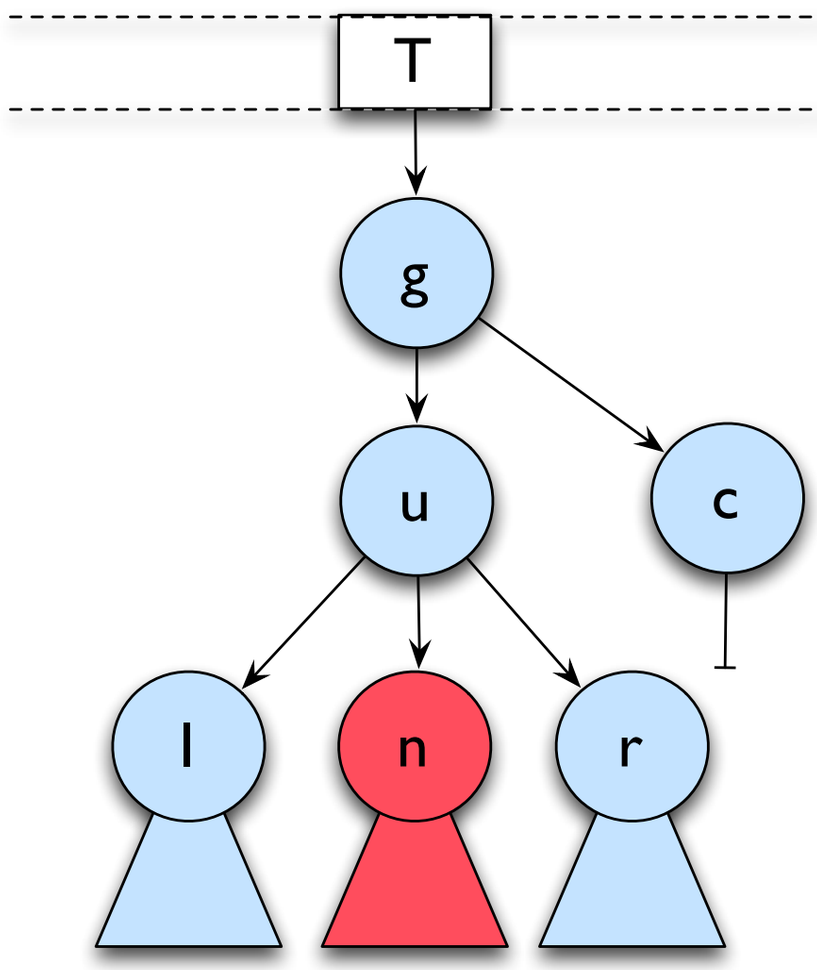
High Level Trees



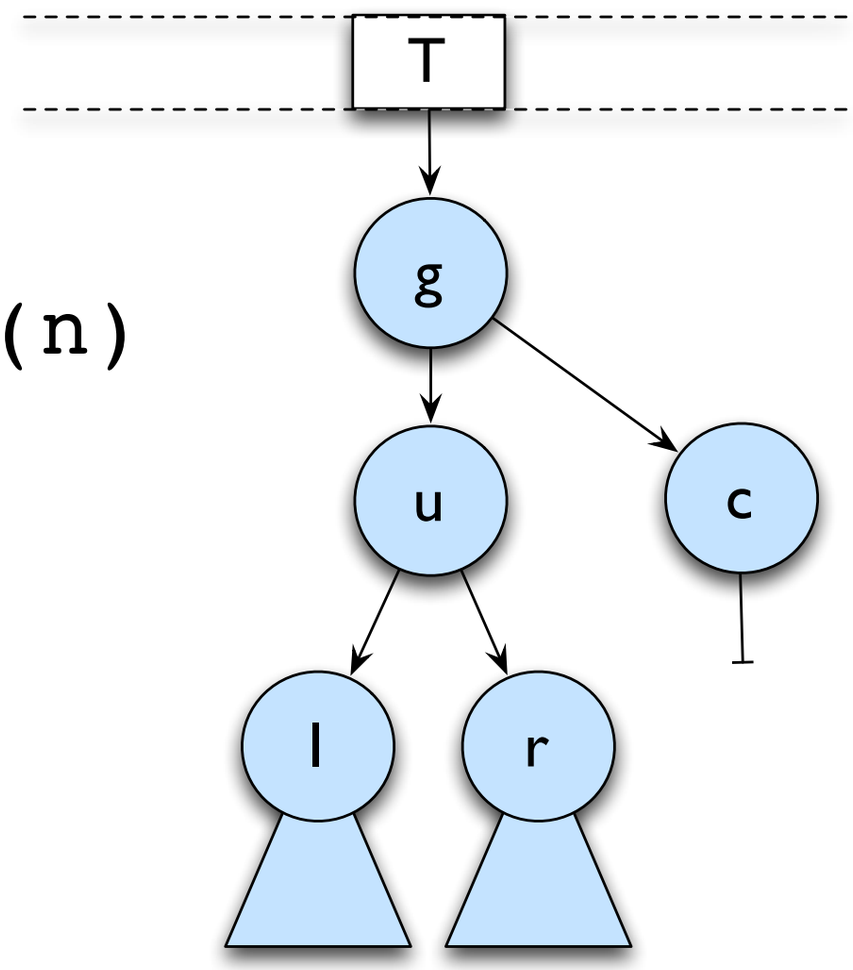
`deleteTree(n)`



High Level Trees



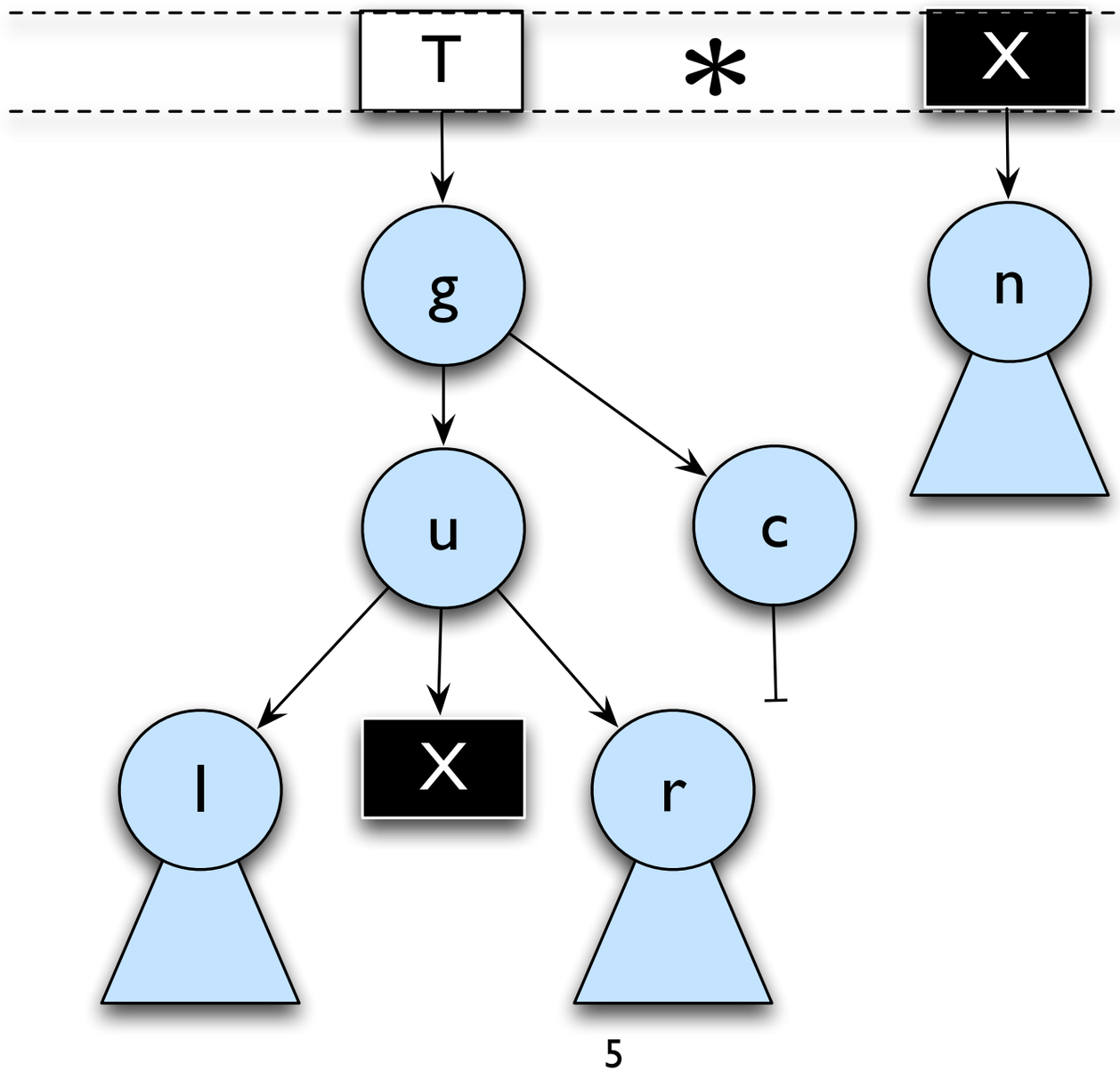
deleteTree(n)



Unnecessarily large footprint!

High Level Trees

Structural Separation Logic to the rescue!



High Level Trees

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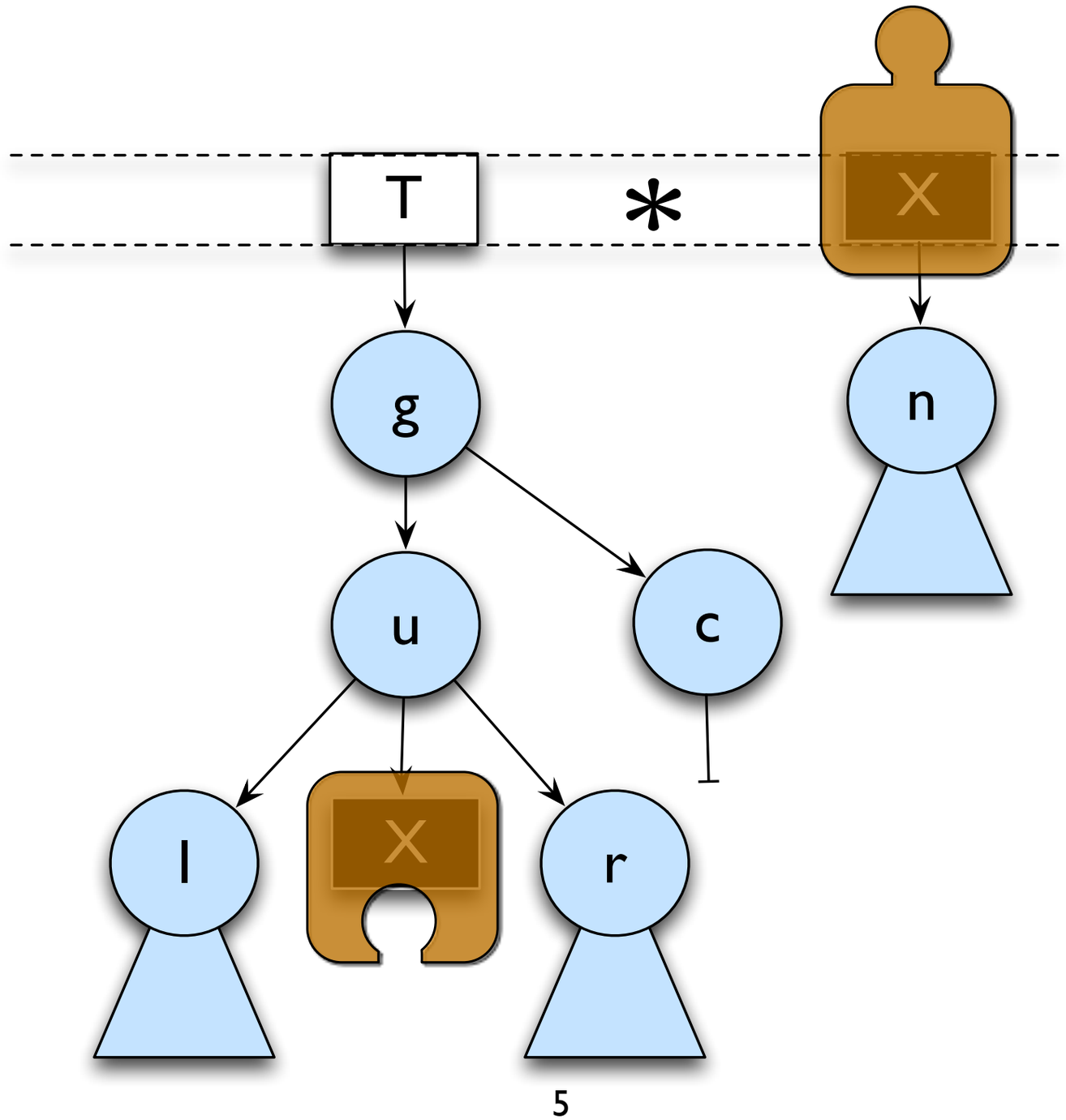

High Level Trees

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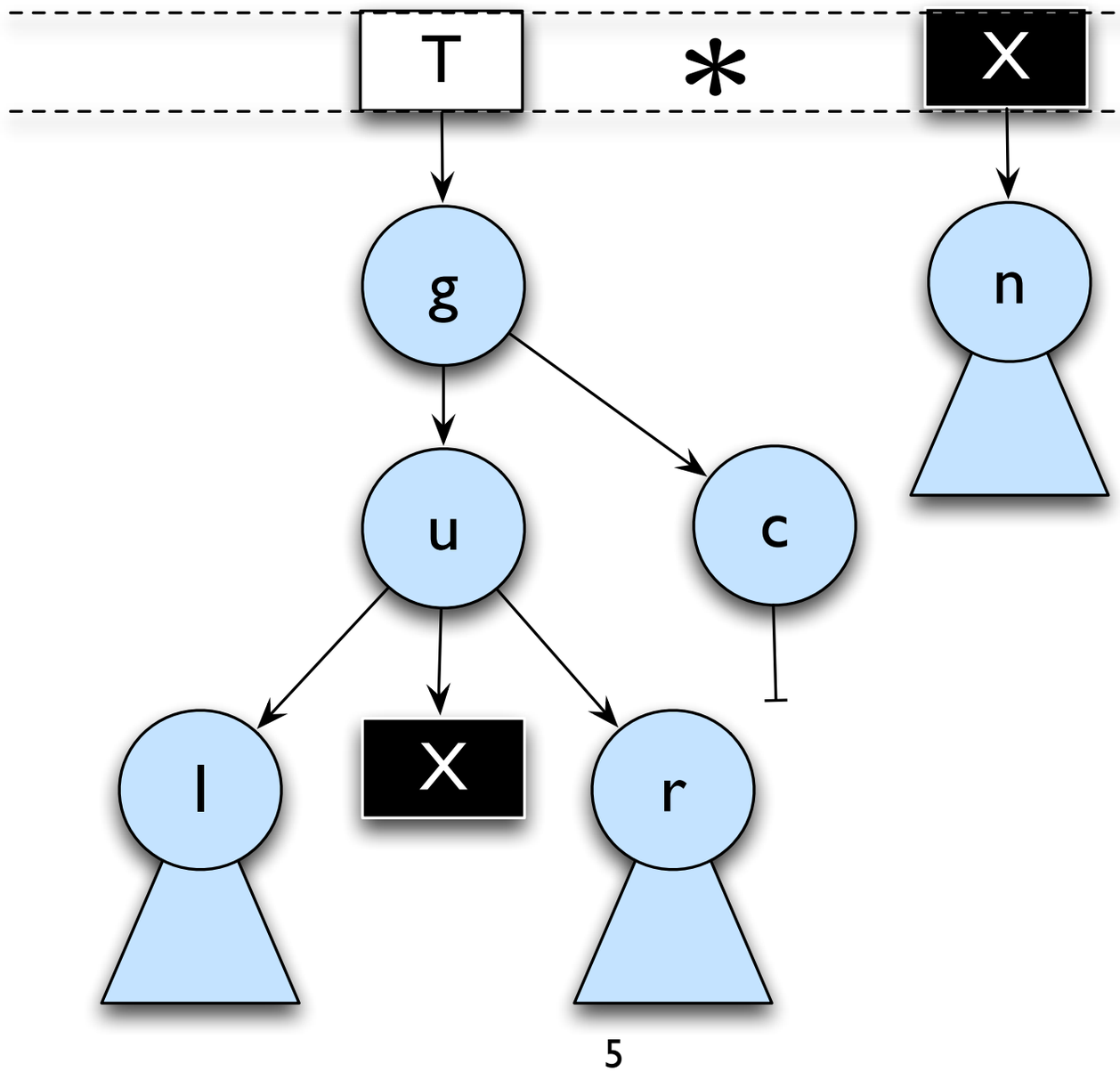

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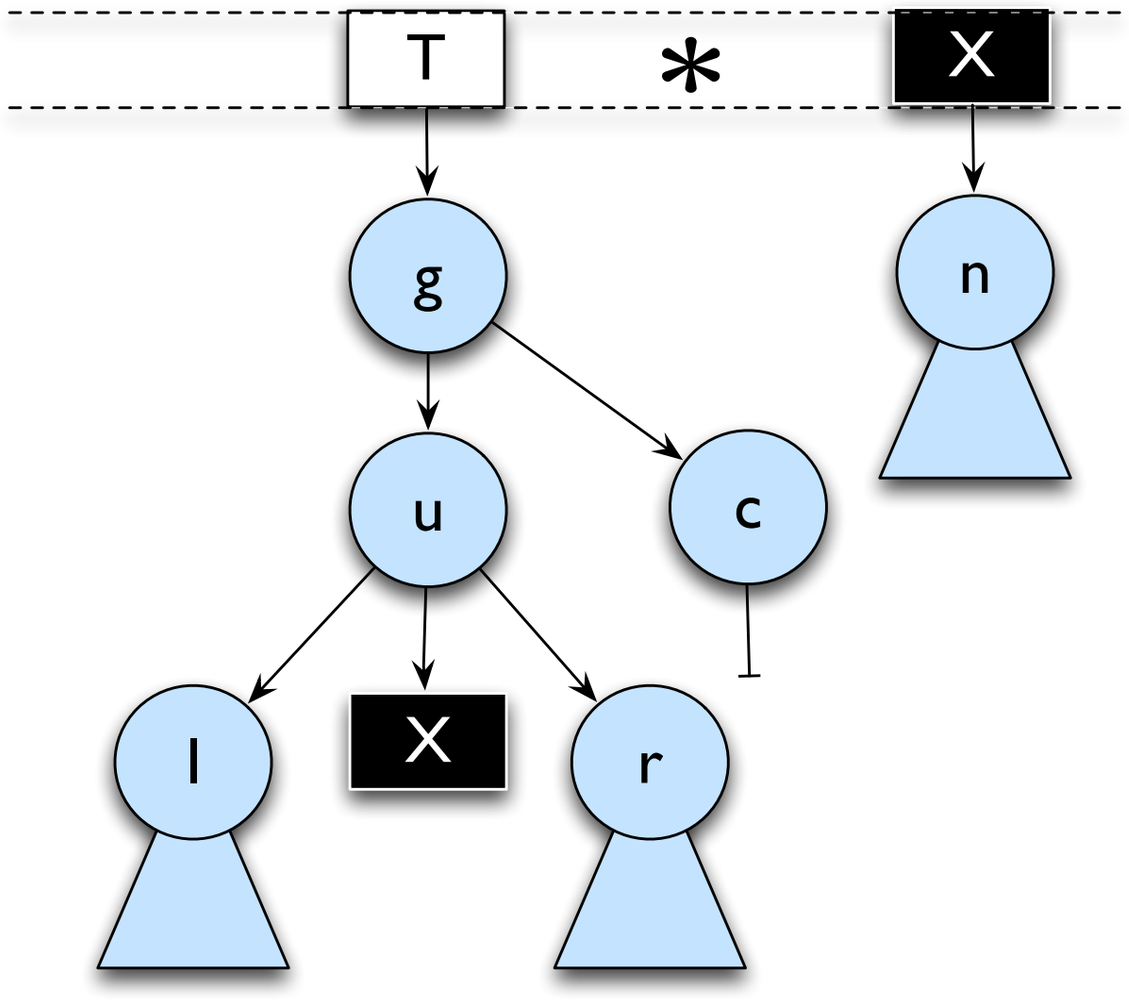
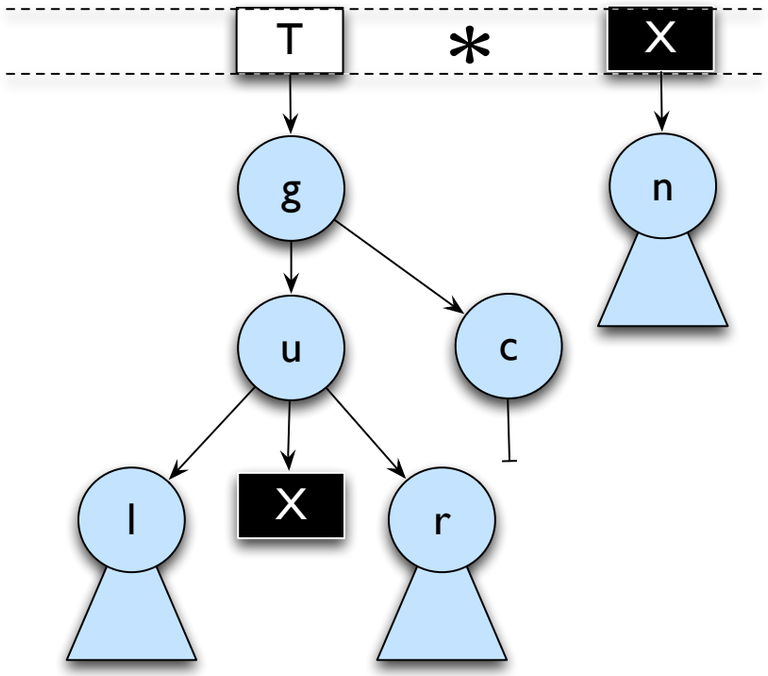


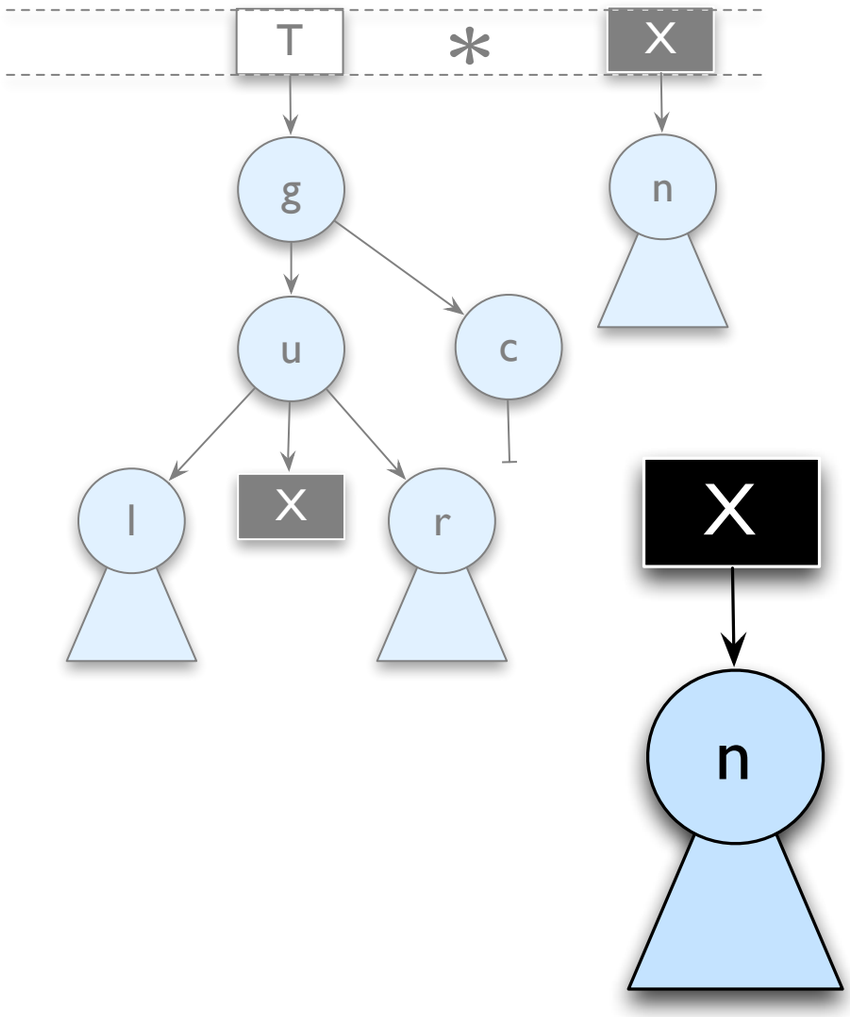
- Careful choice of abstract address set

- Abstract addresses must be preserved

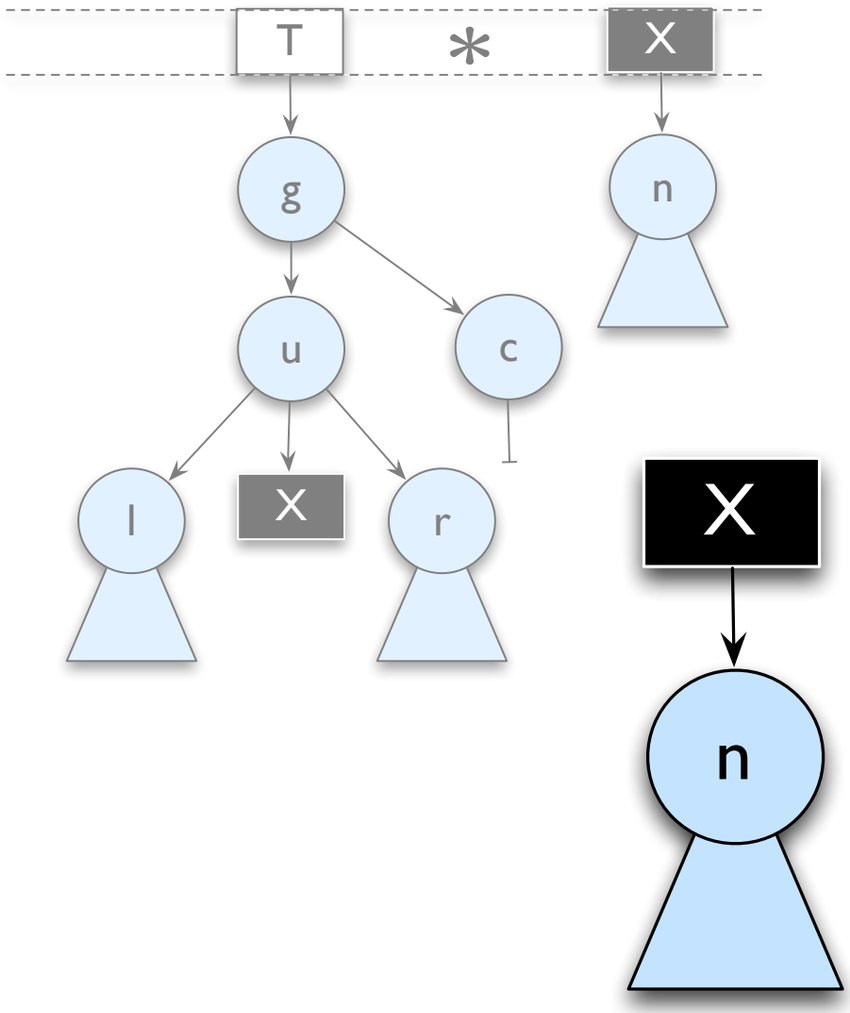
High Level Trees



High Level Trees

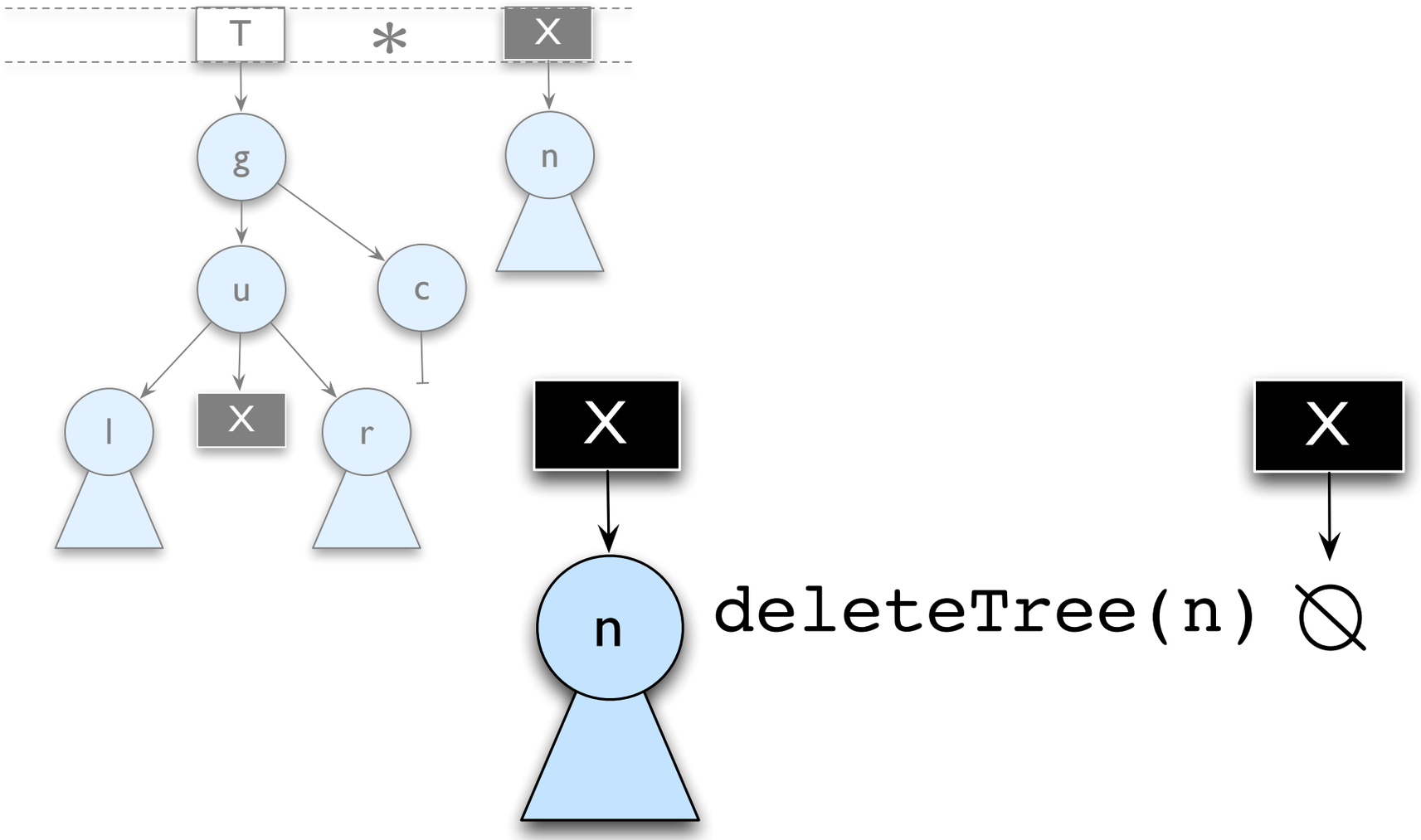


High Level Trees

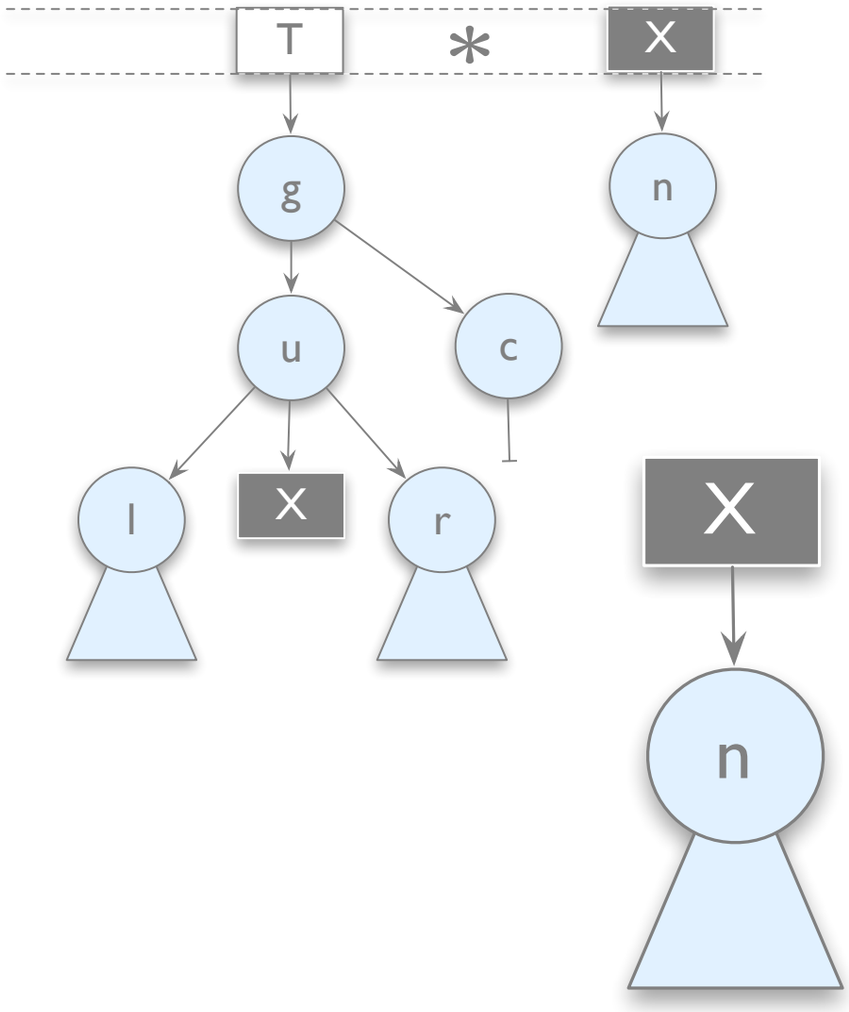



```
deleteTree(n)
```

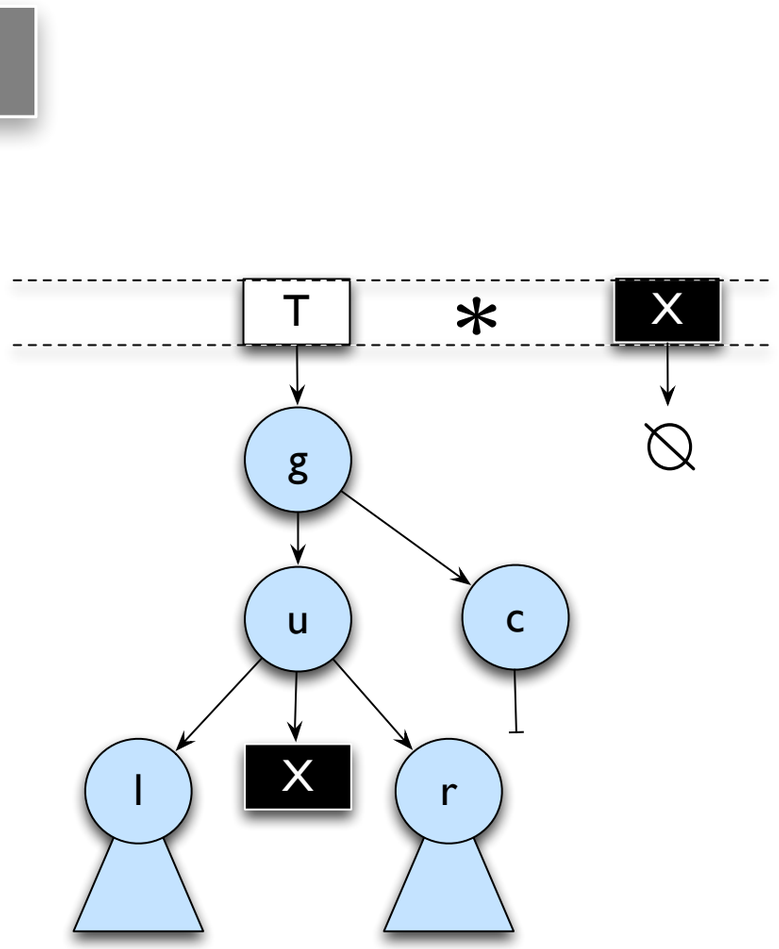
High Level Trees



High Level Trees



deleteTree(n)



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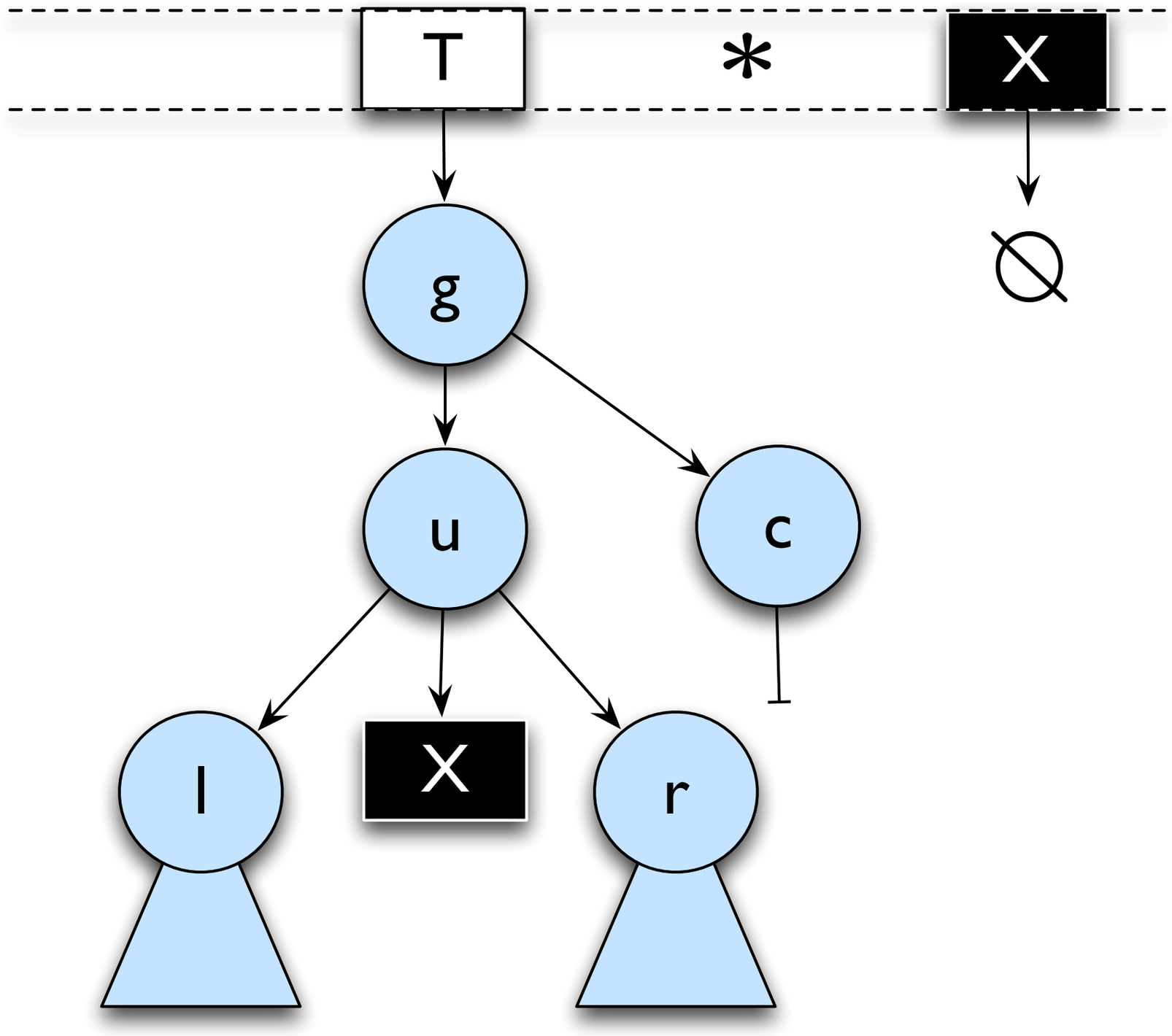
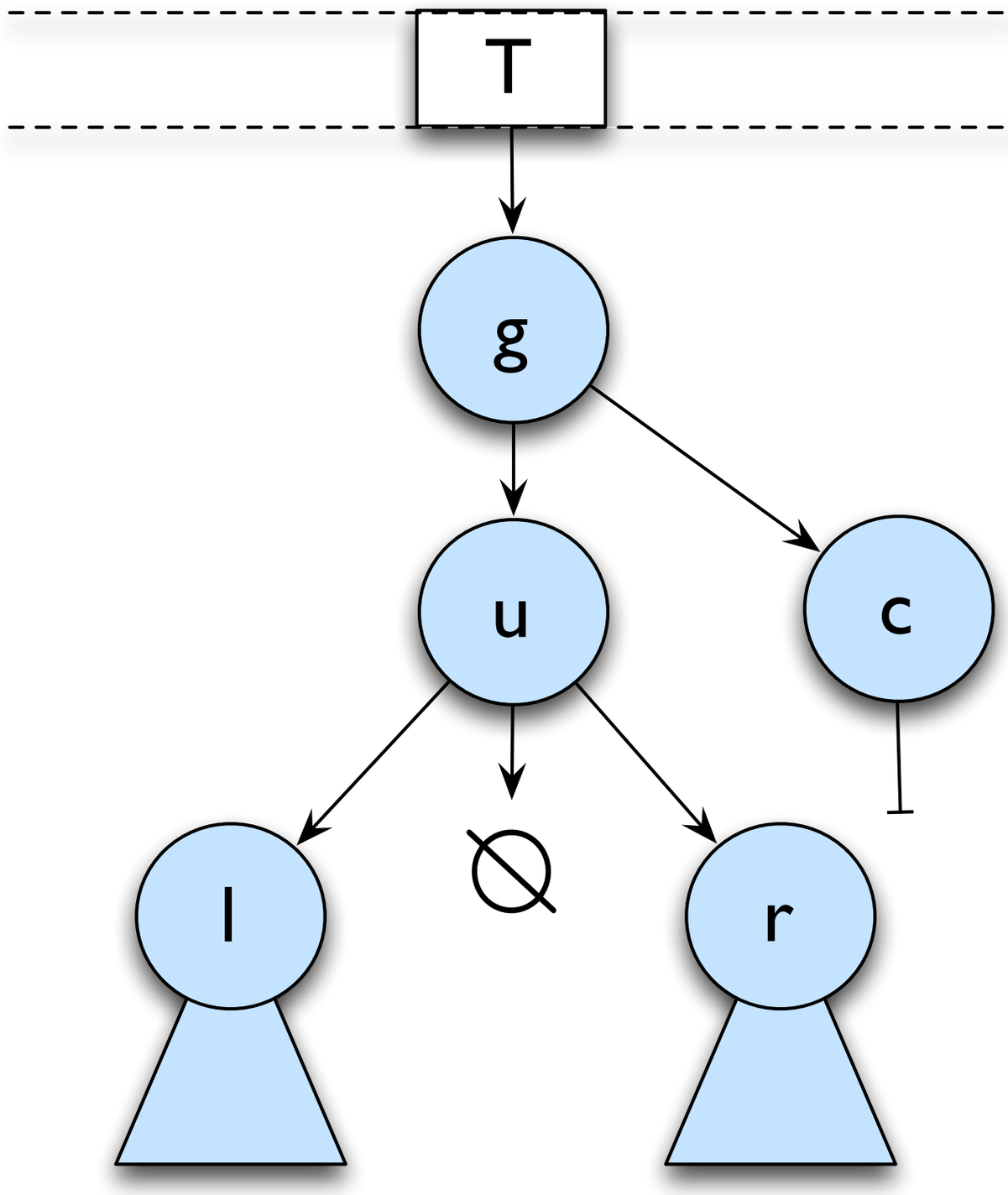


High Level Trees



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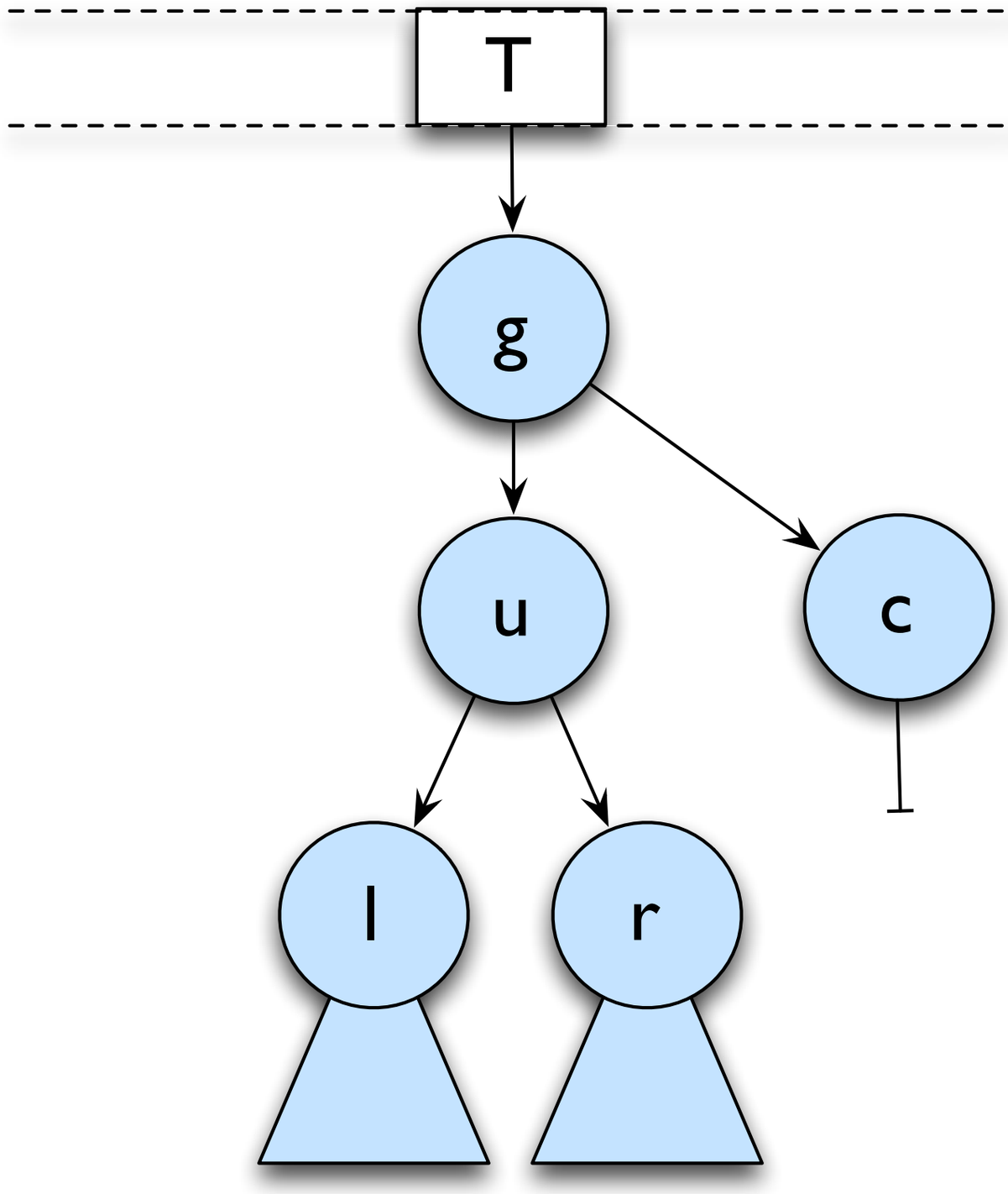
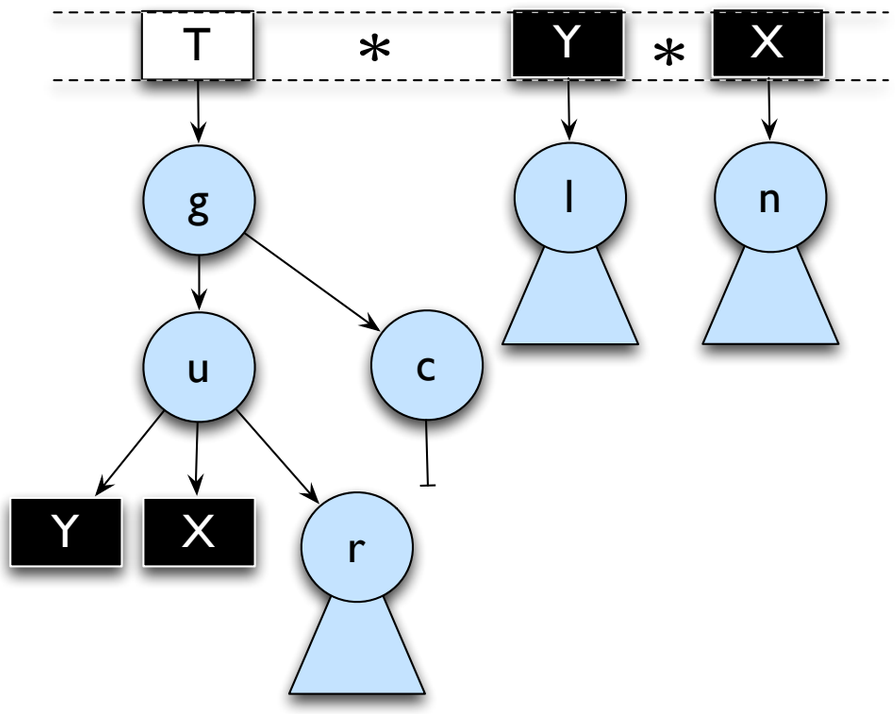


High Level Trees

Separation gives us disjoint concurrency!

$$\frac{\{\,P_1\,\}\ C_1\ \{\,Q_1\,\} \quad \{\,P_2\,\}\ C_2\ \{\,Q_2\,\}}{\{\,P_1 * P_2\,\}\ C_1 || C_2\ \{\,Q_1 * Q_2\,\}}$$

High Level Trees



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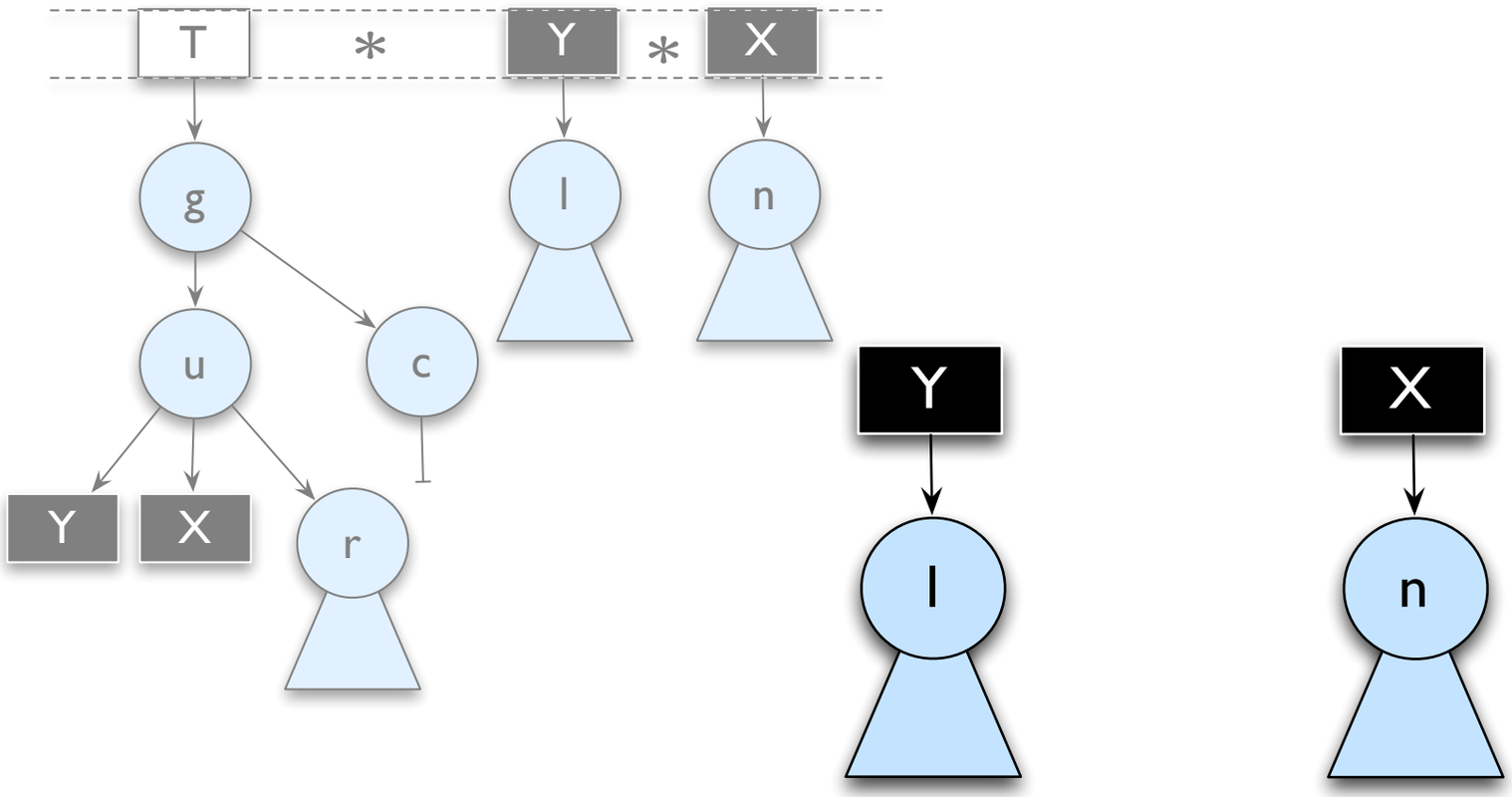

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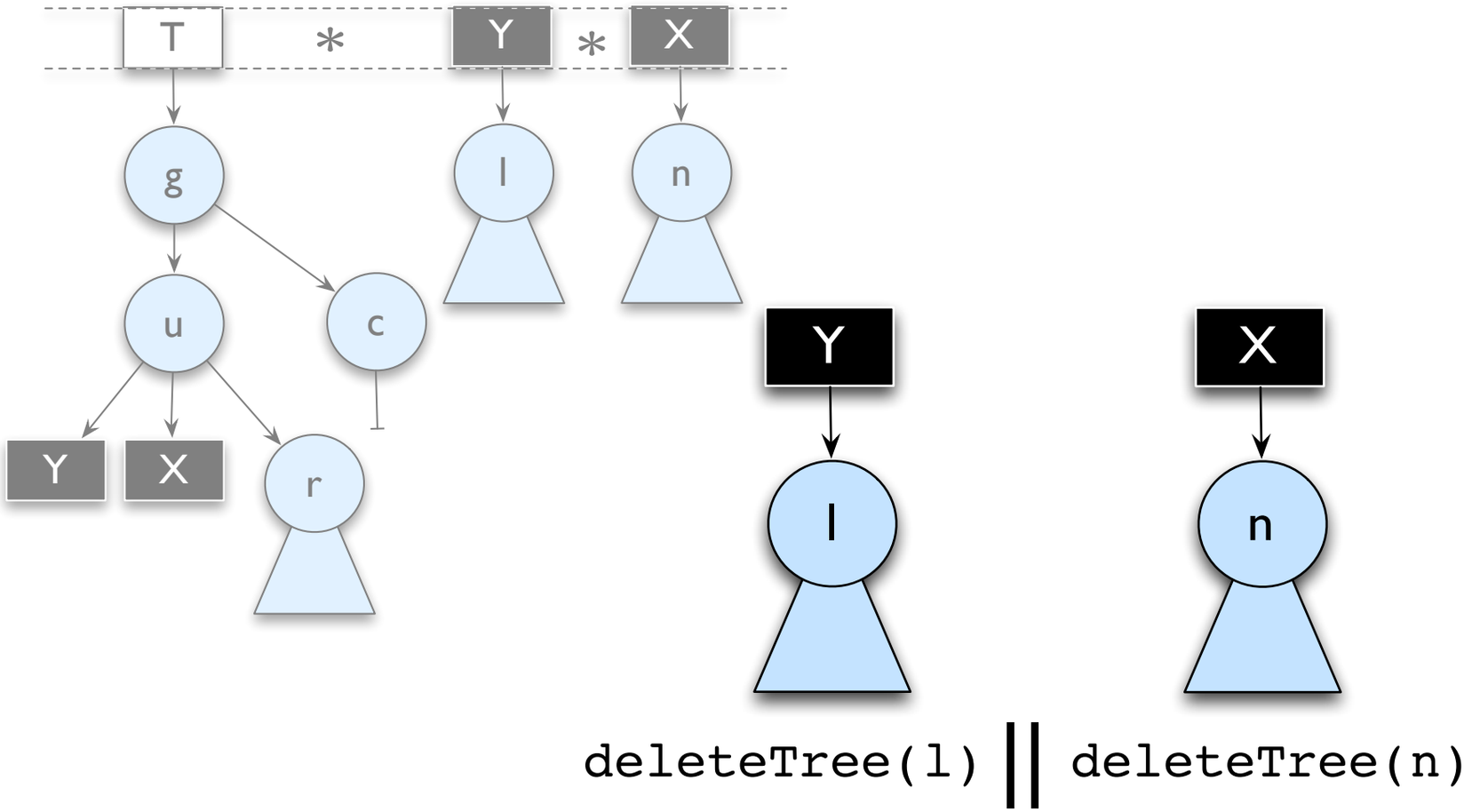
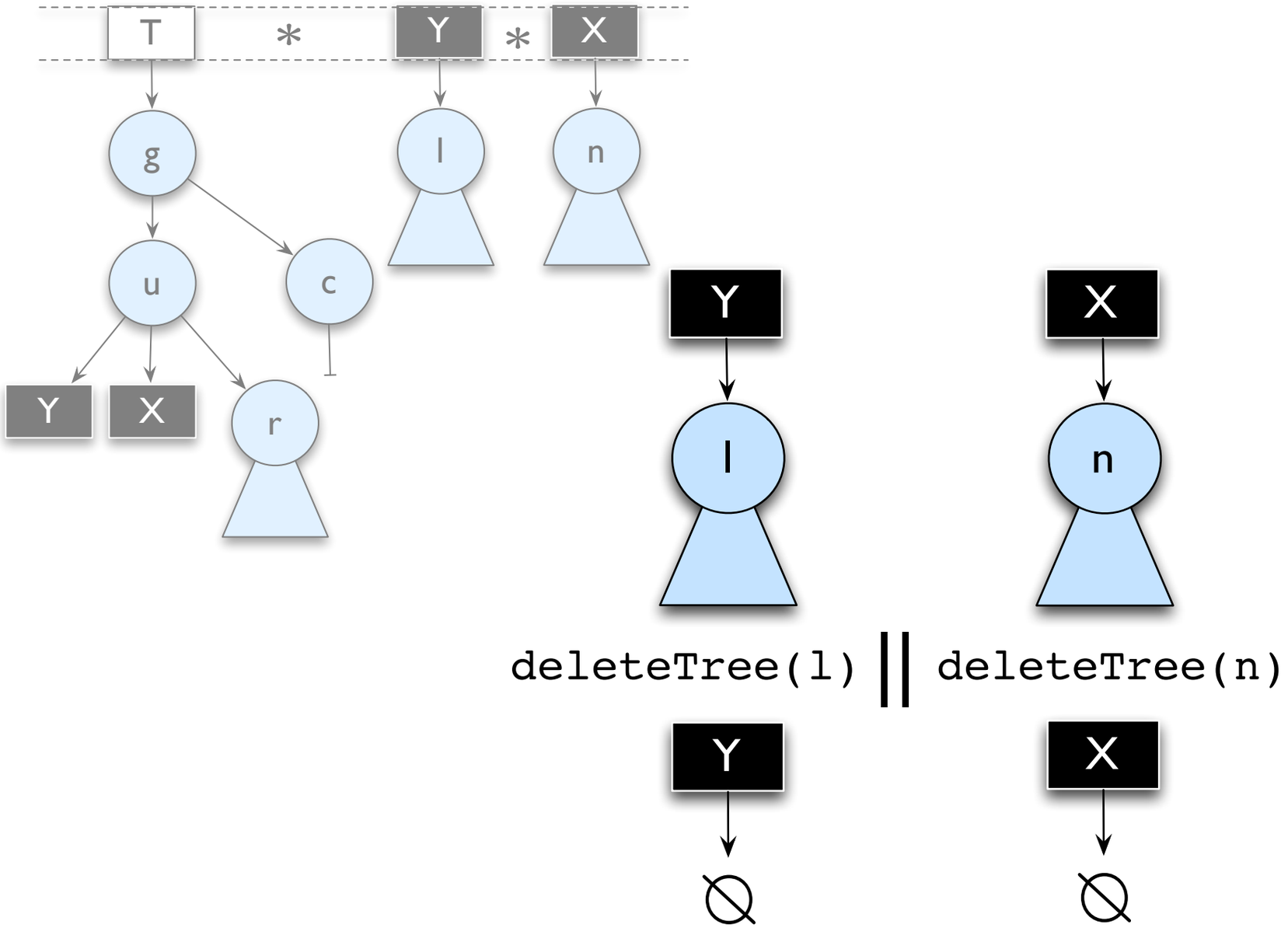



```
deleteTree(l) || deleteTree(n)
```

High Level Trees



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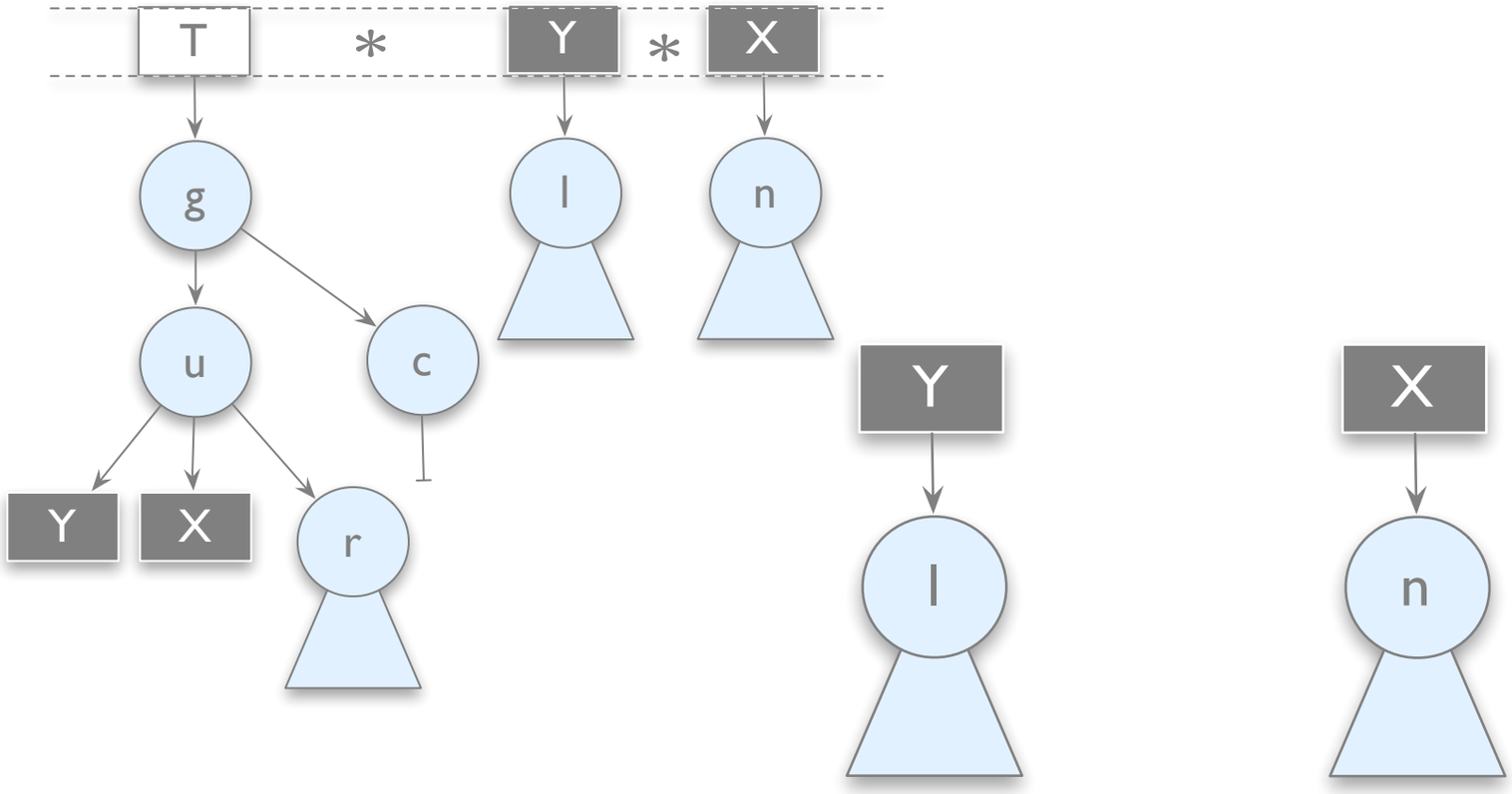
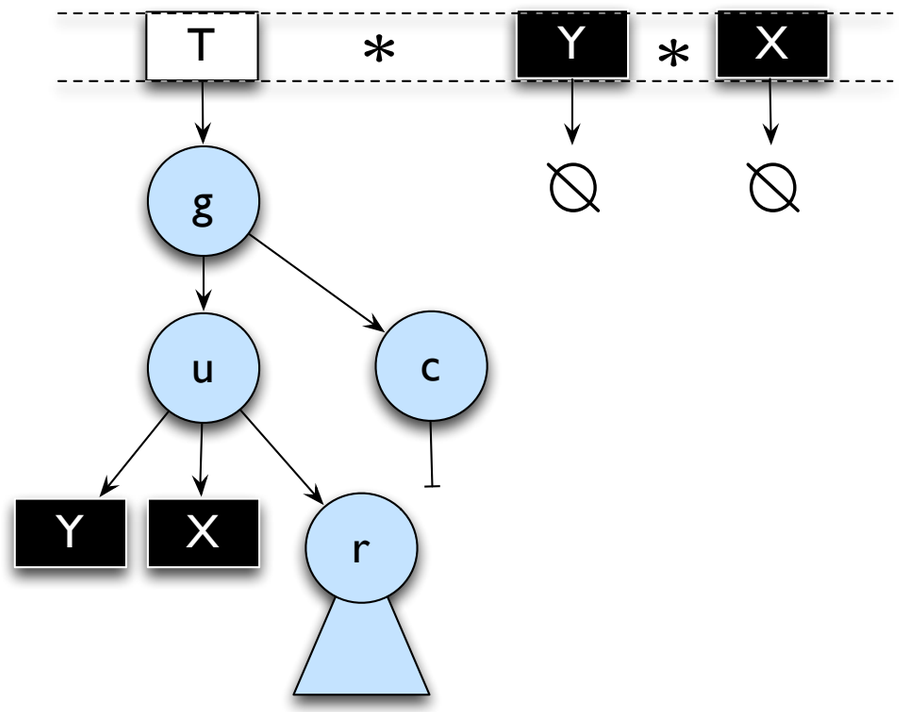
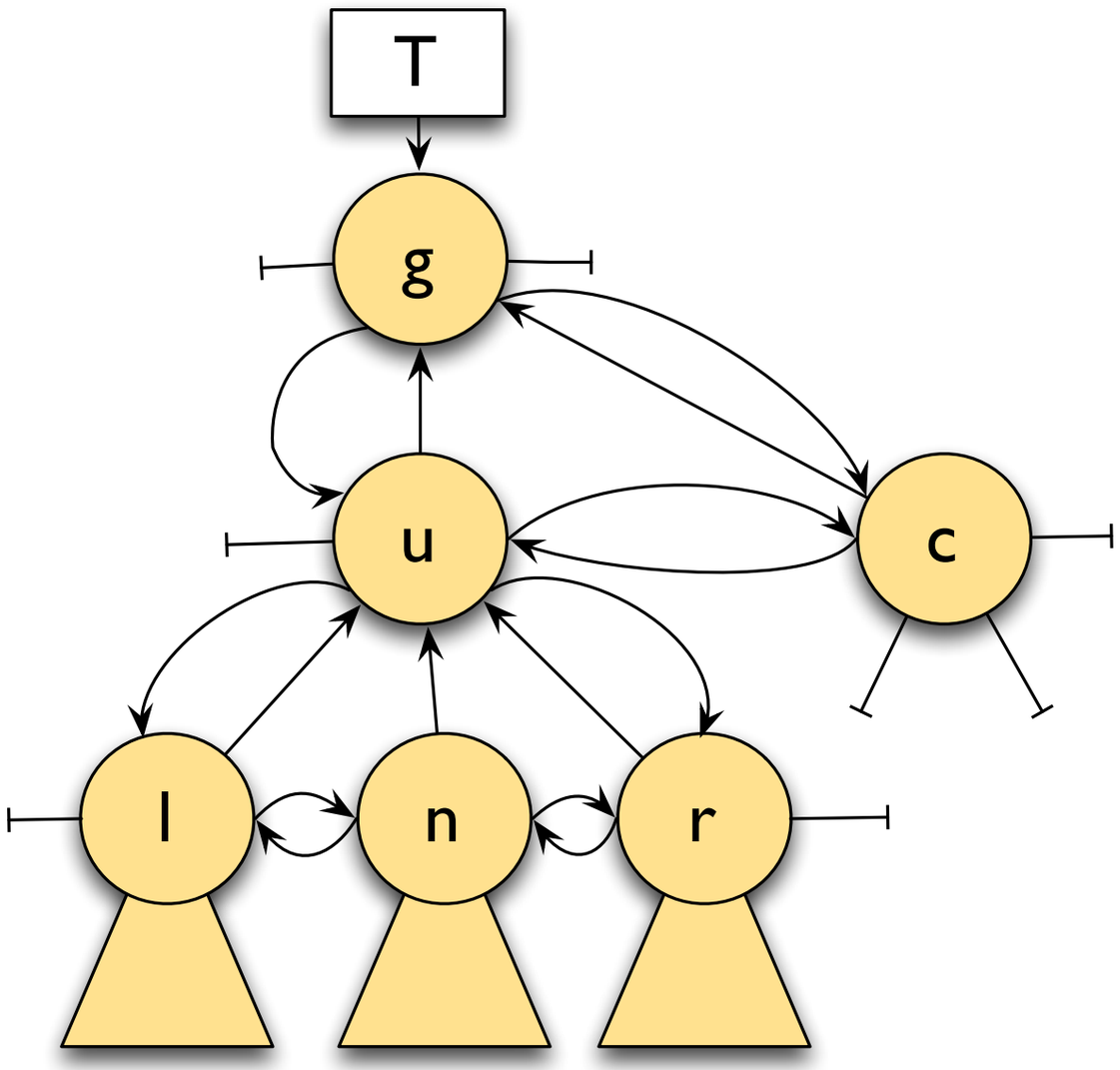


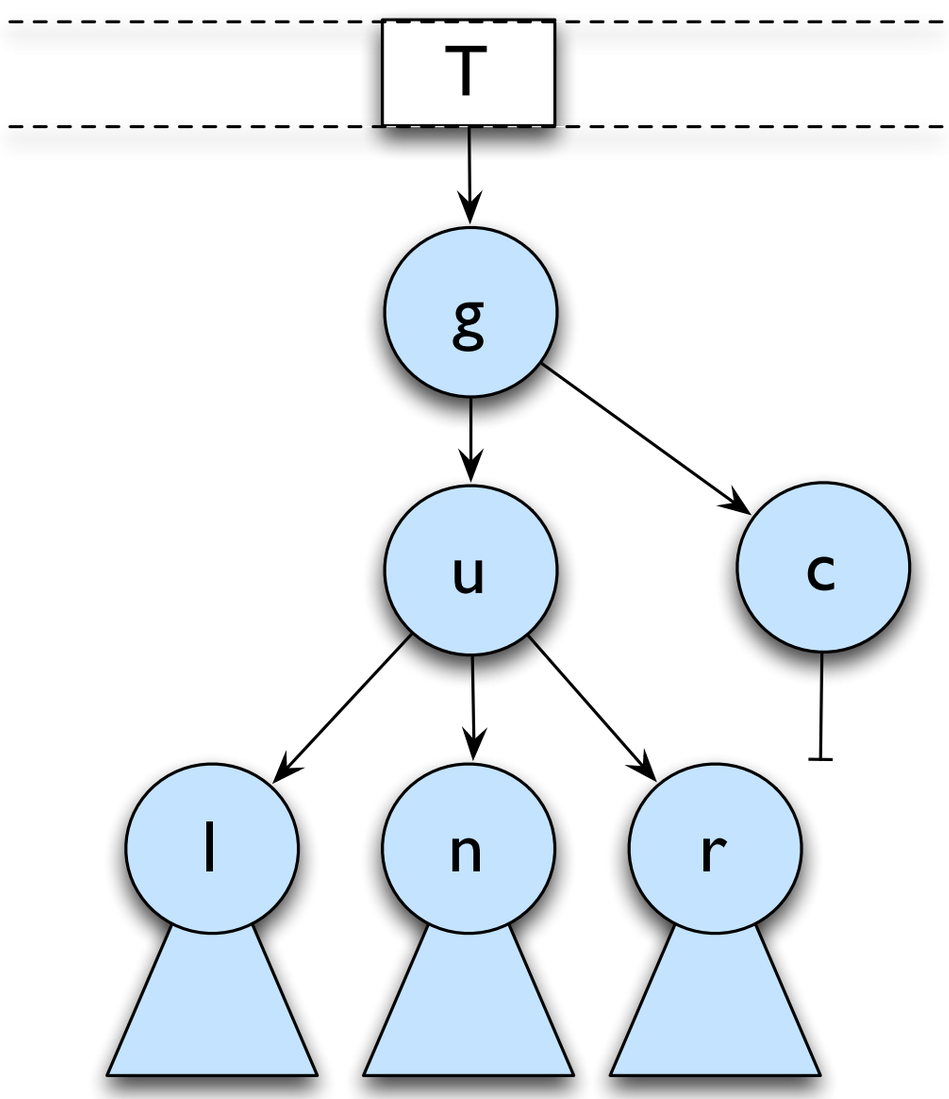
`deleteTree(l)` || `deleteTree(n)`



Low Level Trees

Concrete Tree Representation





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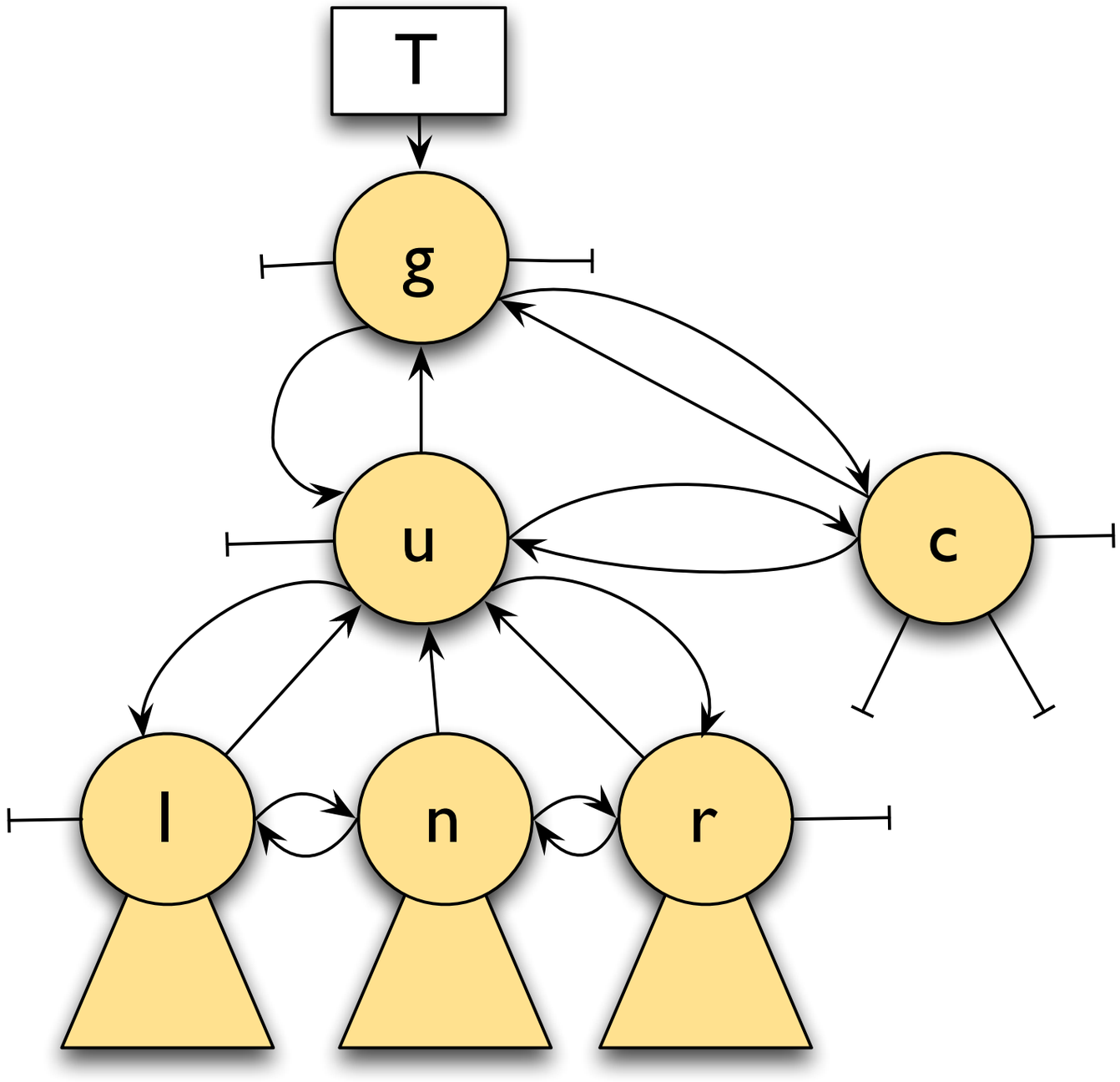

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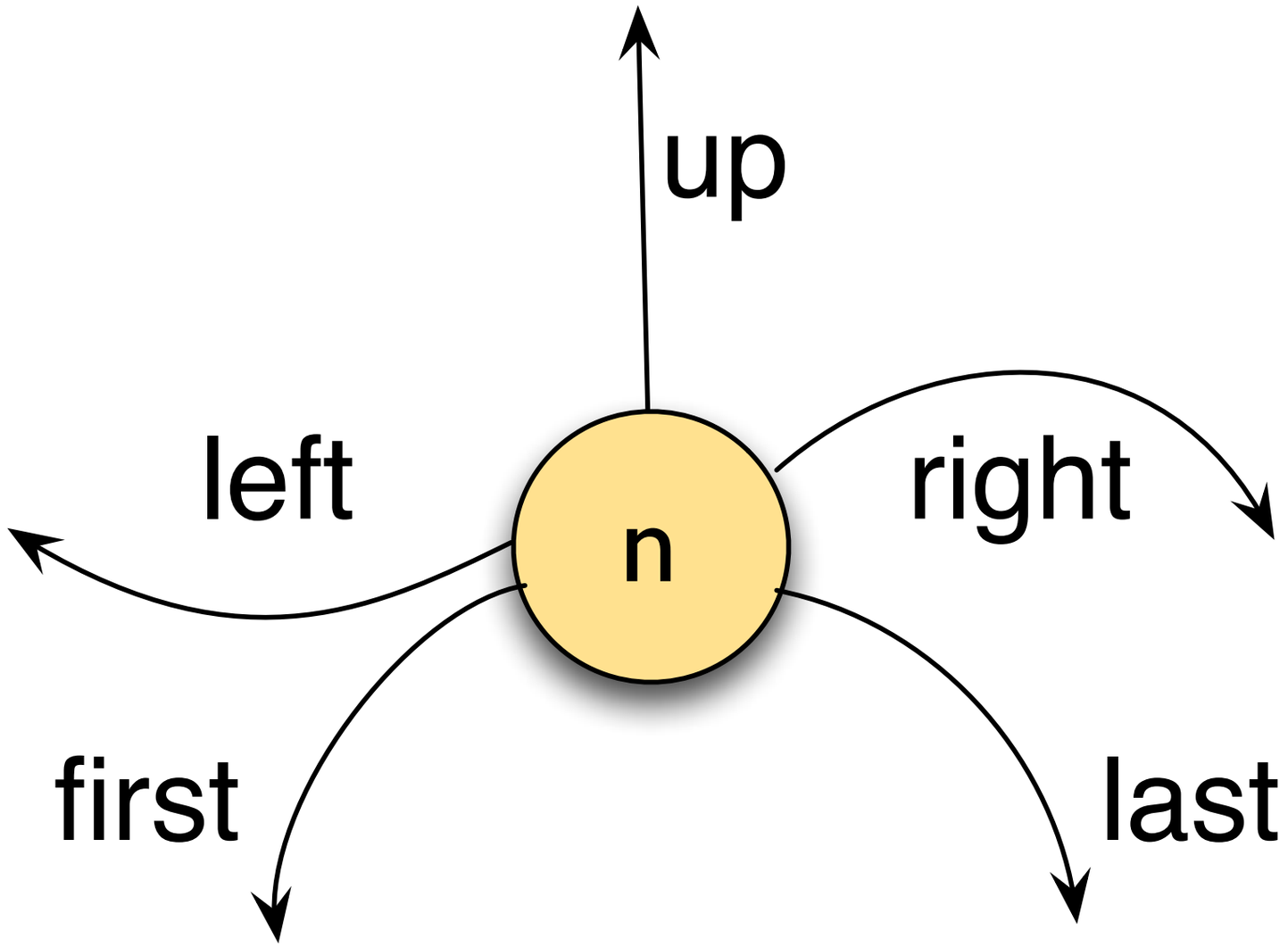
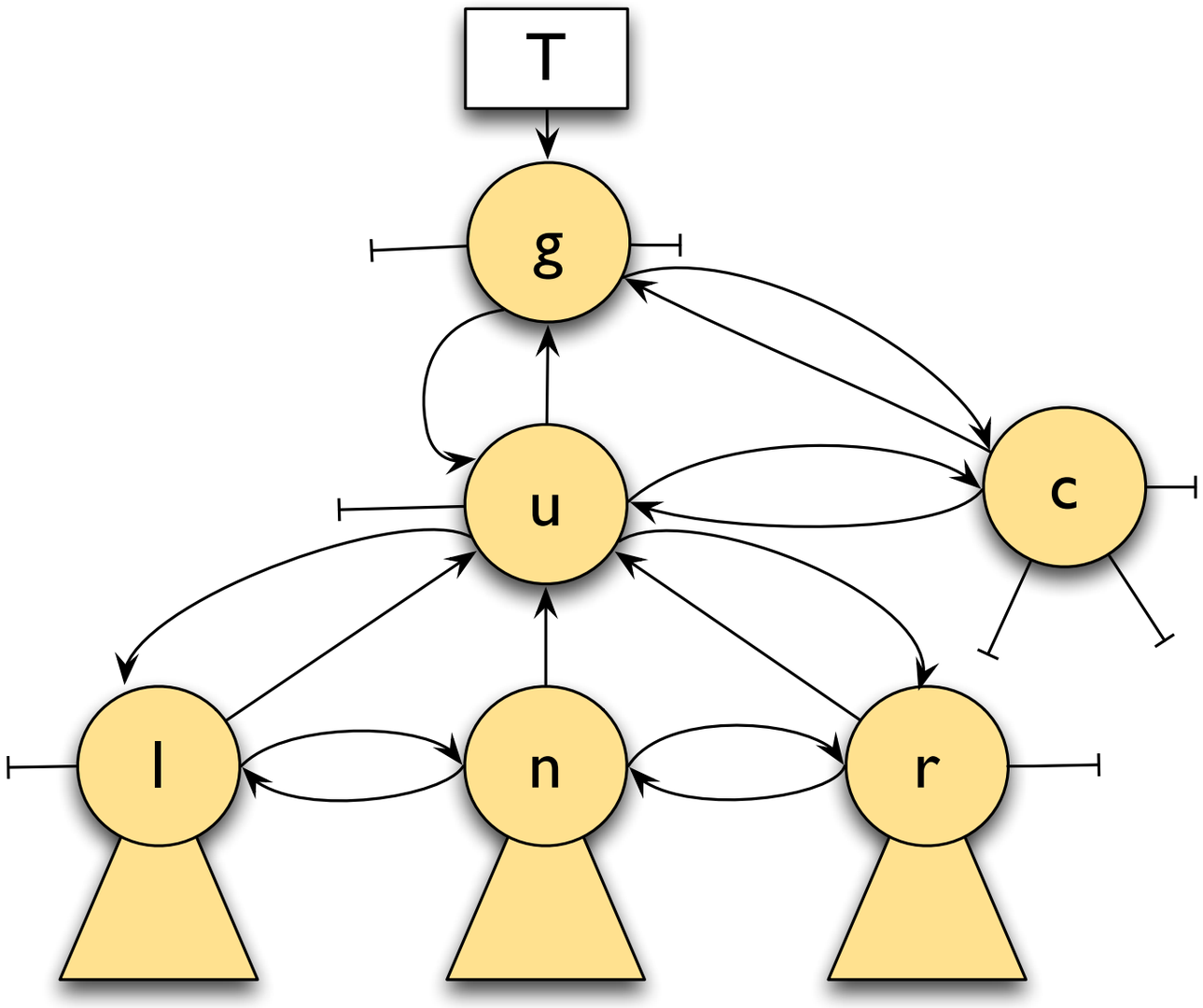


Low Level Trees

`deleteTree` Procedure



Low Level Trees

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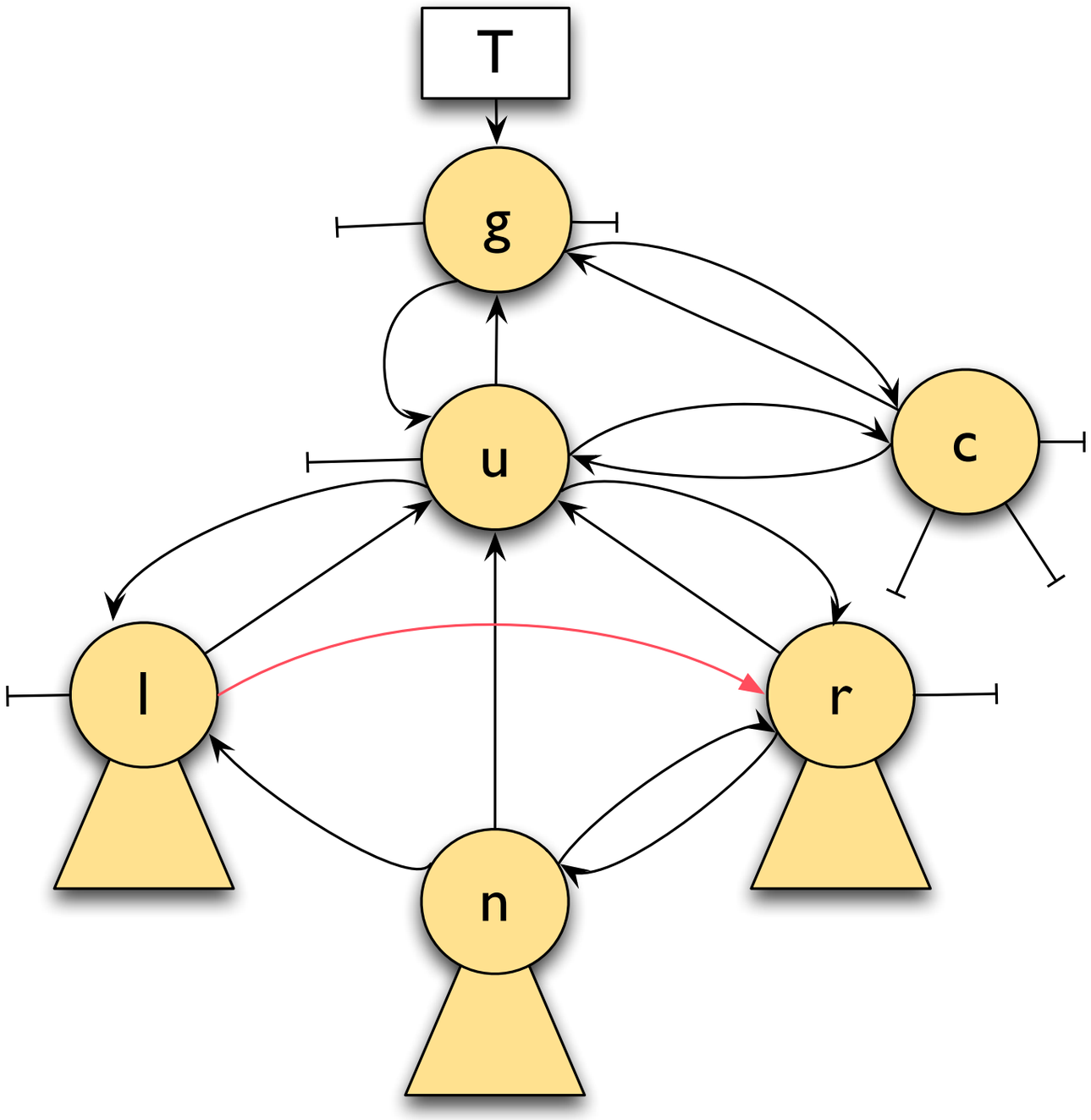

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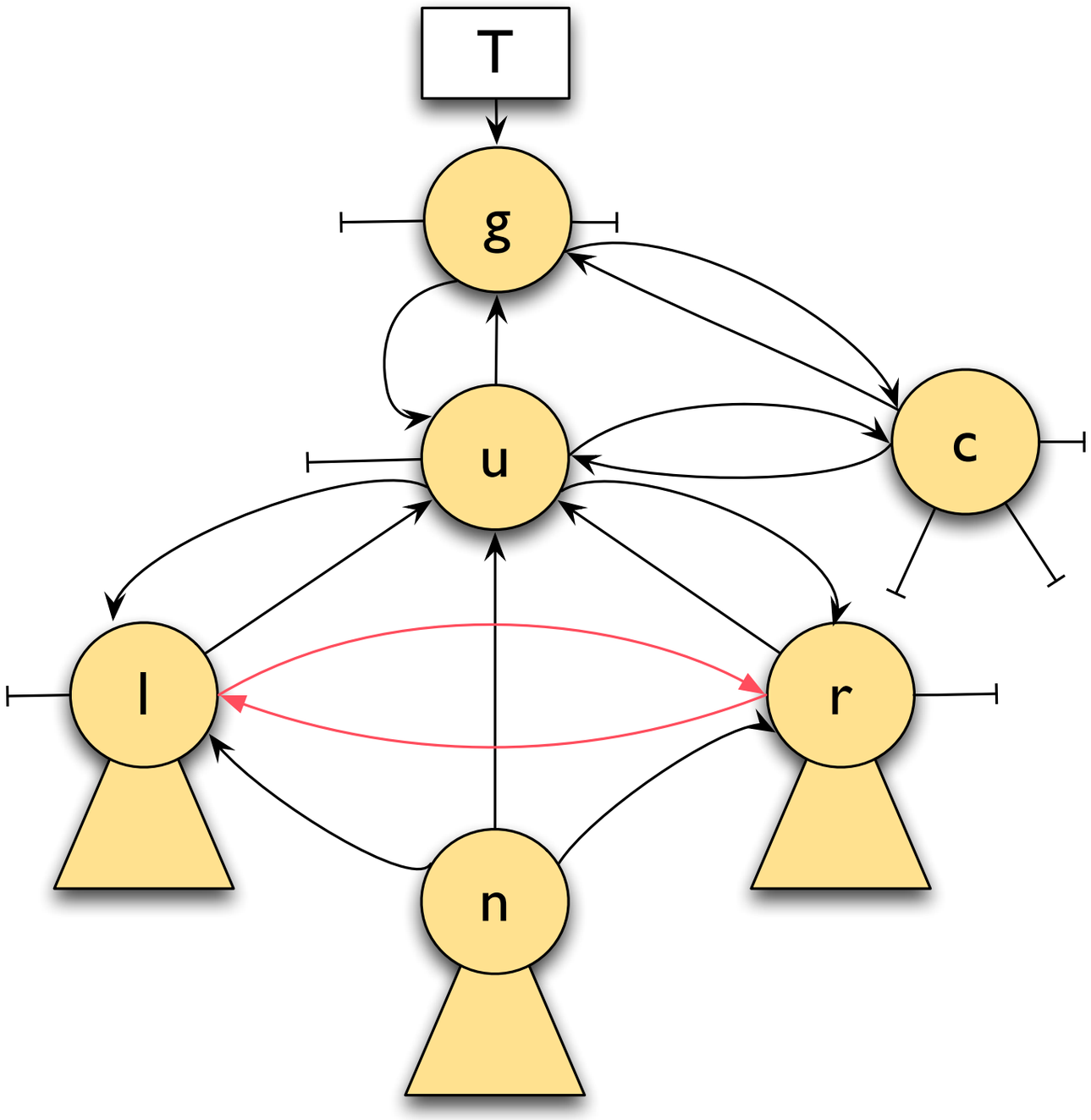

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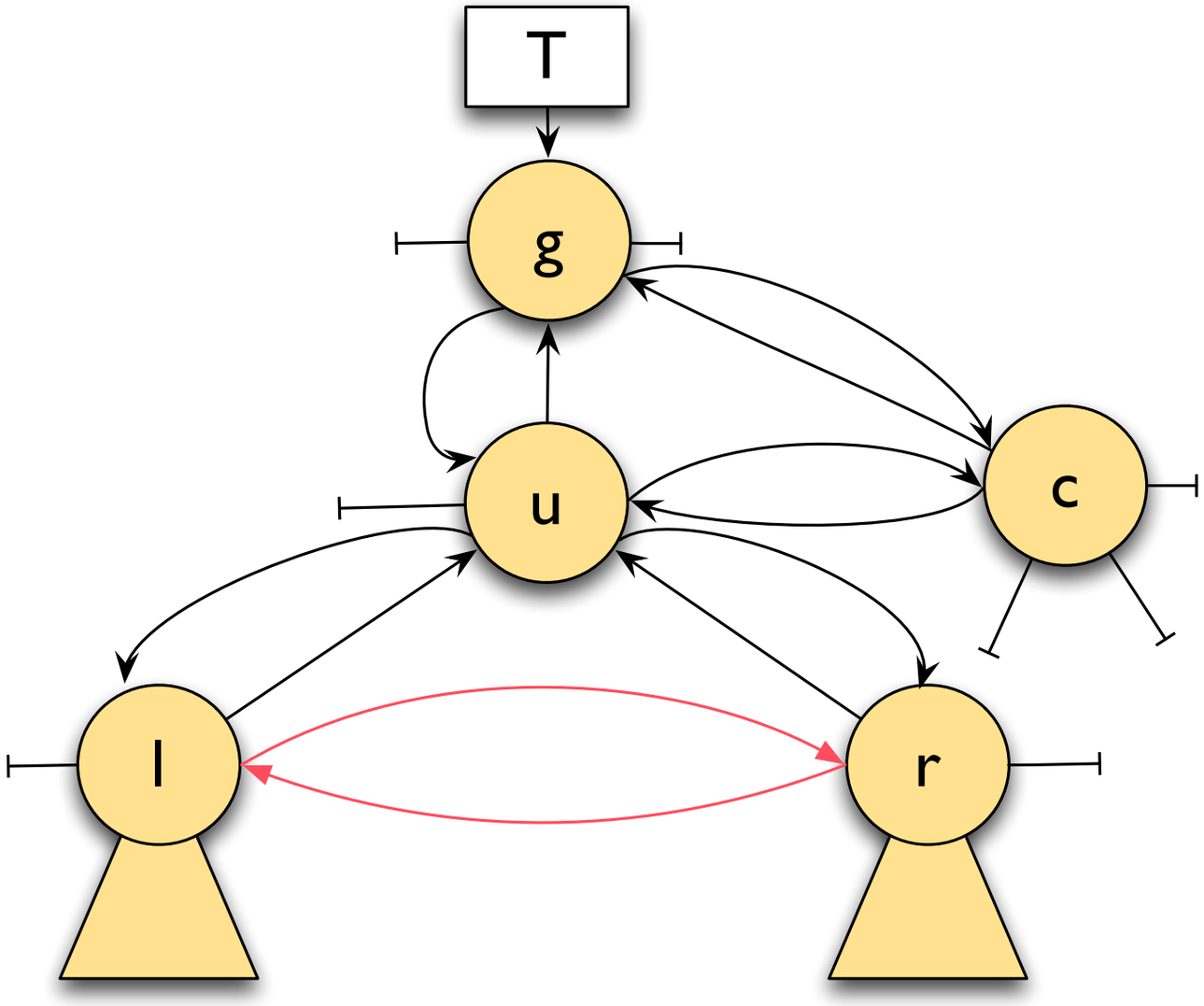
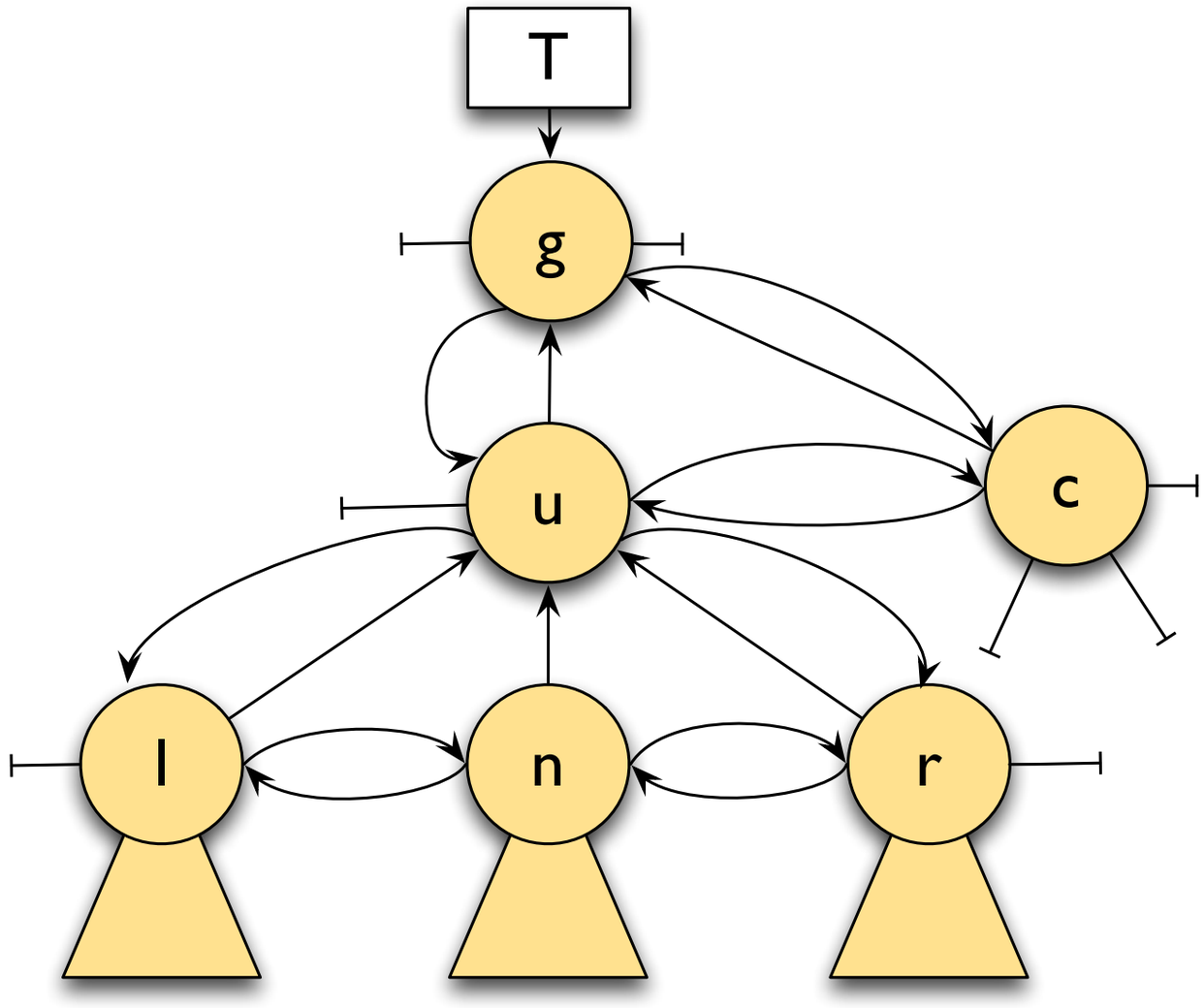


Low Level Trees

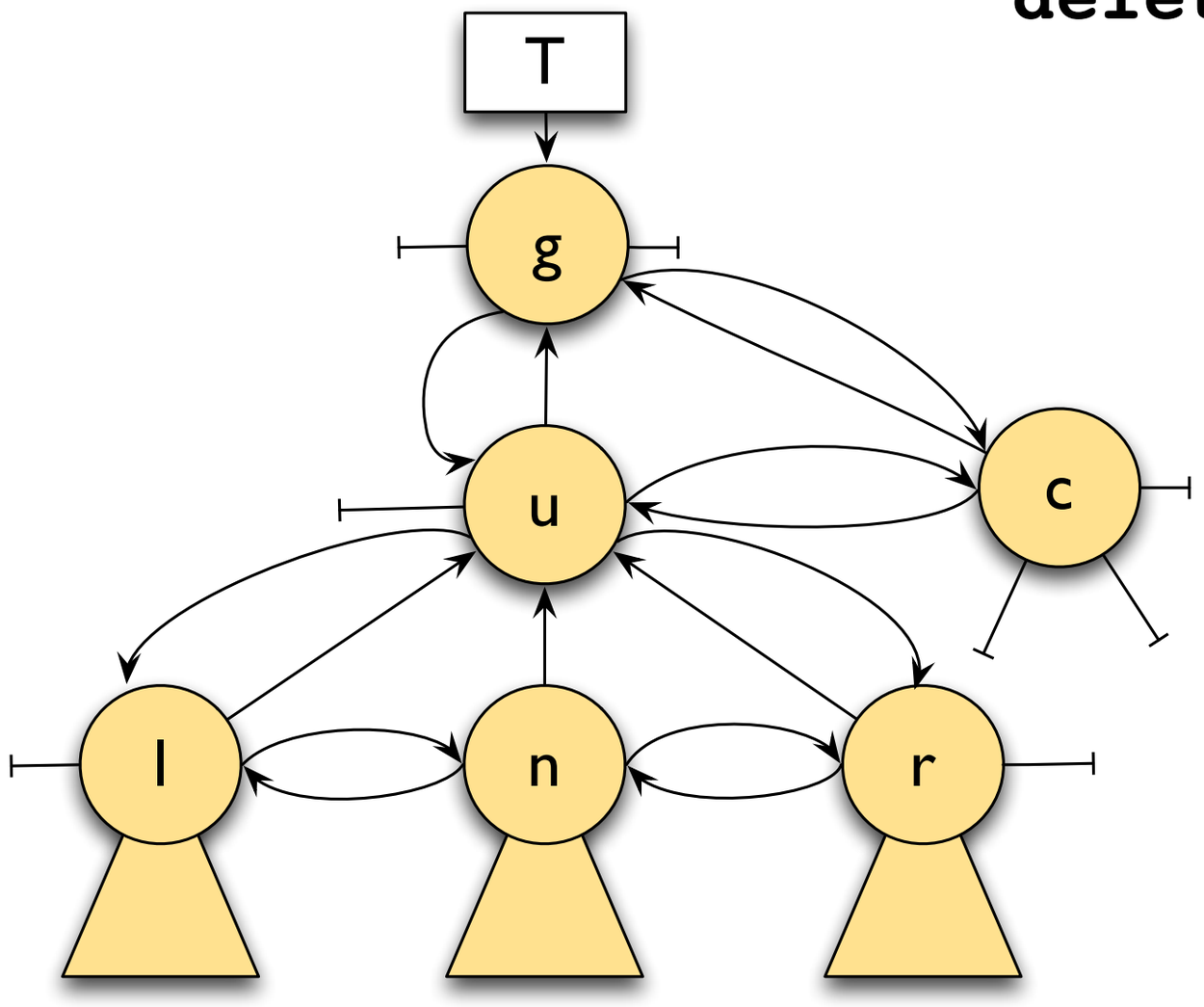
Concurrent `deleteTree` Procedure



Low Level Trees

Concurrent `deleteTree` Procedure

`deleteTree(n)`



Low Level Trees

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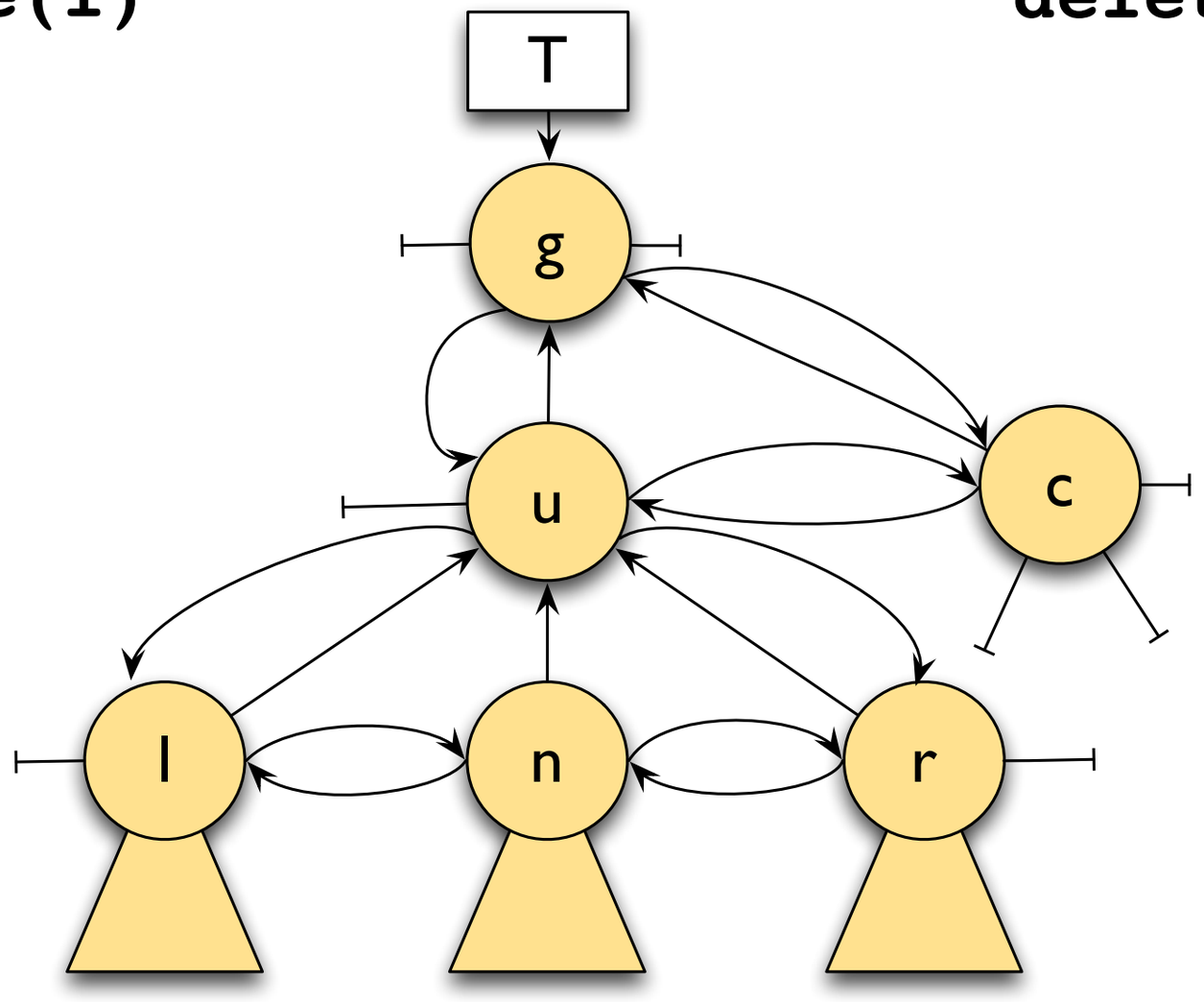

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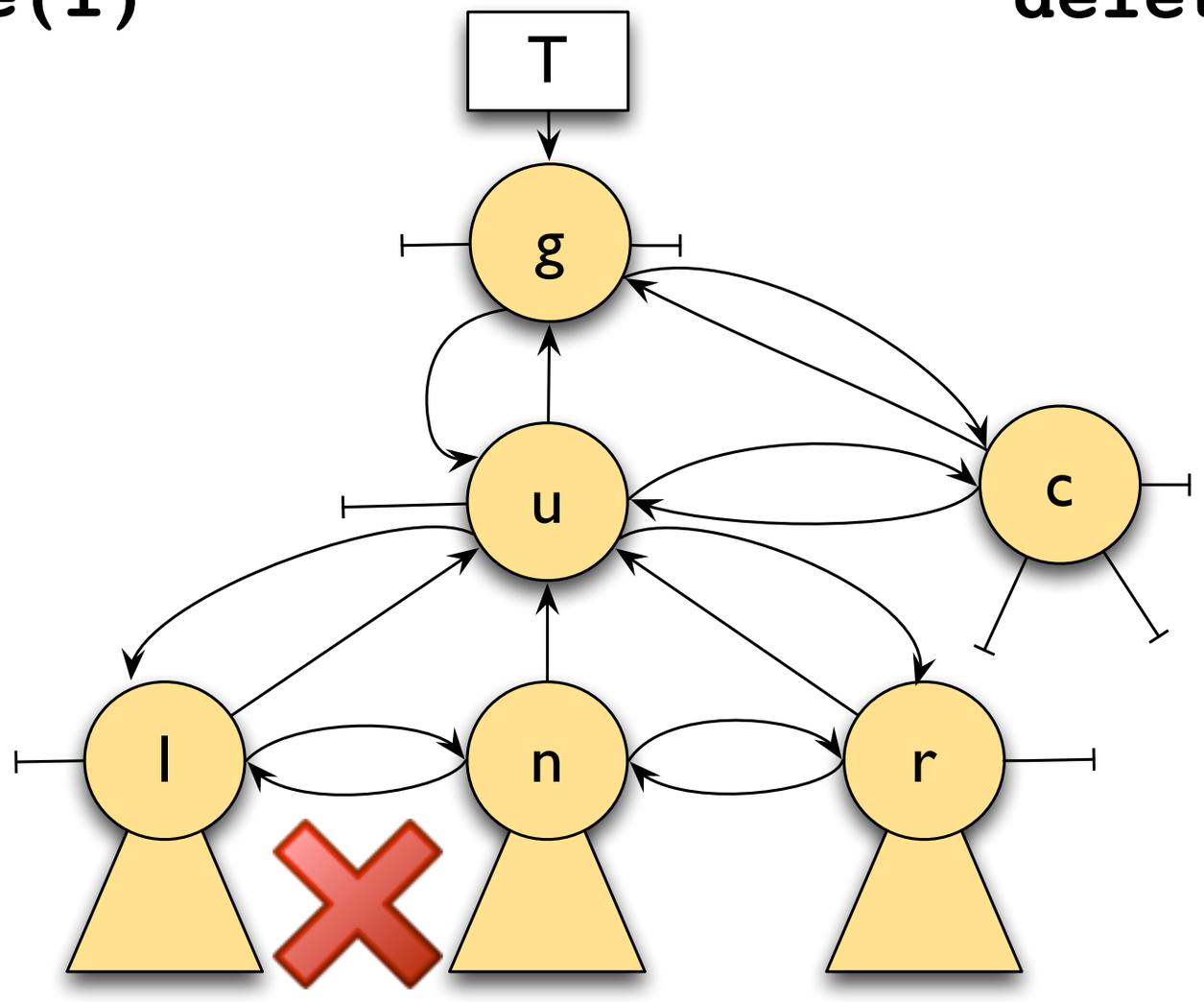

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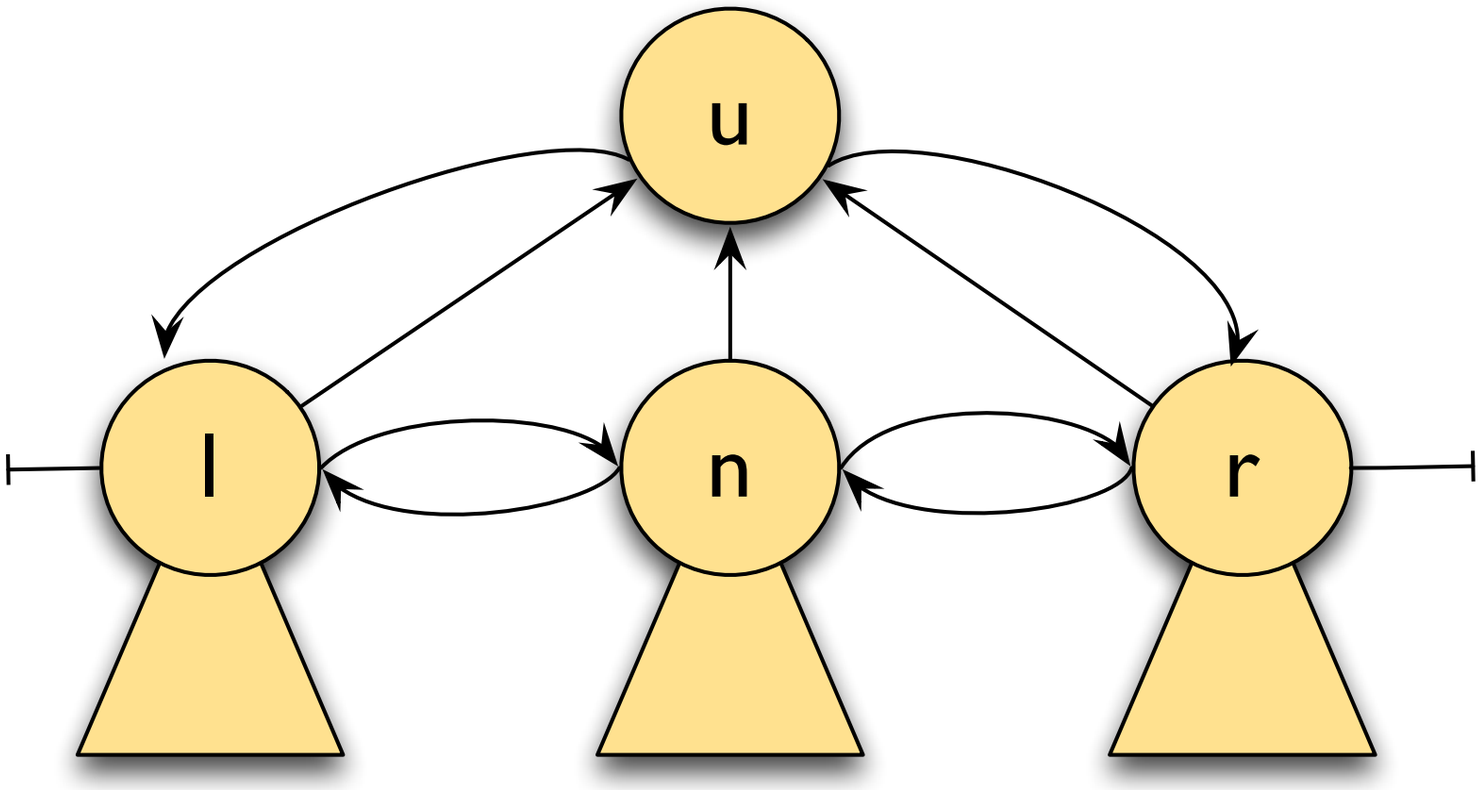

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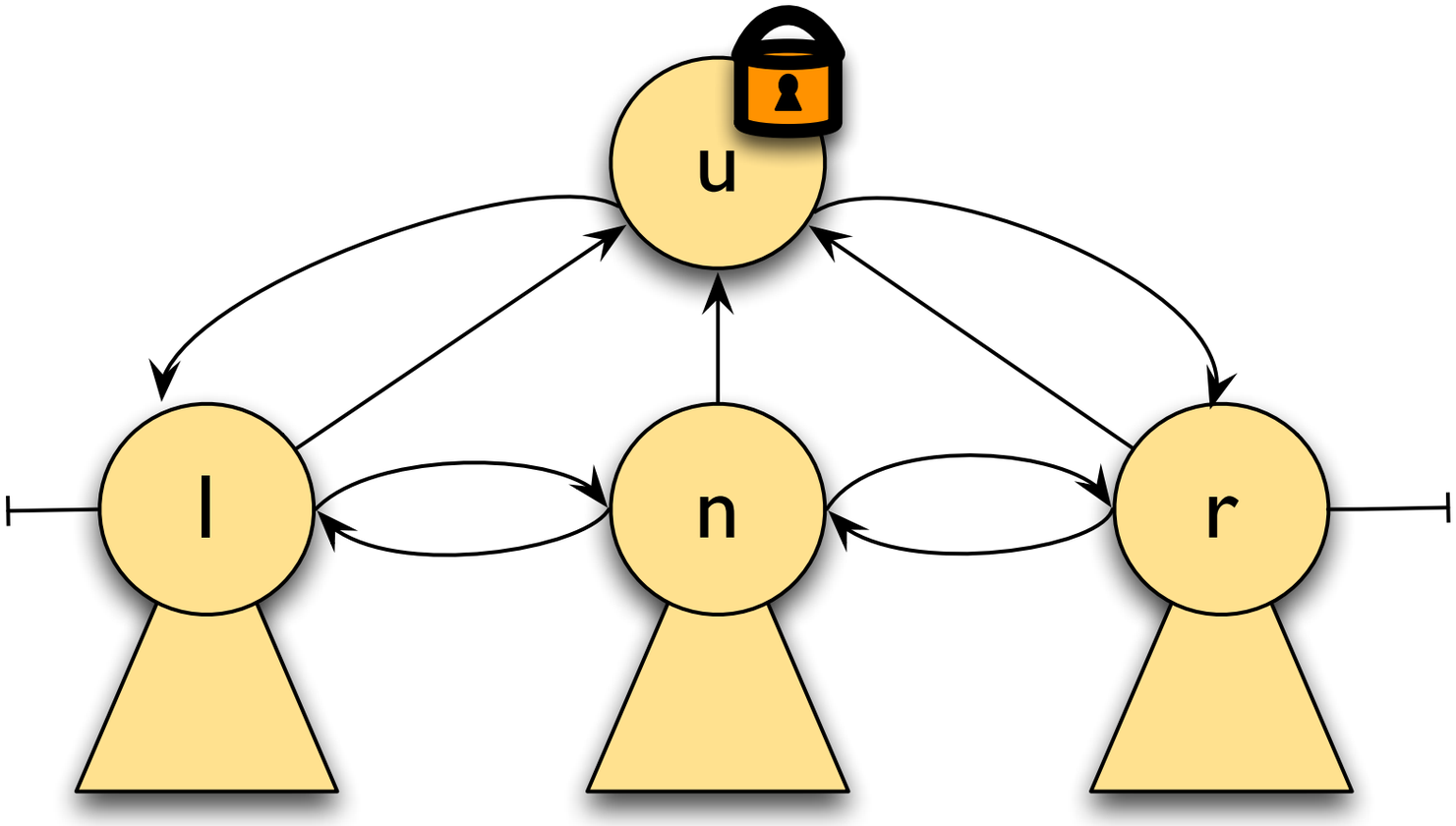

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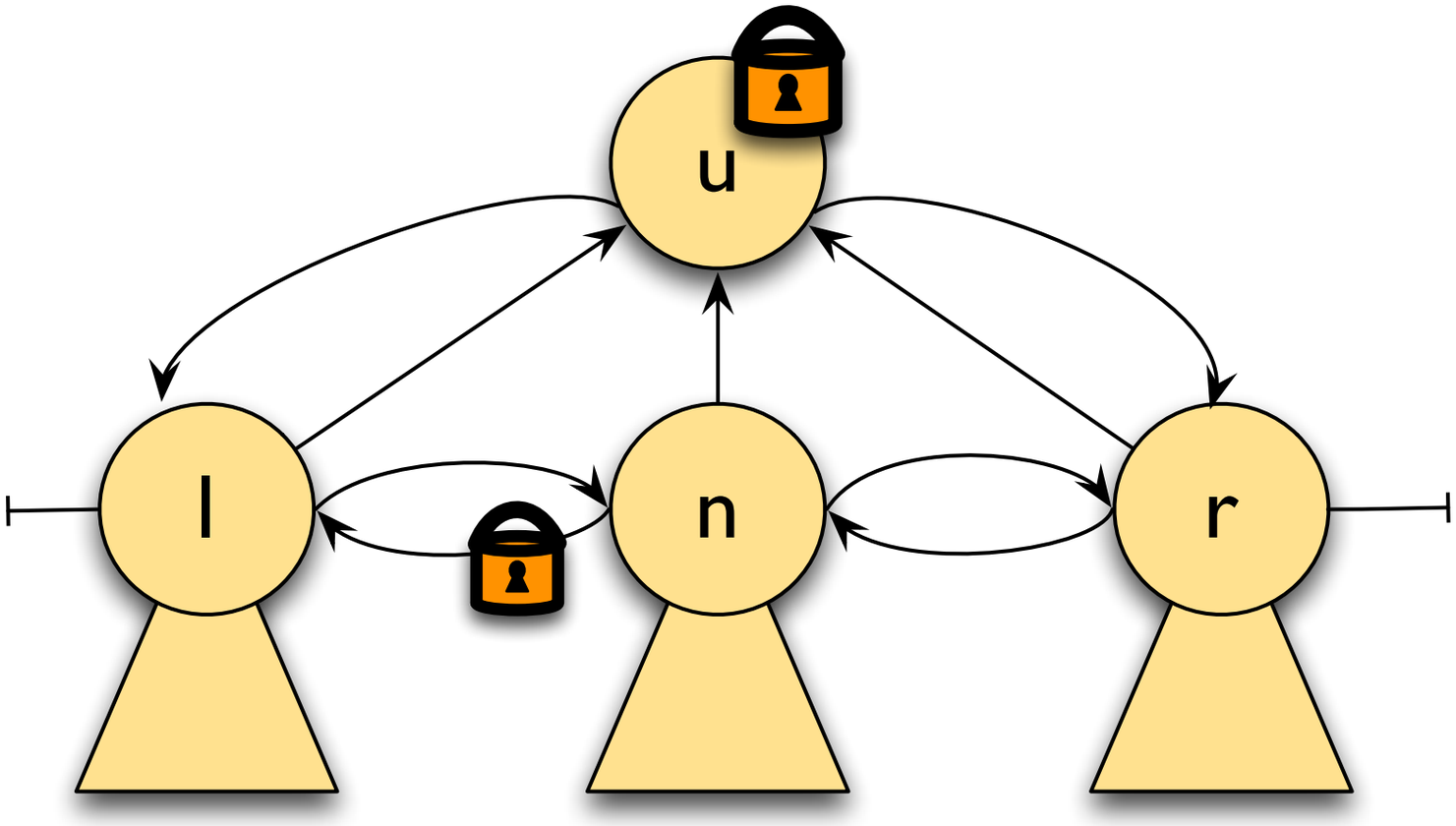

Low Level Trees

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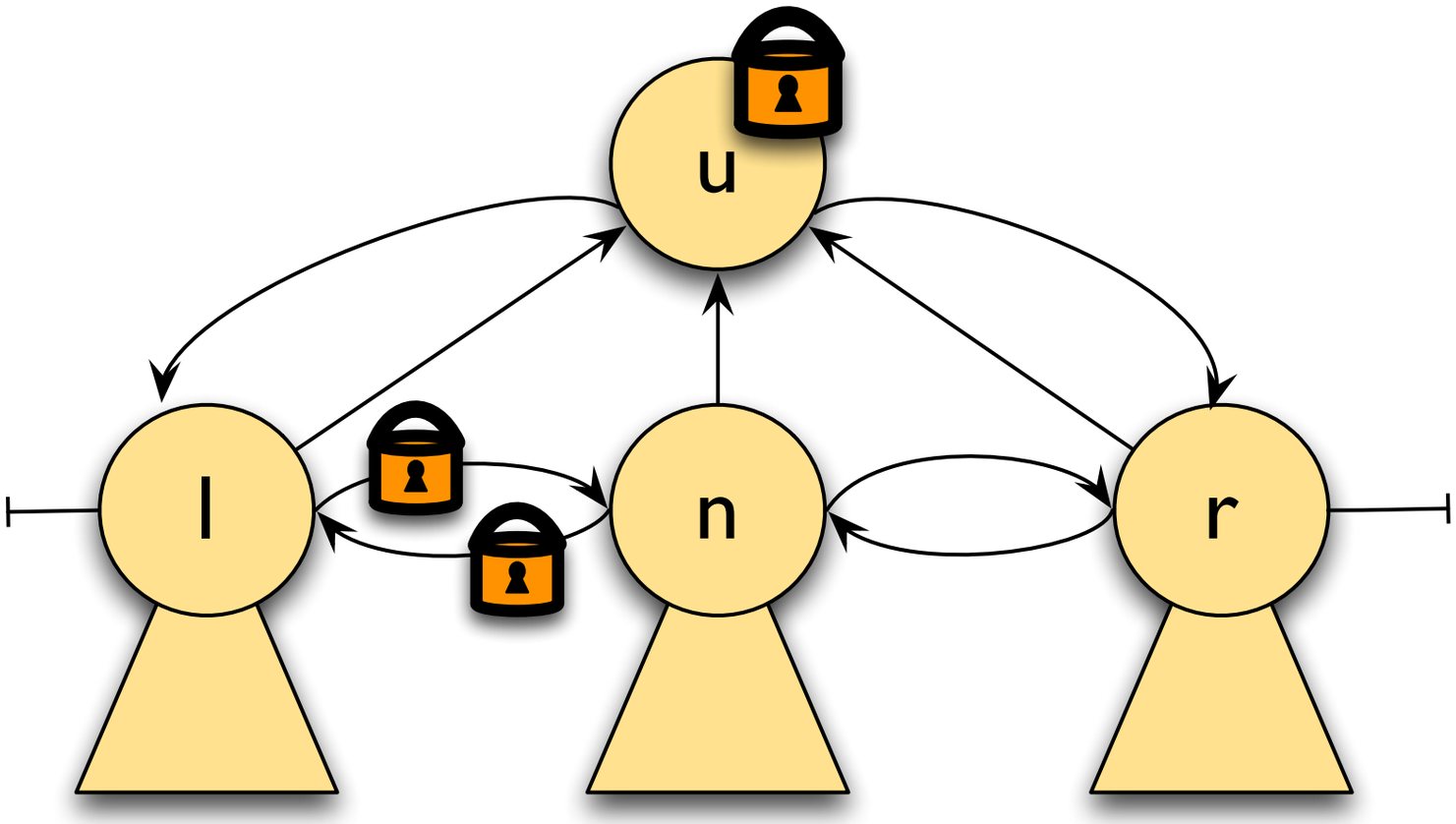

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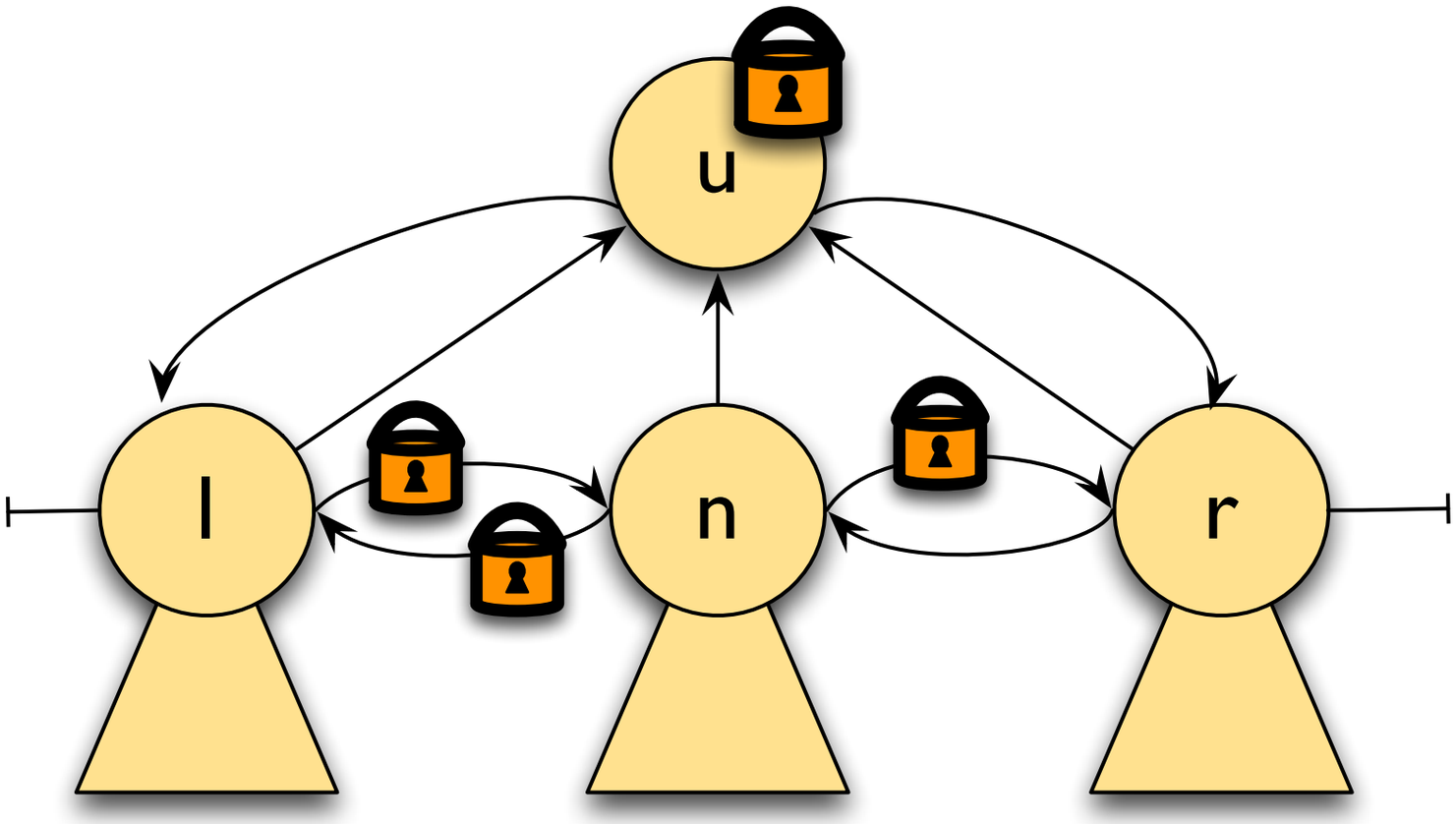
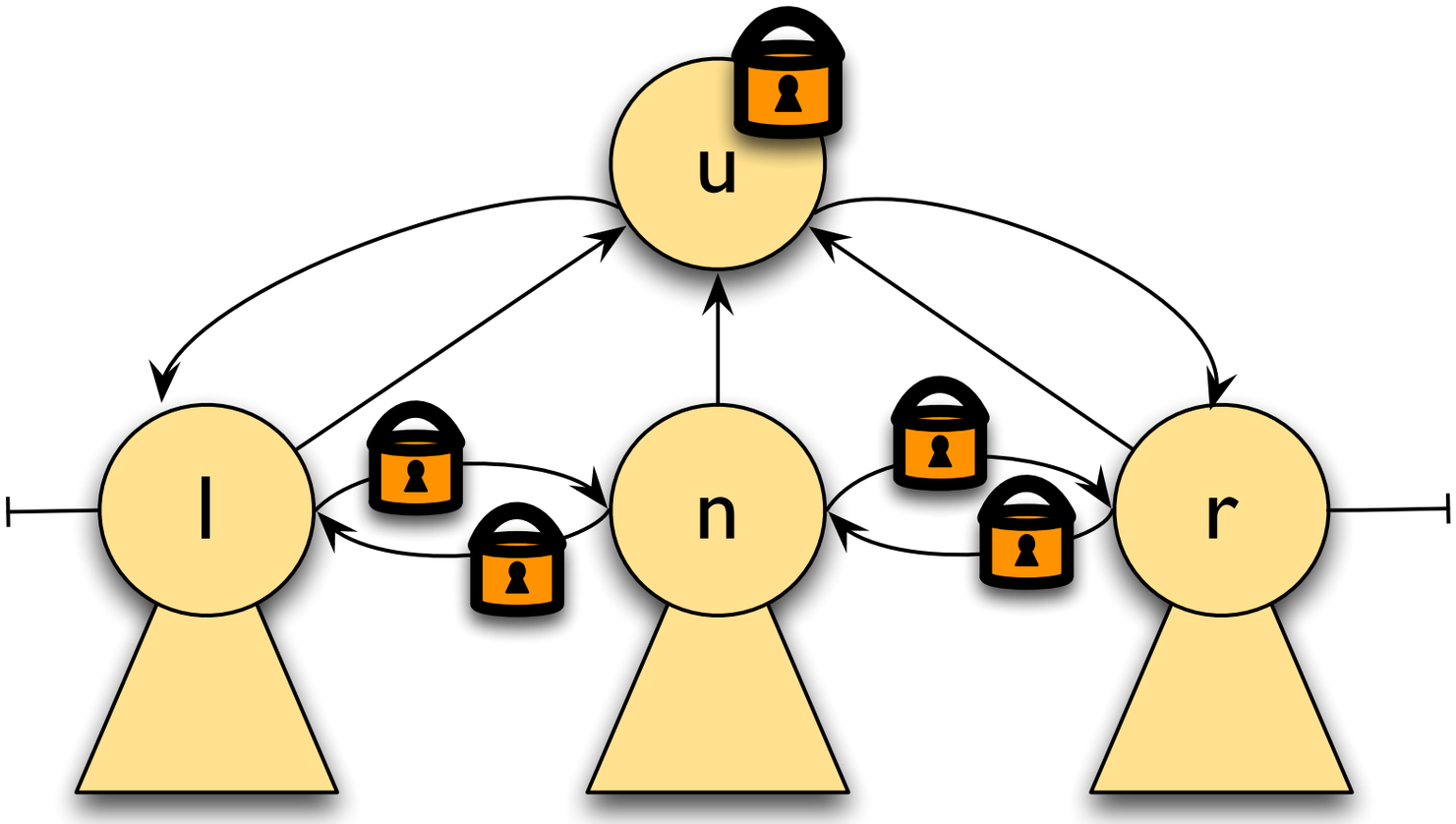


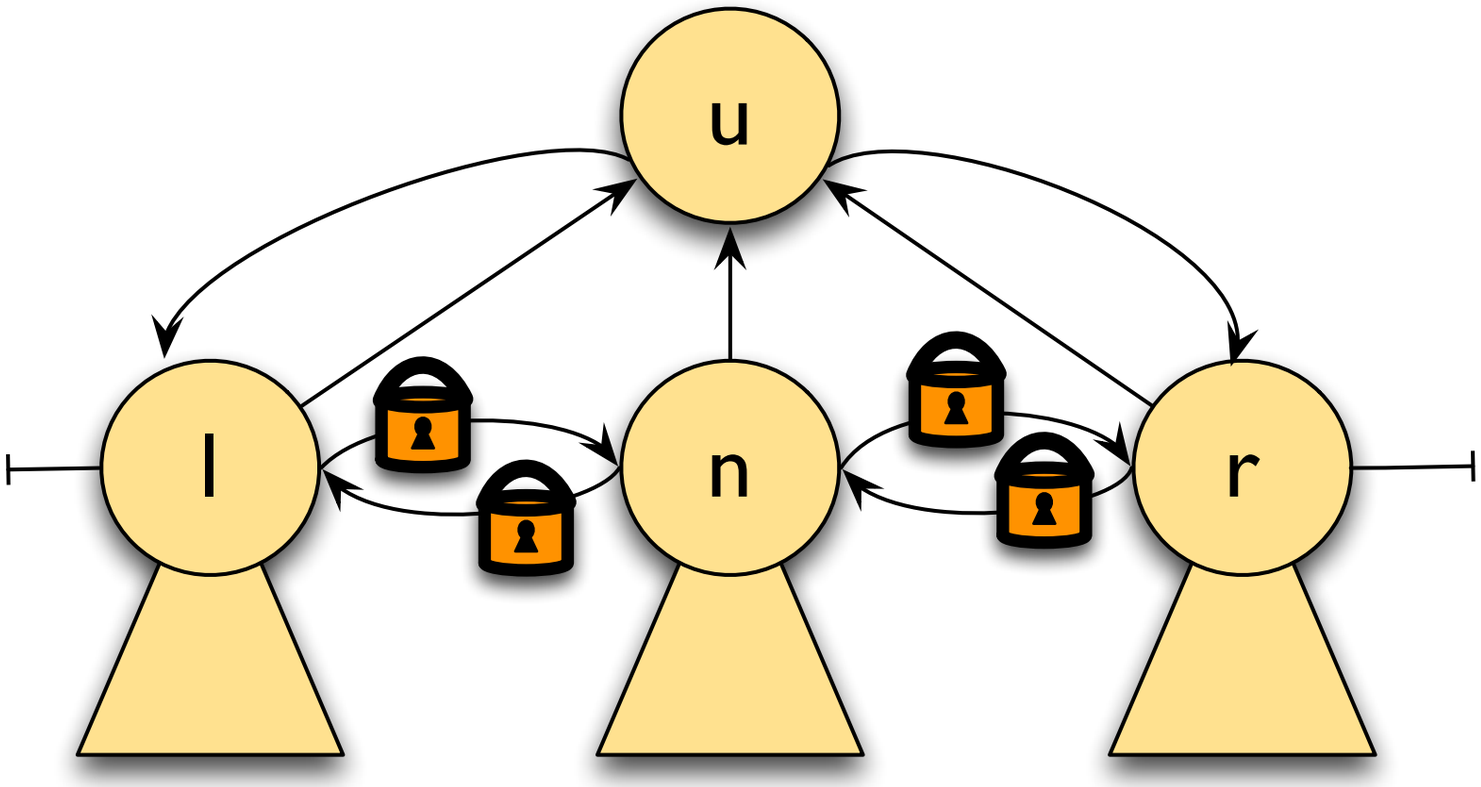
Low Level Trees

Concurrent `deleteTree` Procedure



Low Level Trees

Concurrent `deleteTree` Procedure



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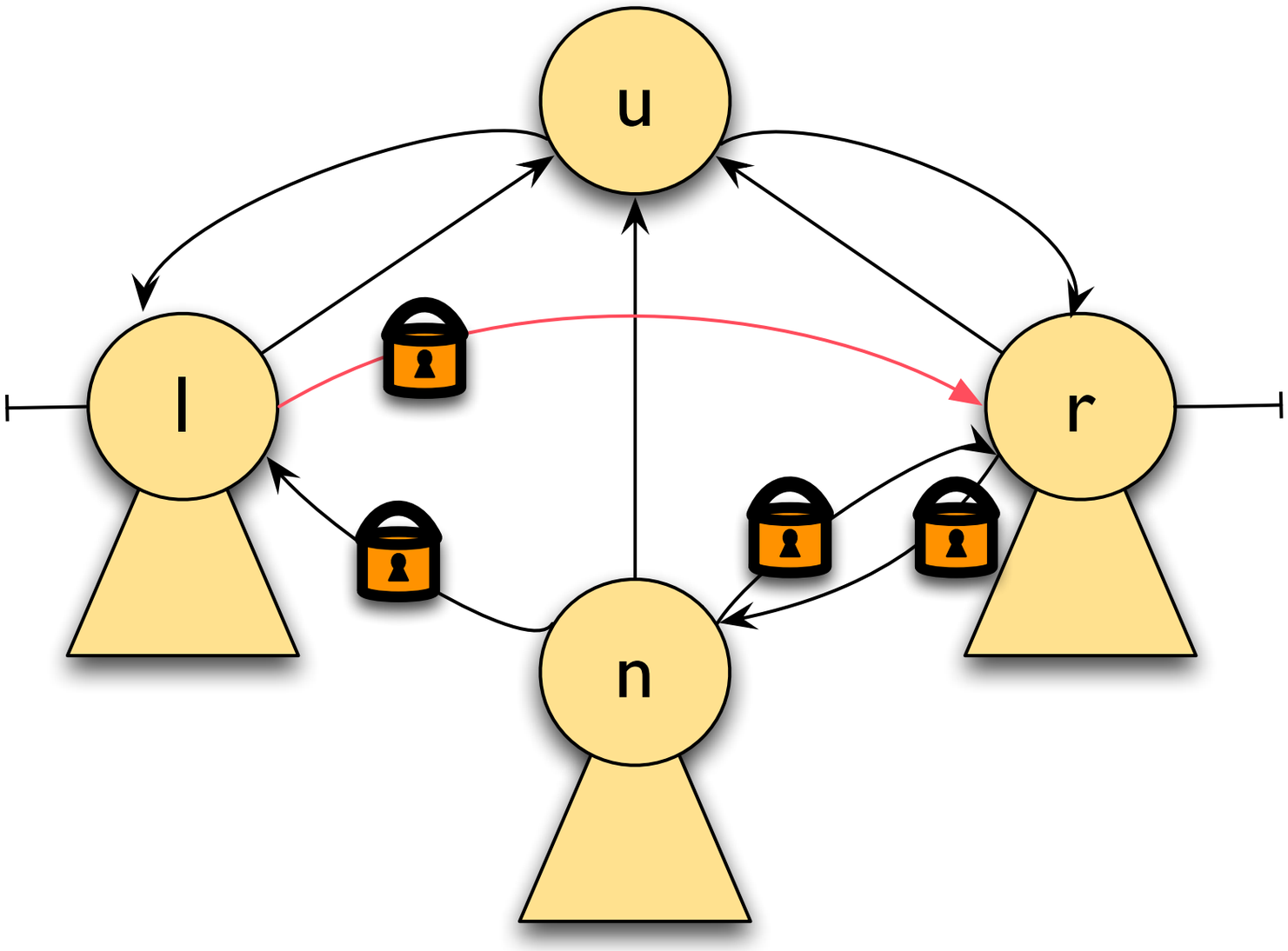

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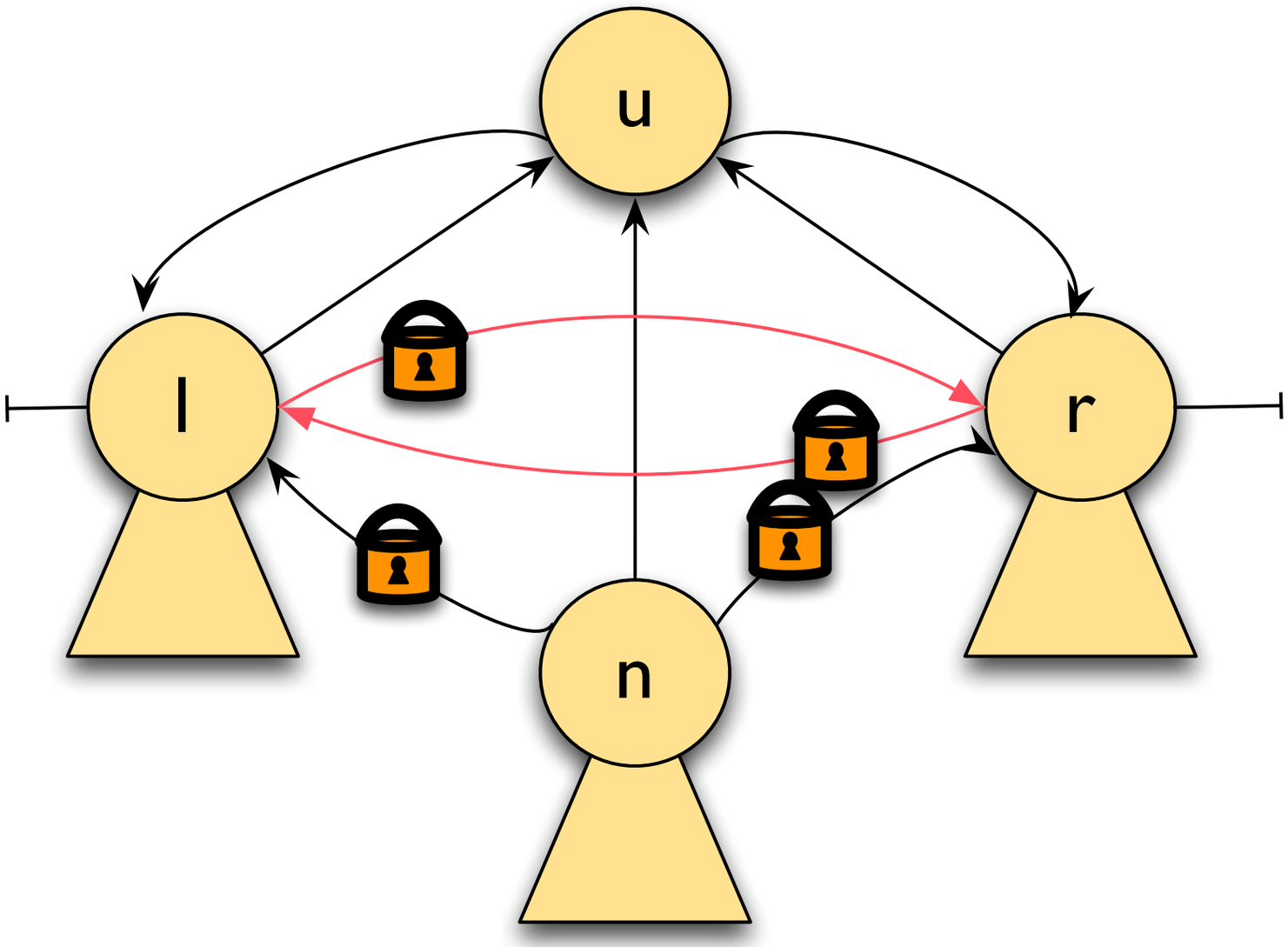

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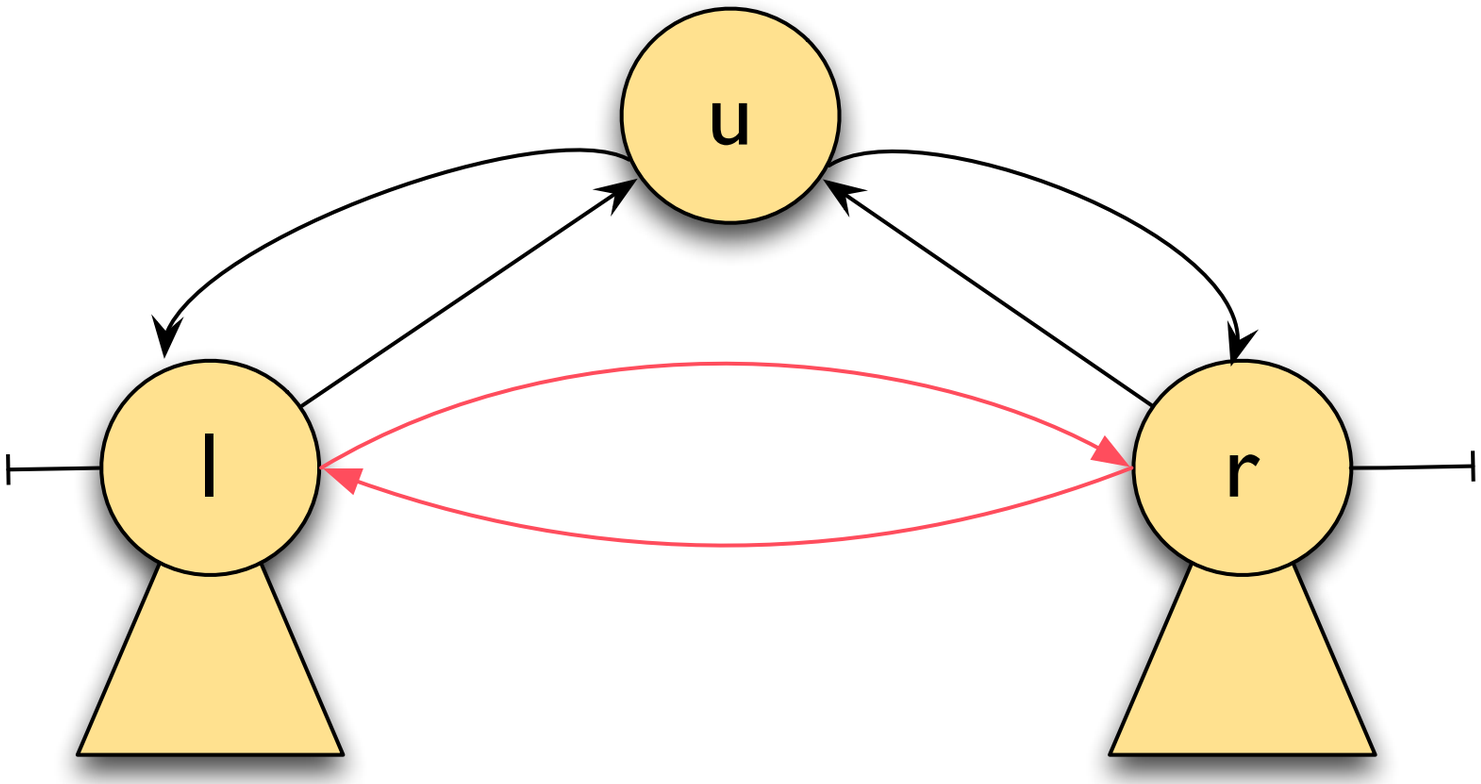
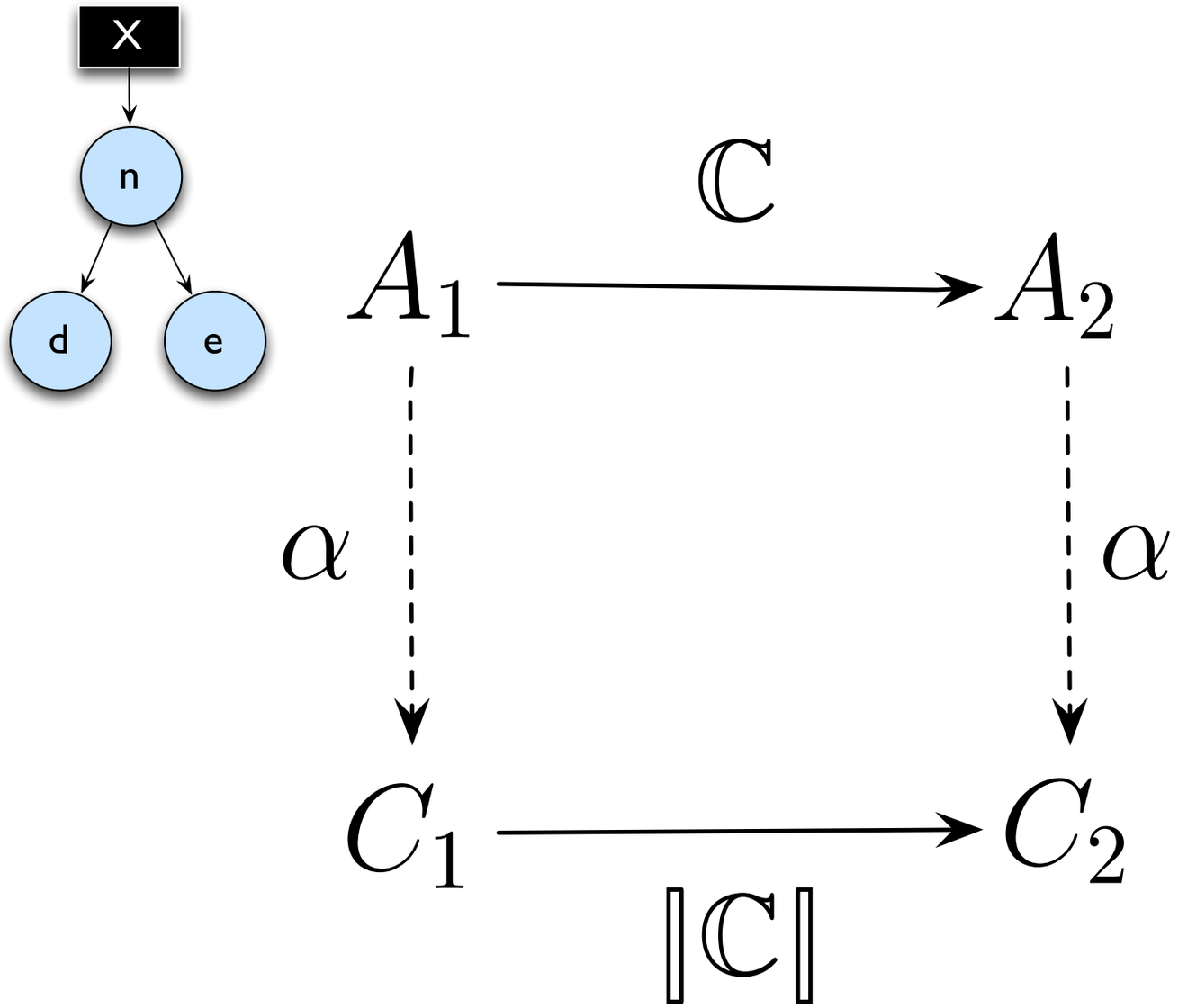


Low Level Trees

Concrete tree representation for partial trees



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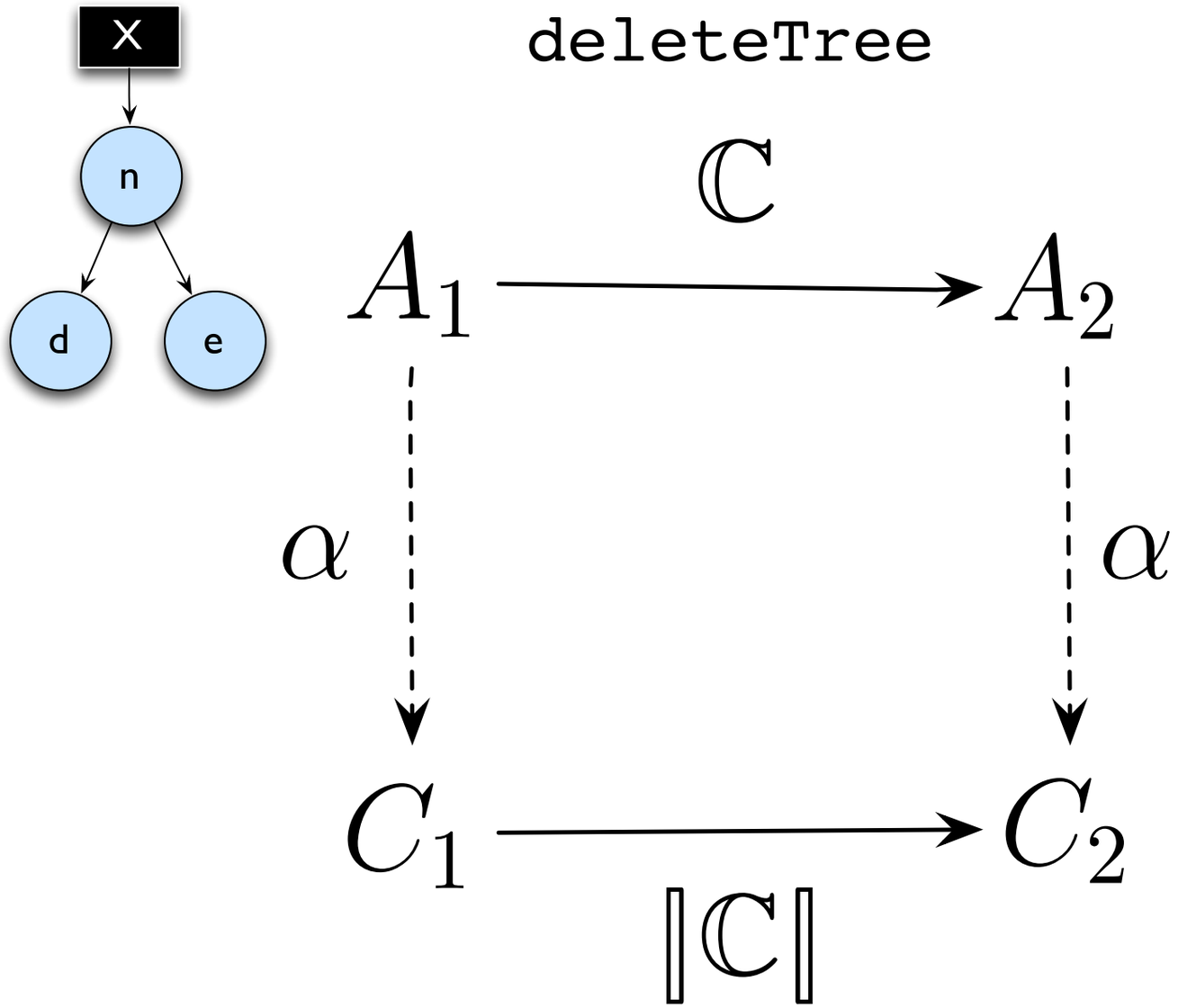
Concrete tree representation for partial trees



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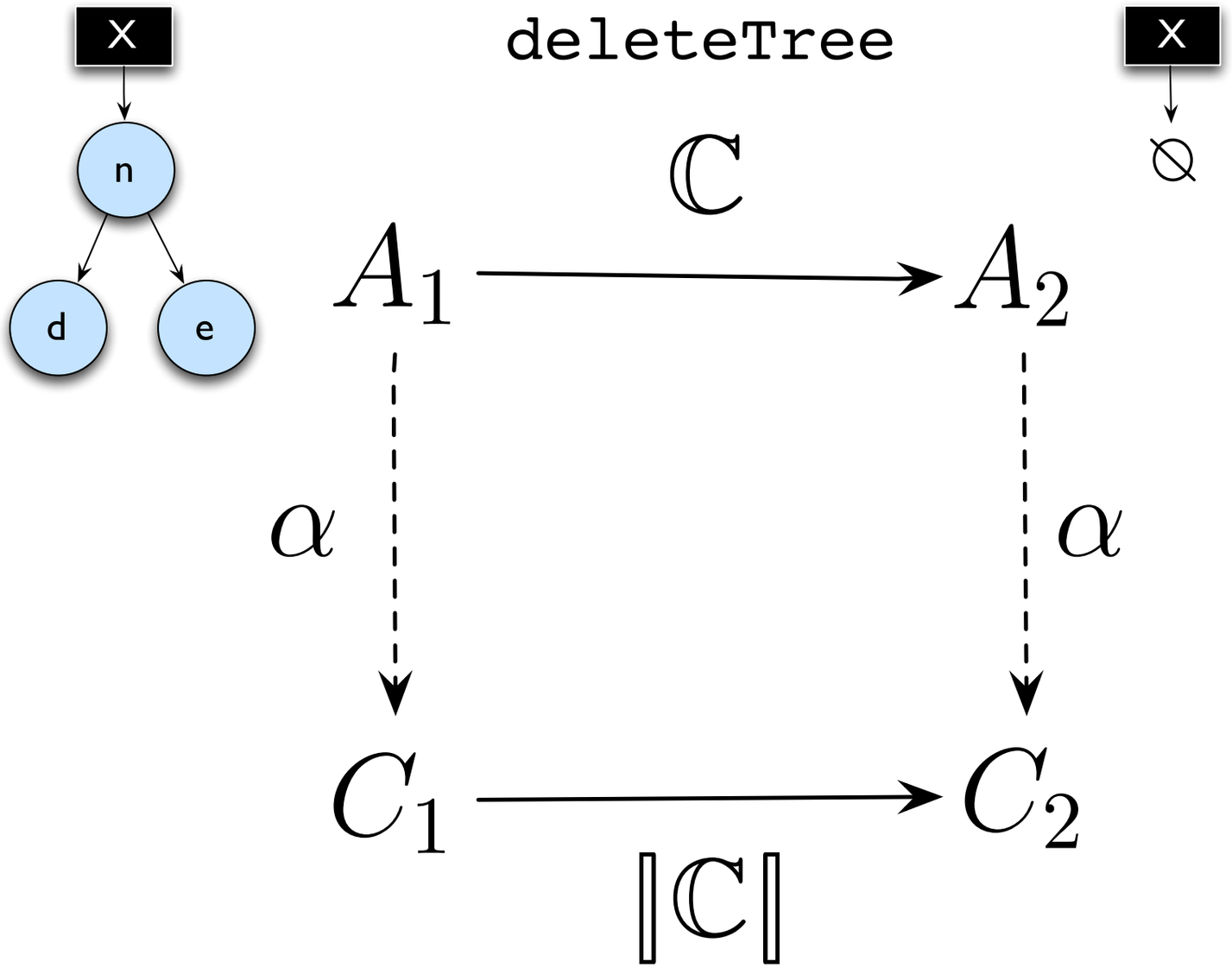
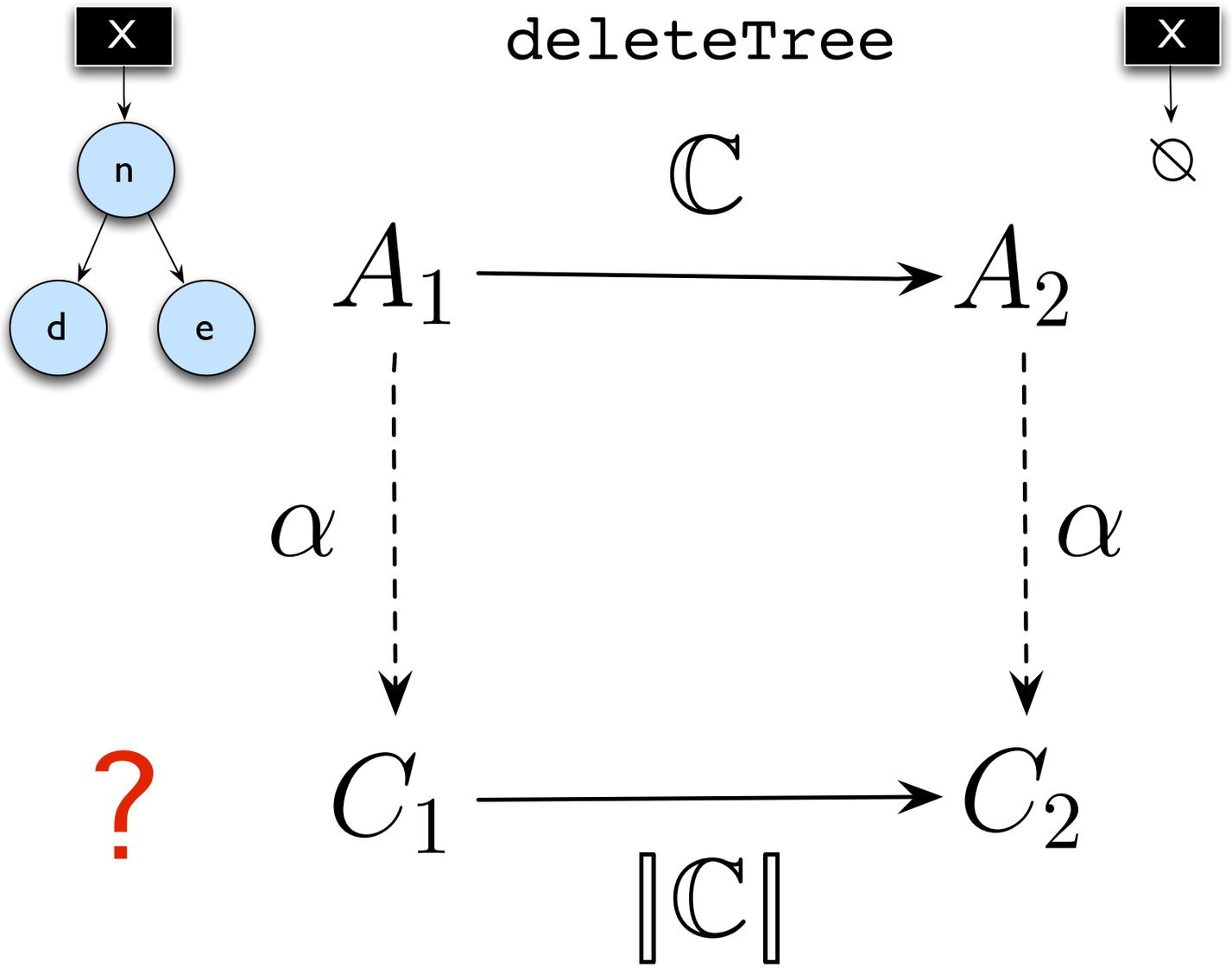
Concrete tree representation for partial trees



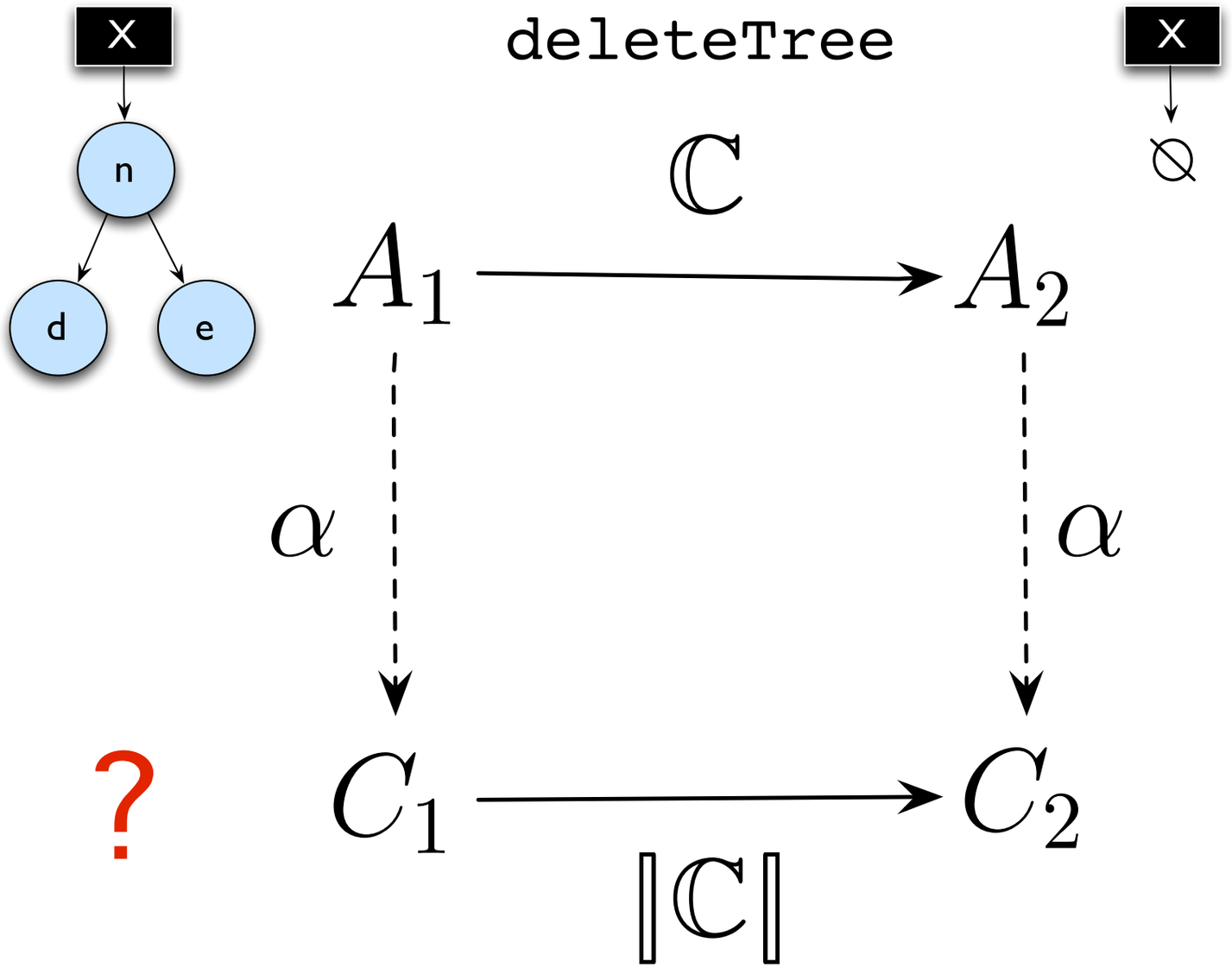
Low Level Trees

Concrete tree representation for partial trees



Low Level Trees

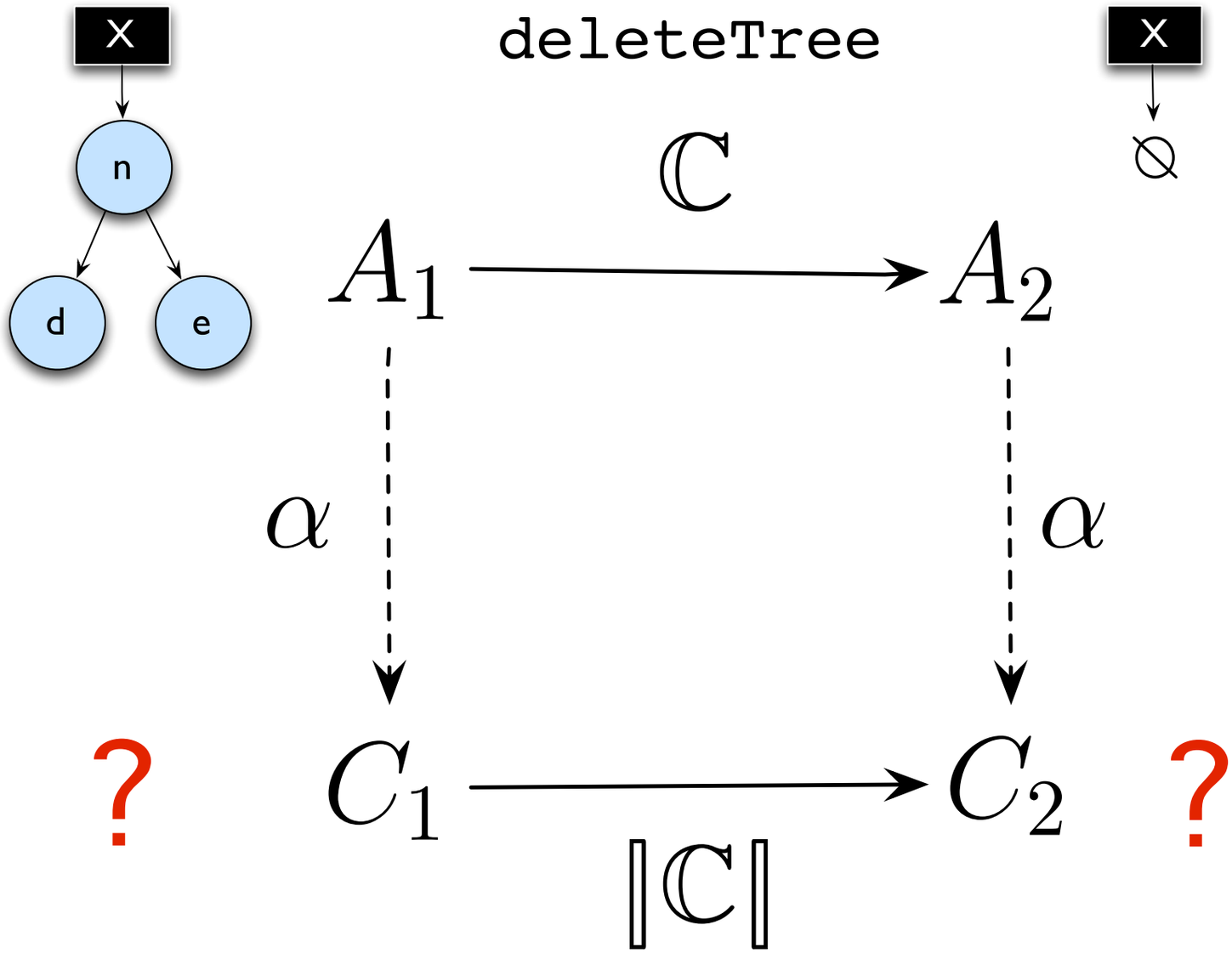
Concrete tree representation for partial trees



(Our locking implementation)

Low Level Trees

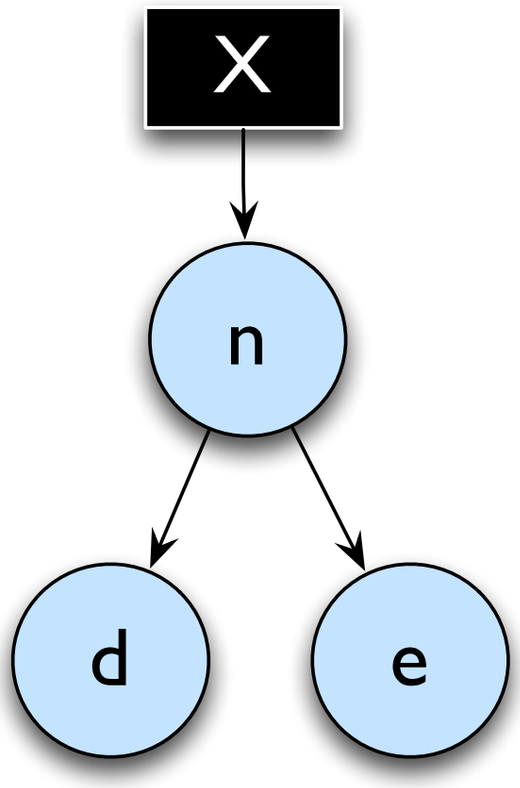
Concrete tree representation for partial trees



(Our locking implementation)

Low Level Trees

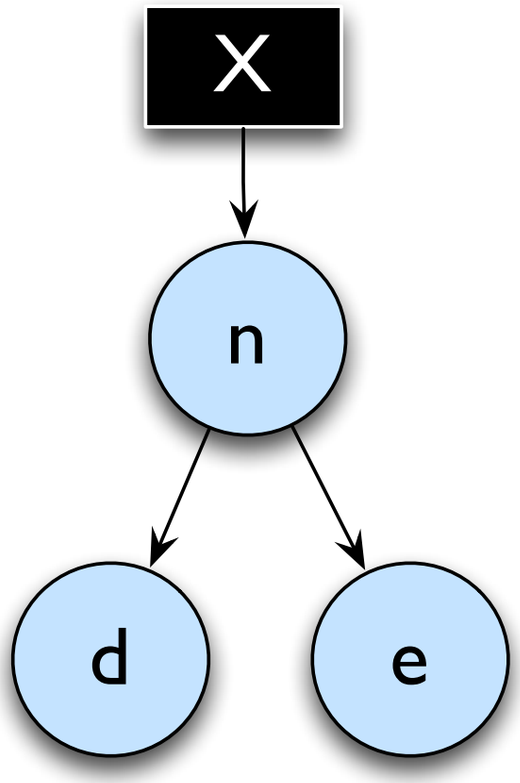
Concrete tree representation for partial trees

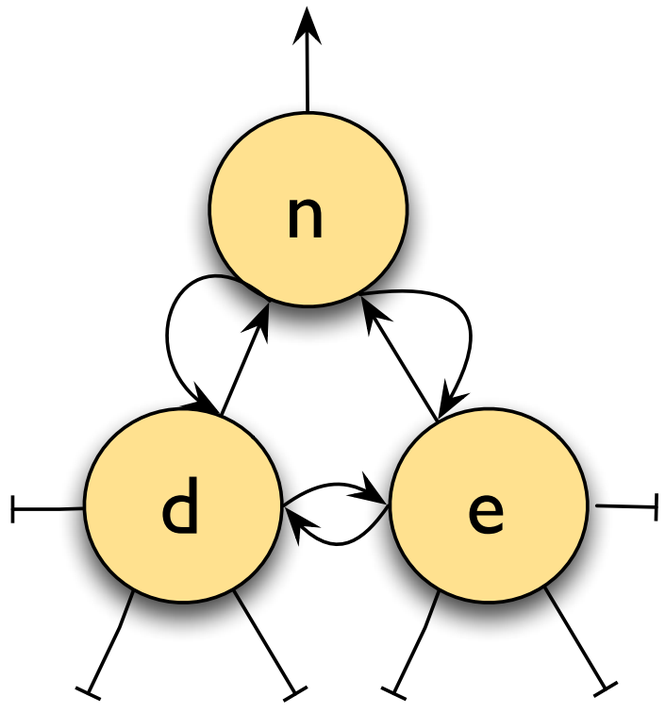


Abstract address: x

Low Level Trees

Concrete tree representation for partial trees

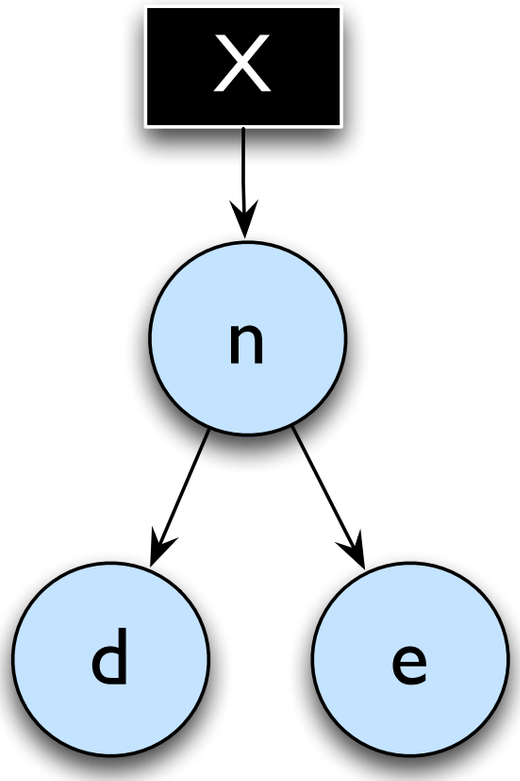


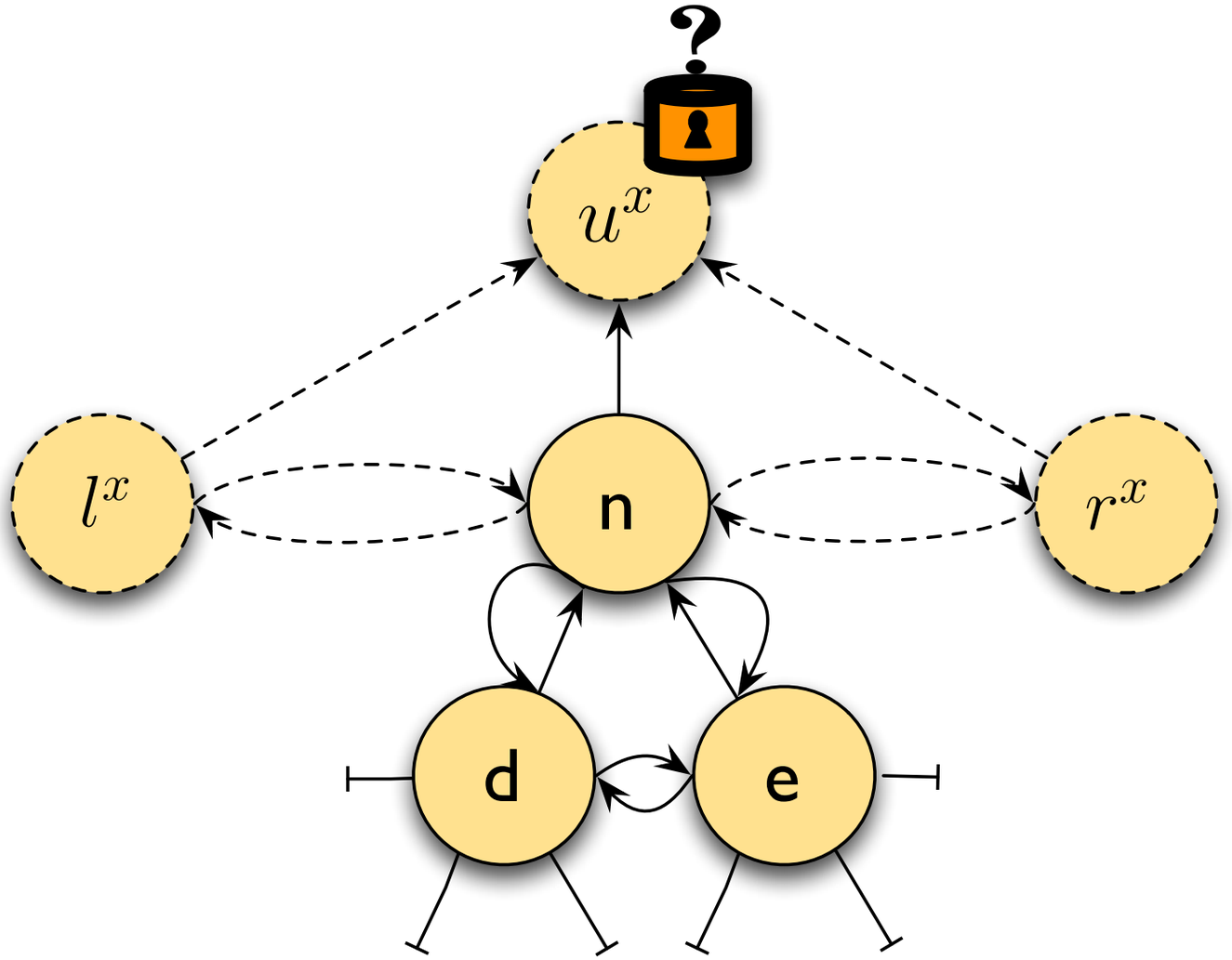


Abstract address: x

Low Level Trees

Concrete tree representation for partial trees





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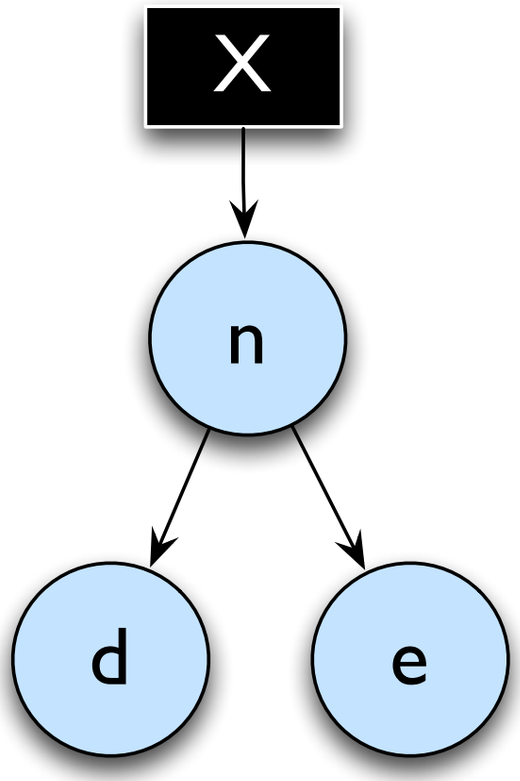
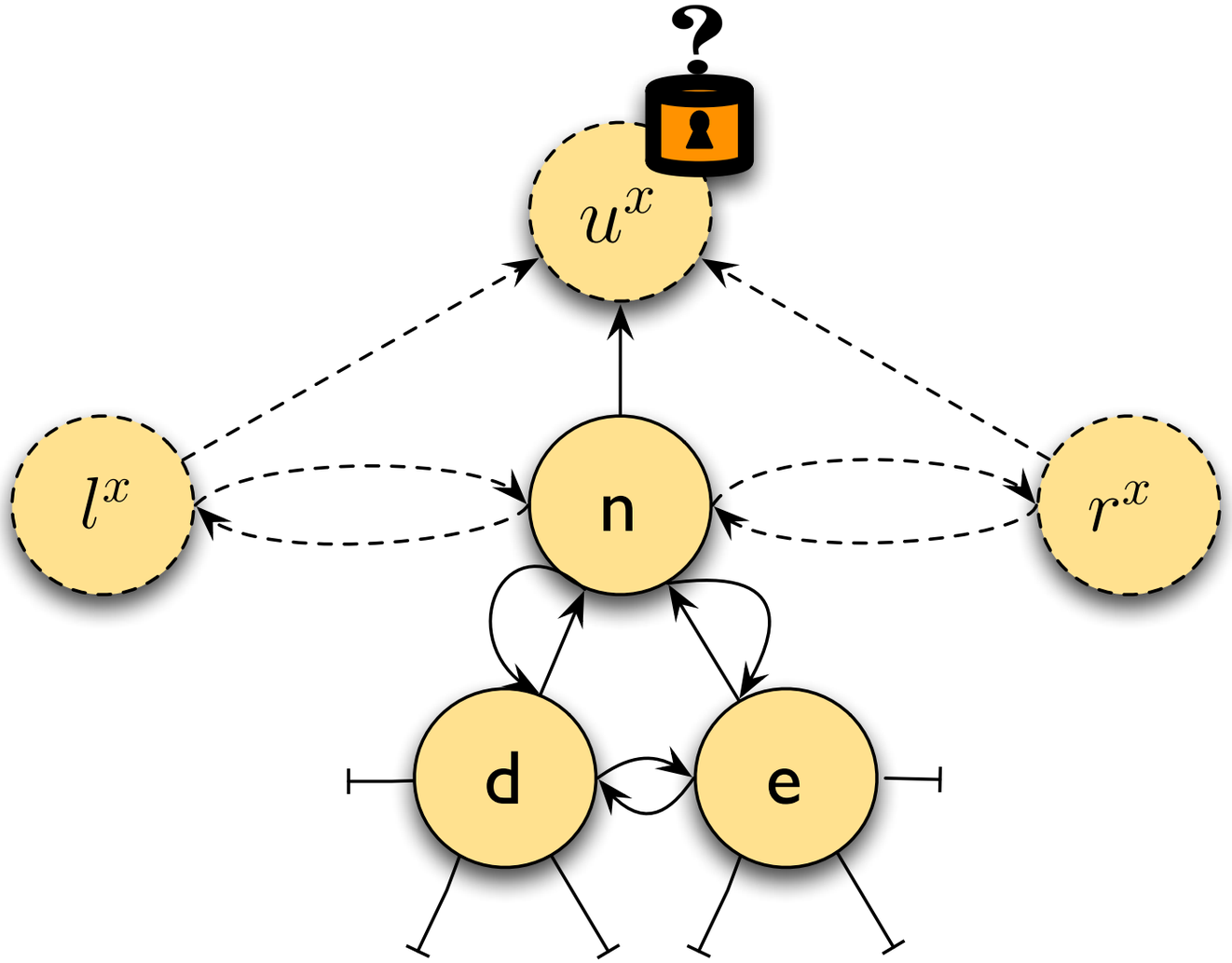


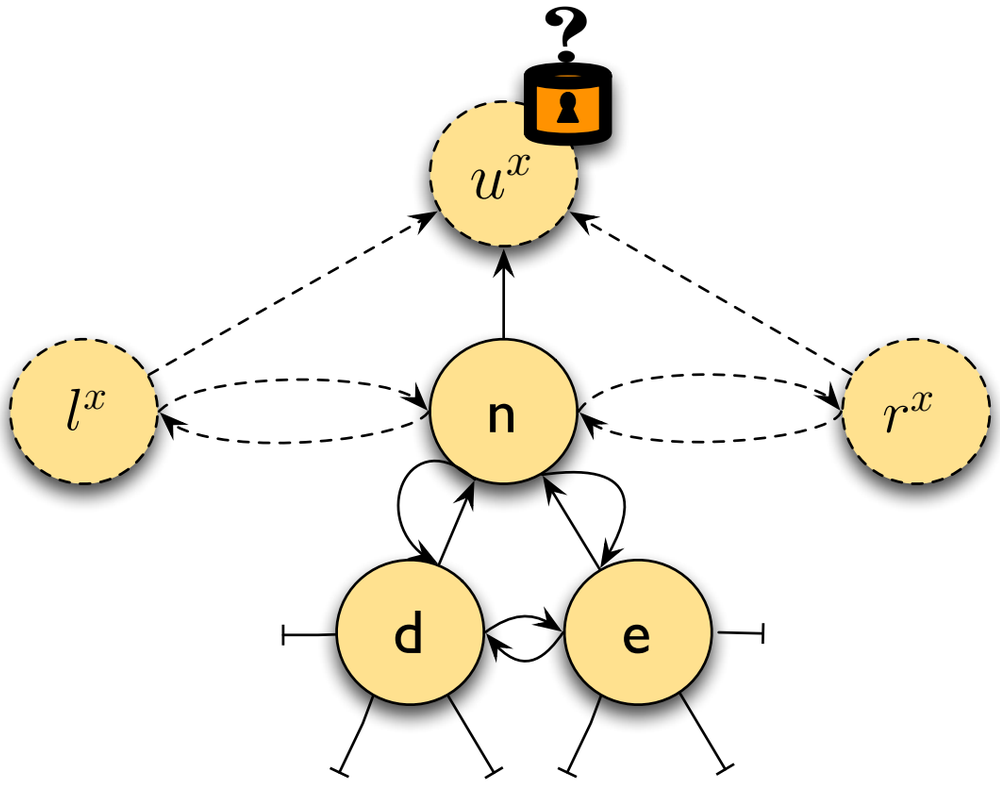


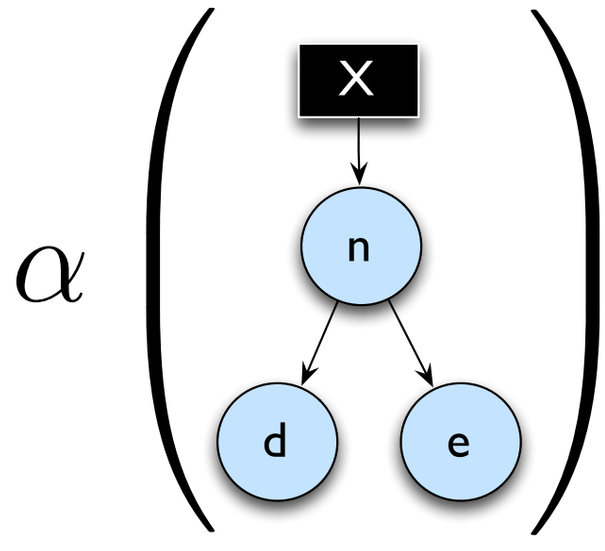
Abstract address: x

$$I_x^{out} = (l^x, u^x, r^x)$$

Refinement (Axiomatic Correctness)

```
proc deleteTree(n){
  local l,u,d,r,ul in
    u := [n.up] ; d := [n.first] ; ul:= [n.upL] ;
    //Acquiring the necessary locks.
    lock(ul) ;
    lock(n.leftL) ; l:= [n.left] ;
    if l ≠ null then lock(l.rightL)
    else if u ≠ null then lock(u.firstL) ;
    lock(n.rightL) ; r:= [n.right] ;
    if r ≠ null then lock(r.leftL) ;
    else if u ≠ null then lock(u.lastL) ;
    unlock(ul) ;
    //Pointer Swinging.
    if l ≠ null then      [l.right] := r ;
    else if u ≠ null then [u.first] := r ;
    if r ≠ null then [r.left] := l ;
    else if u ≠ null then [u.last] := l ;
    //Unlocking the acquired locks.
    if l ≠ null then unlock(l.rightL) ;
    else if u ≠ null then unlock(u.firstL) ;
    if r ≠ null then unlock(r.leftL) ;
    else if u ≠ null then unlock(u.lastL) ;
    call disposeForest(d) ;
    disposeNode(n) ;
}
```





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    if l ≠ null then lock(l.rightL)
    else if u ≠ null then lock(u.firstL) ;
    lock(n.rightL) ; r:= [n.right] ;
    if r ≠ null then lock(r.leftL) ;
    else if u ≠ null then lock(u.lastL) ;
    unlock(ul) ;
    //Pointer Swinging.
    if l ≠ null then      [l.right] := r ;
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}
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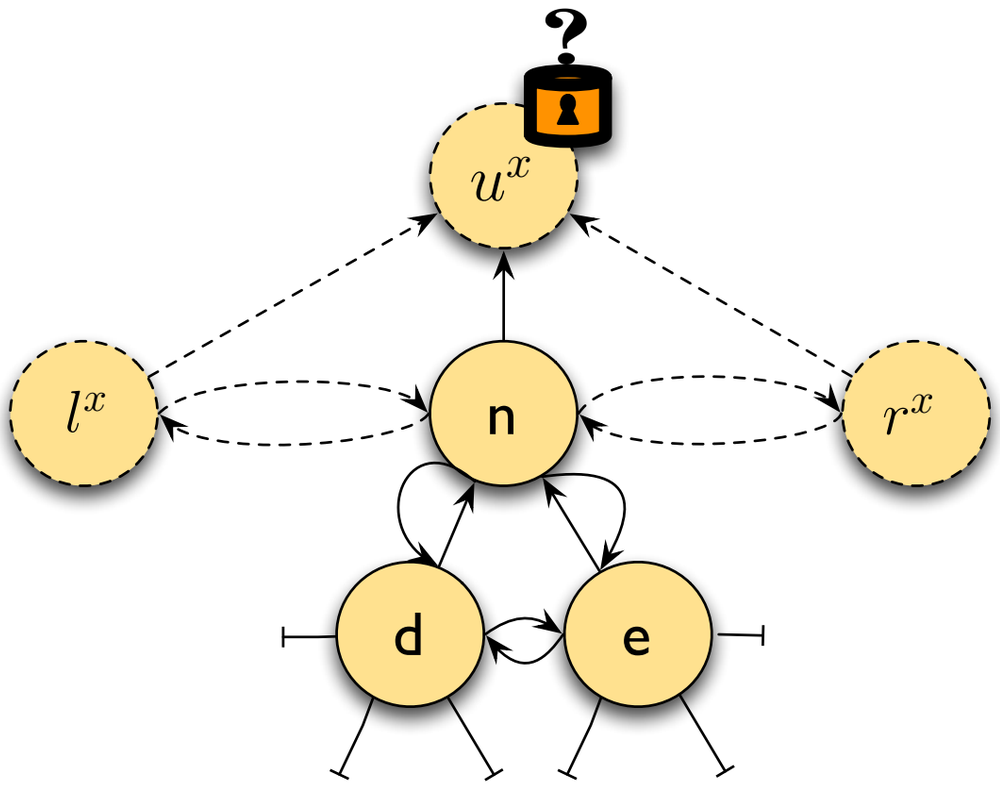
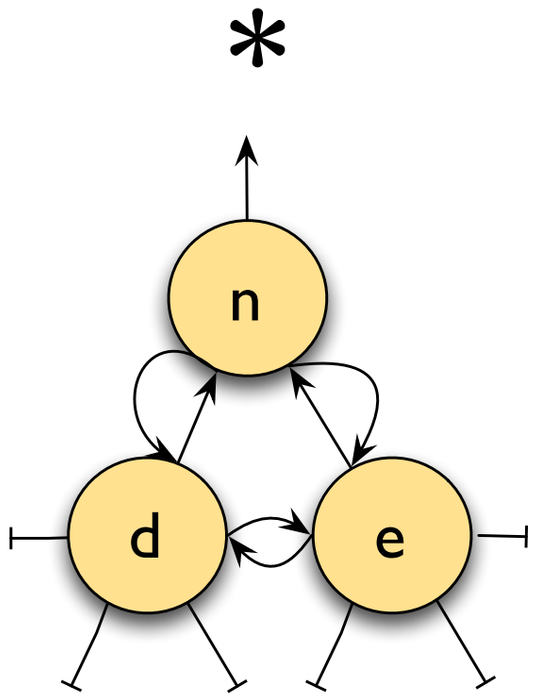


isParentLock(u^x)



*

Crust$(n, n)(l^x, u^x, r^x)$

Refinement (Axiomatic Correctness)

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    if r ≠ null then lock(r.leftL) ;
    else if u ≠ null then lock(u.lastL) ;
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    call disposeForest(d) ;
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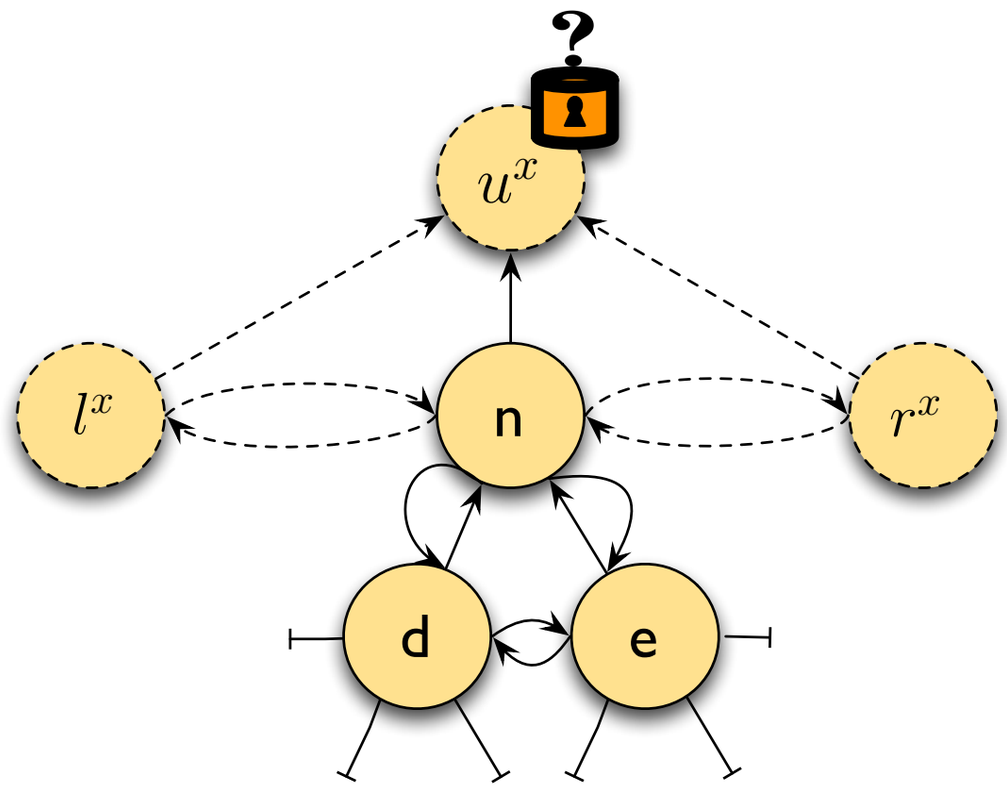

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$*$

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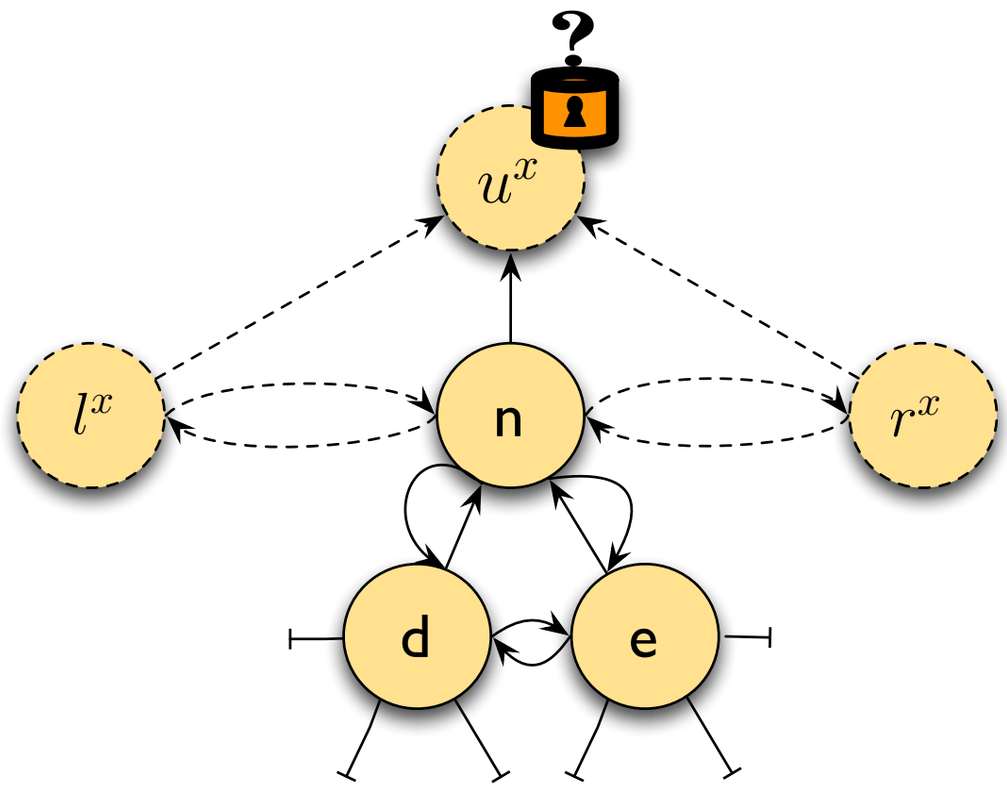

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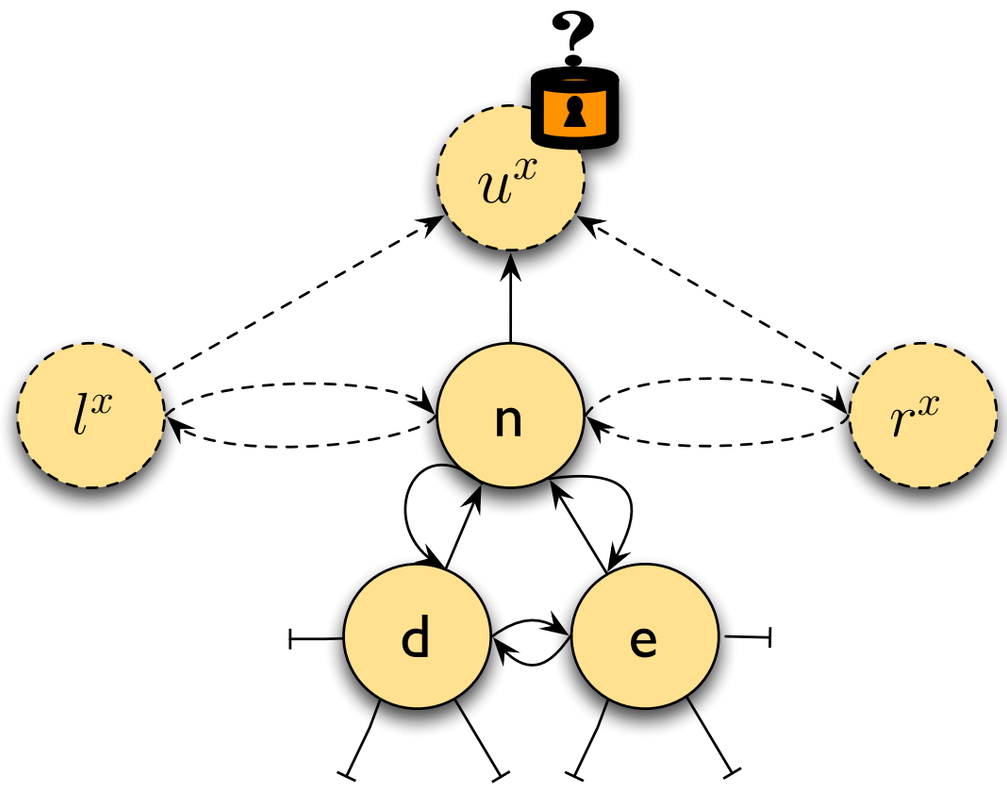

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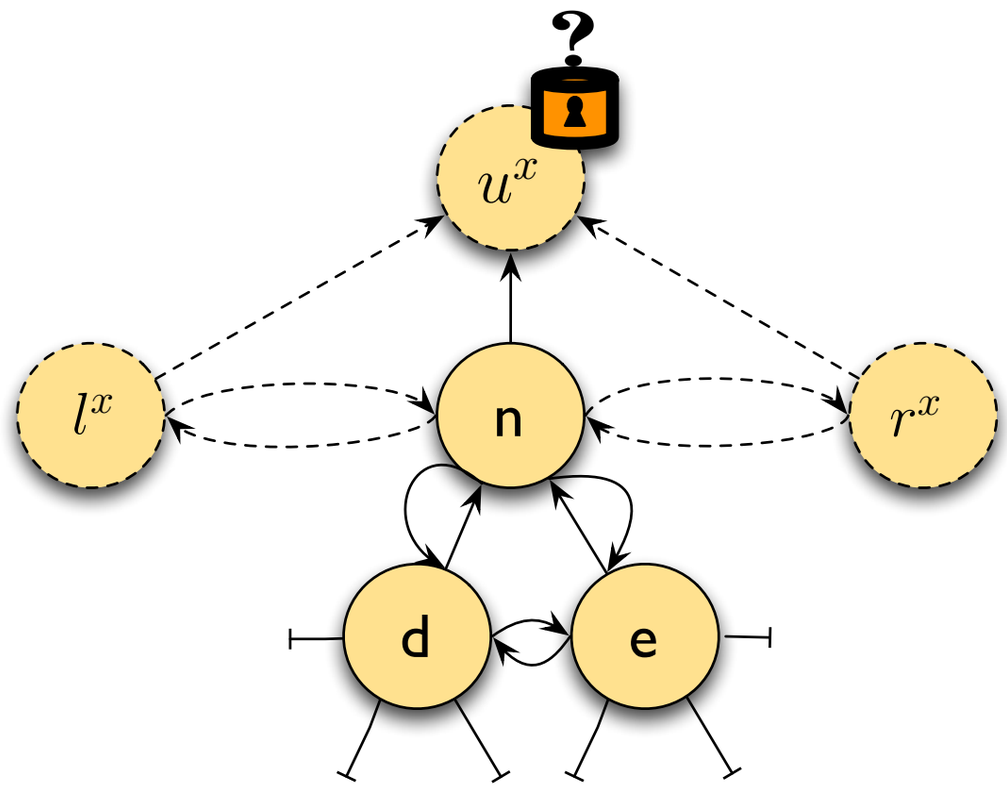

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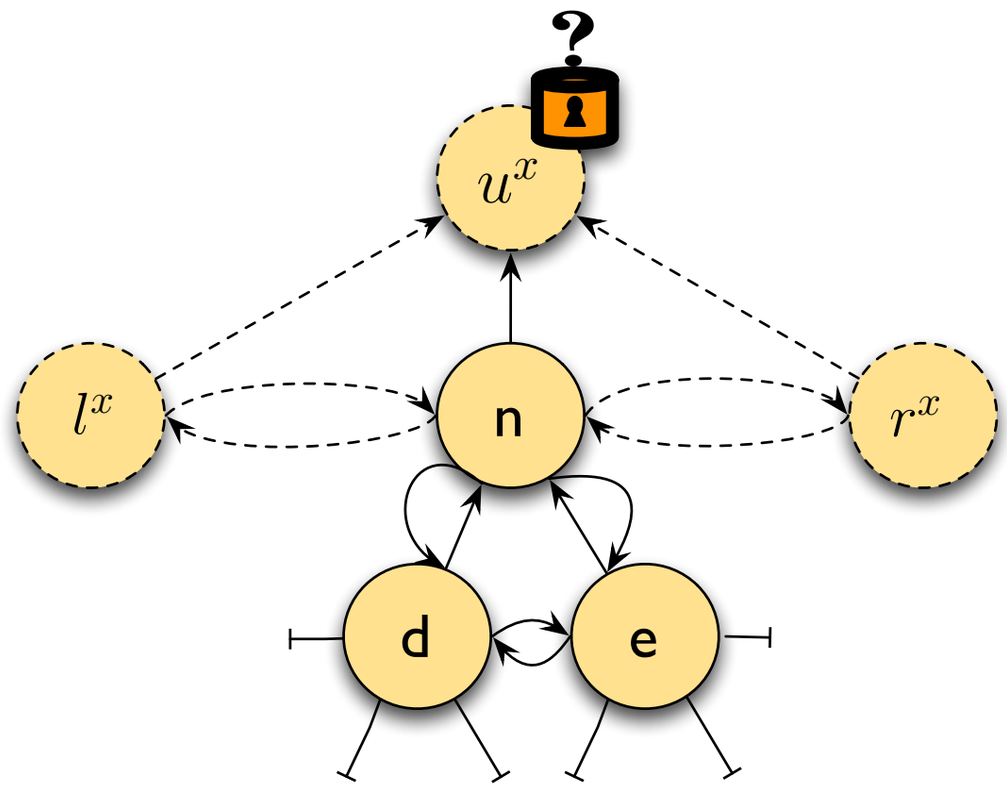

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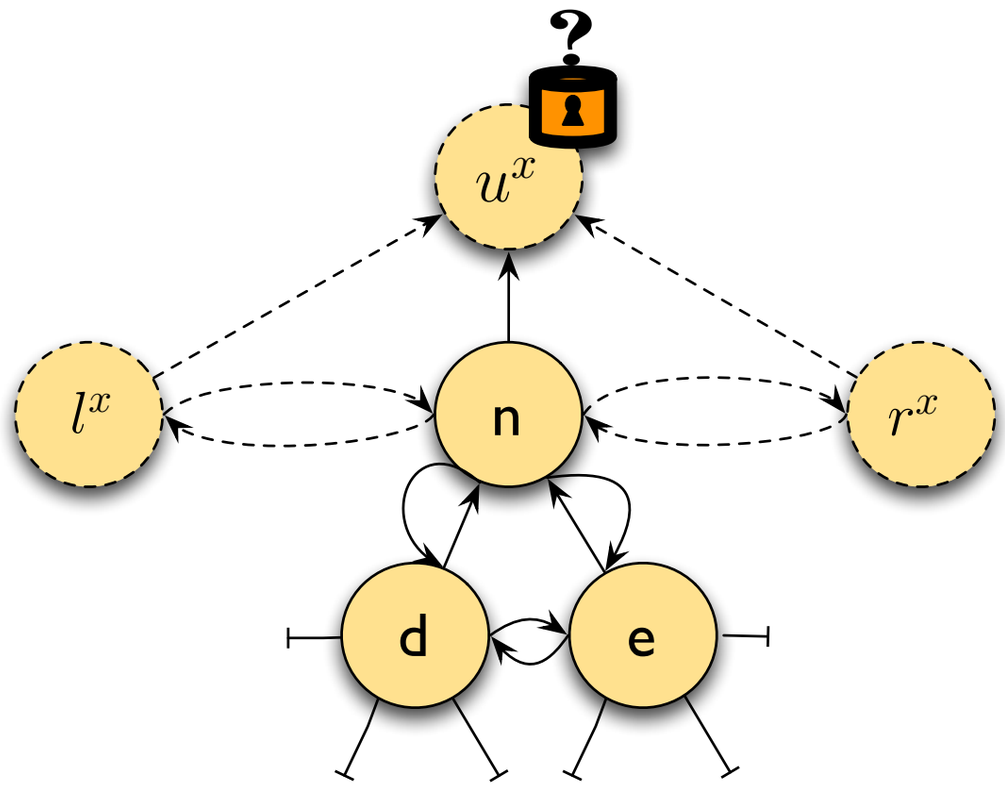
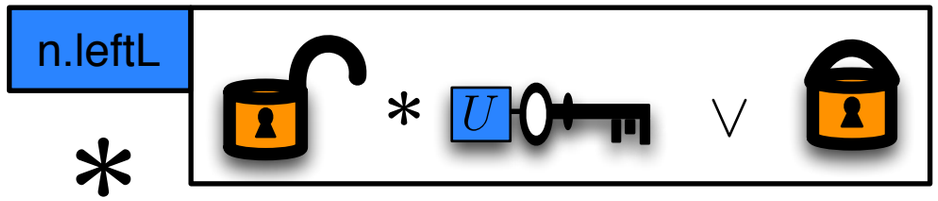


isParentLock(u^x)



Refinement (Axiomatic Correctness)

```
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  local l,u,d,r,ul in
    u := [n.up] ; d := [n.first] ; ul:= [n.upL] ;
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    if l ≠ null then lock(l.rightL)
    else if u ≠ null then lock(u.firstL) ;
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    unlock(ul) ;
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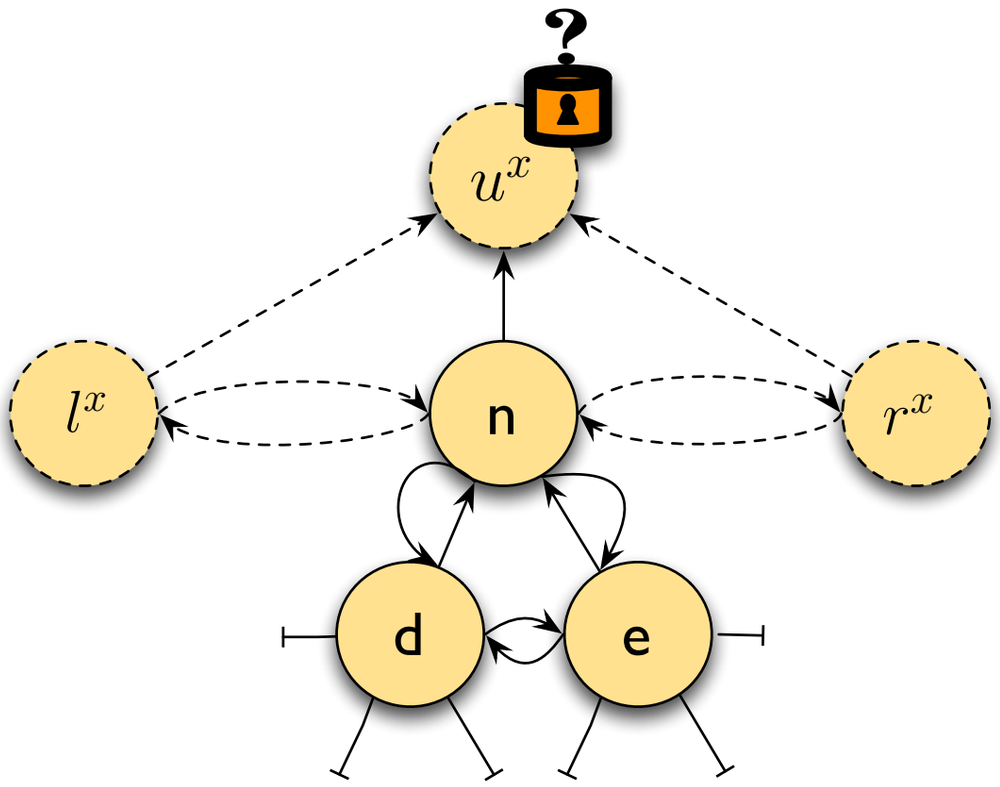

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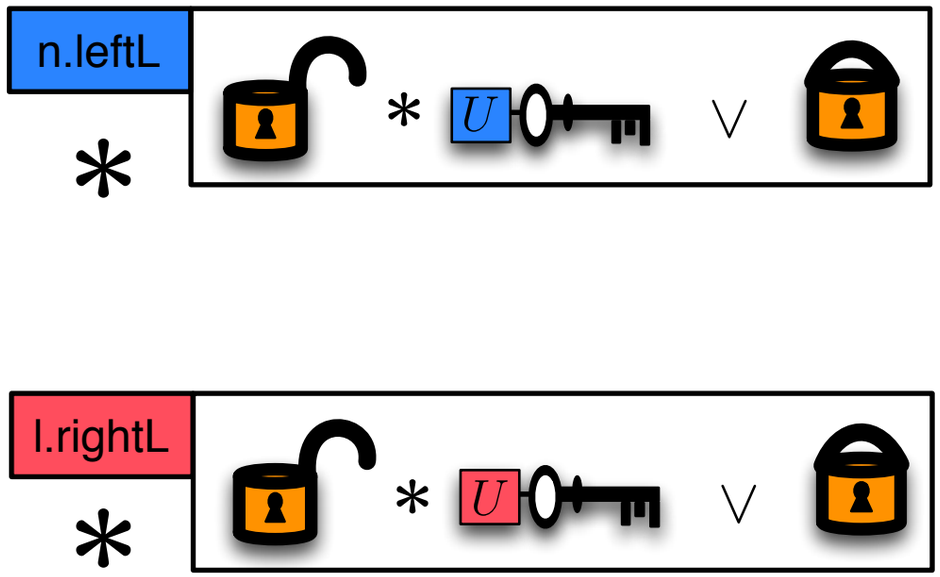

Refinement (Axiomatic Correctness)

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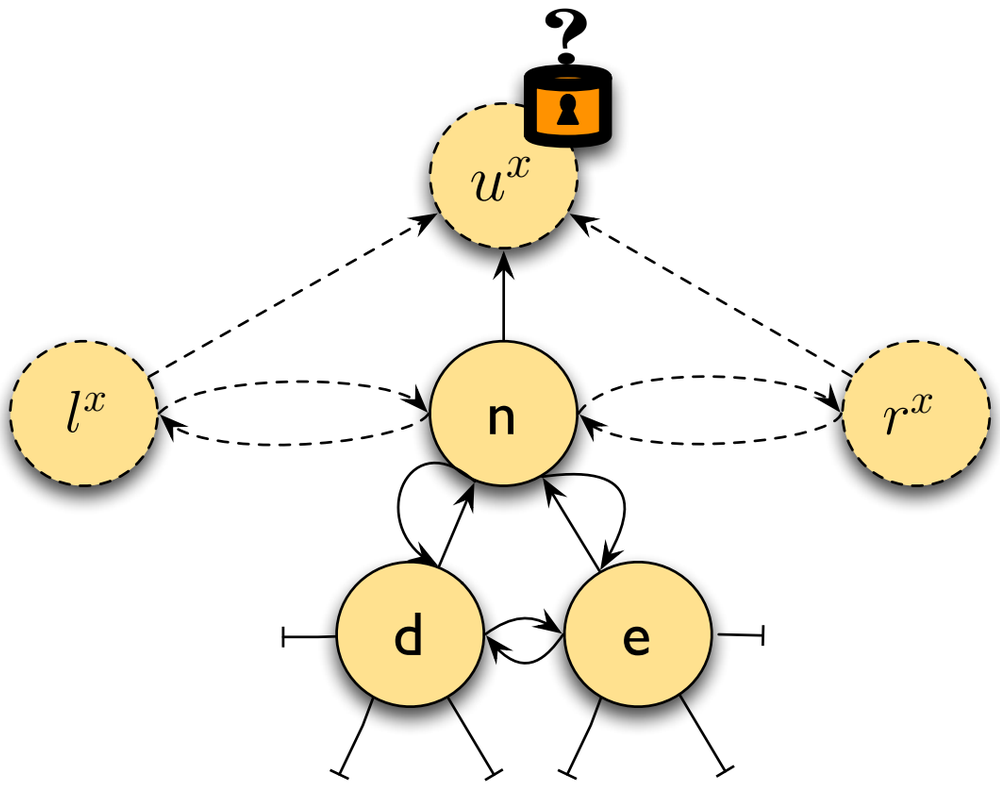
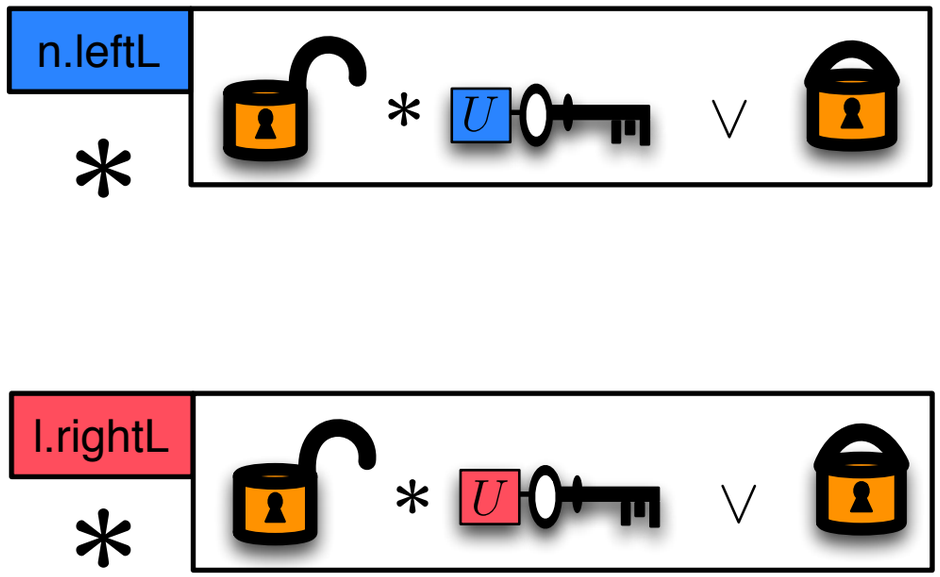


isParentLock(u^x)



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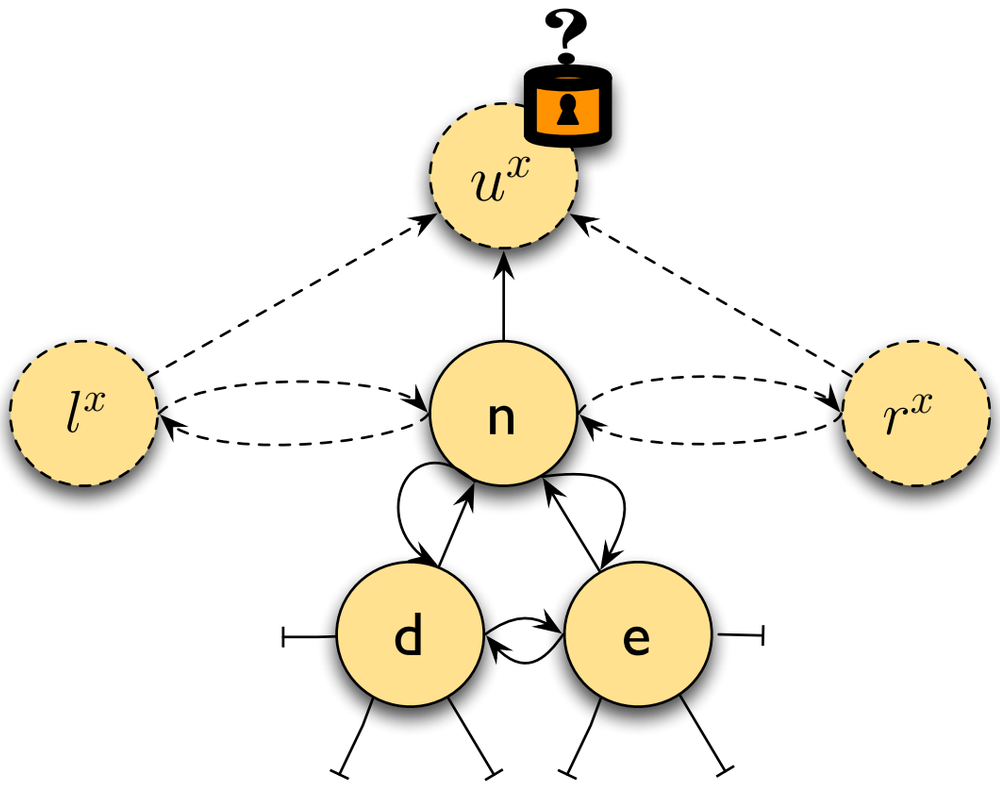
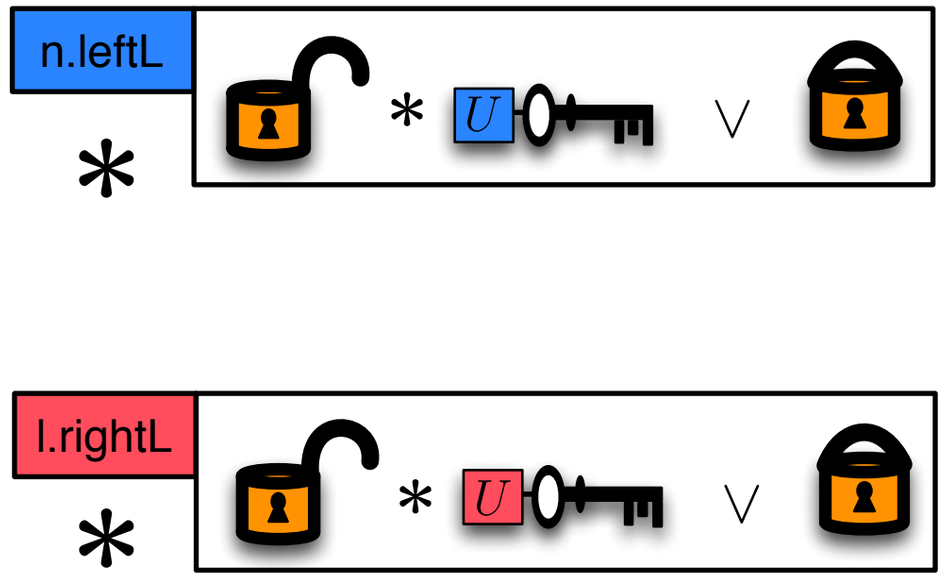
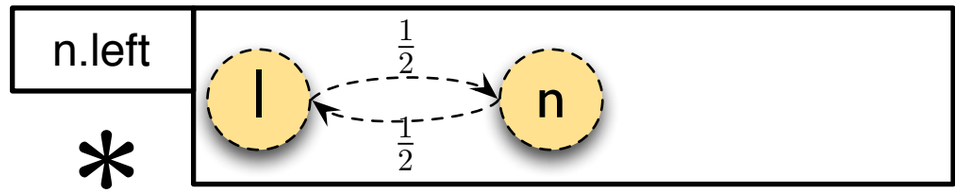
    if l ≠ null then unlock(l.rightL) ;
    else if u ≠ null then unlock(u.firstL) ;
    if r ≠ null then unlock(r.leftL) ;
    else if u ≠ null then unlock(u.lastL) ;
    call disposeForest(d) ;
    disposeNode(n) ;
}
```



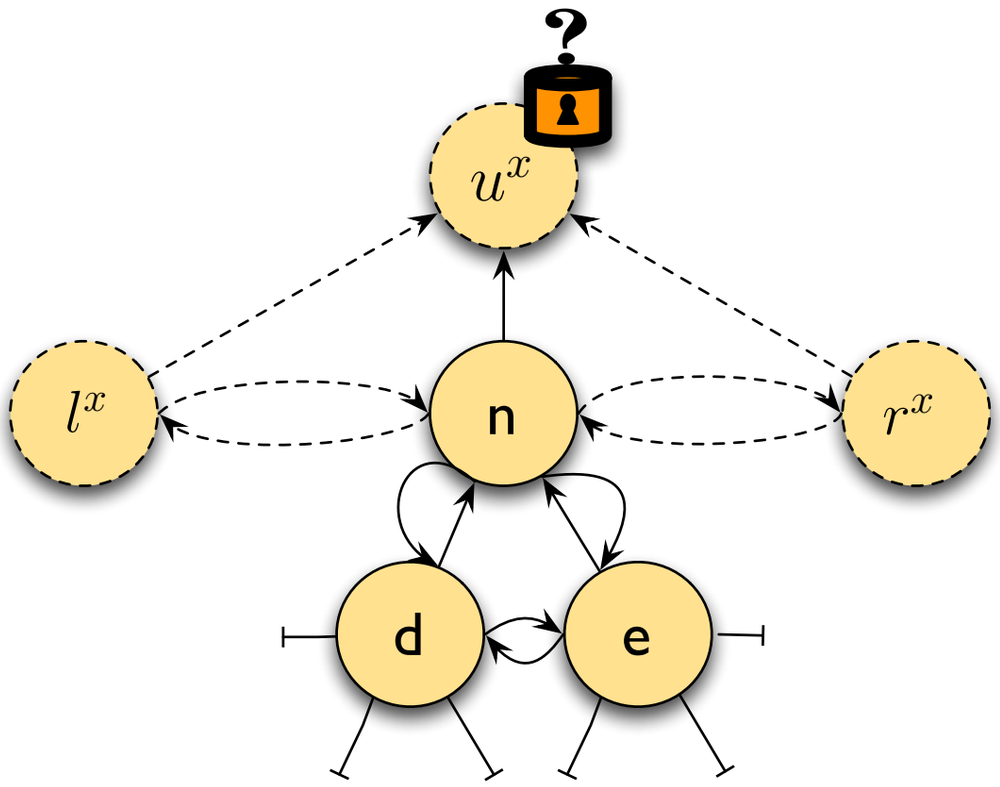
isParentLock(u^x)



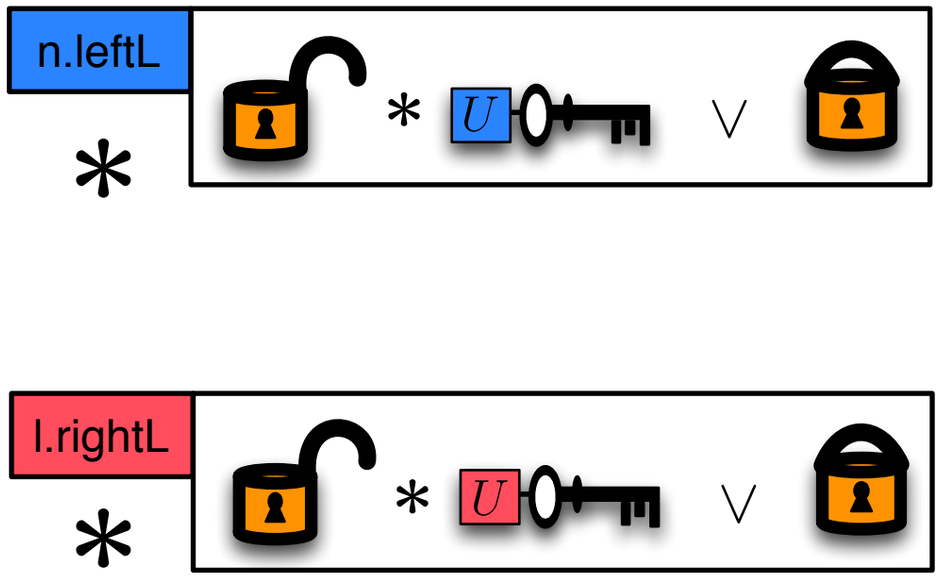


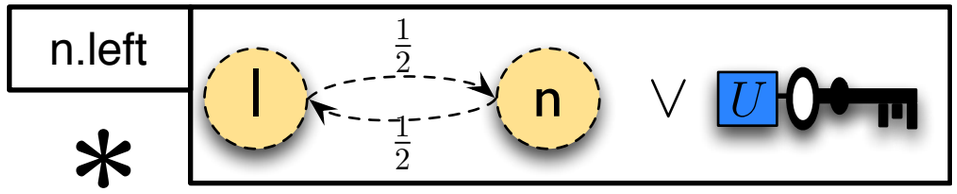
Refinement (Axiomatic Correctness)

```
proc deleteTree(n){
  local l,u,d,r,ul in
    u := [n.up] ; d := [n.first] ; ul:= [n.upL] ;
    //Acquiring the necessary locks.
    lock(ul) ;
    lock(n.leftL) ; l:= [n.left] ;
    if l ≠ null then lock(l.rightL)
    else if u ≠ null then lock(u.firstL) ;
    lock(n.rightL) ; r:= [n.right] ;
    if r ≠ null then lock(r.leftL) ;
    else if u ≠ null then lock(u.lastL) ;
    unlock(ul) ;
    //Pointer Swinging.
    if l ≠ null then      [l.right] := r ;
    else if u ≠ null then [u.first] := r ;
    if r ≠ null then [r.left] := l ;
    else if u ≠ null then [u.last] := l ;
    //Unlocking the acquired locks.
    if l ≠ null then unlock(l.rightL) ;
    else if u ≠ null then unlock(u.firstL) ;
    if r ≠ null then unlock(r.leftL) ;
    else if u ≠ null then unlock(u.lastL) ;
    call disposeForest(d) ;
    disposeNode(n) ;
}
```



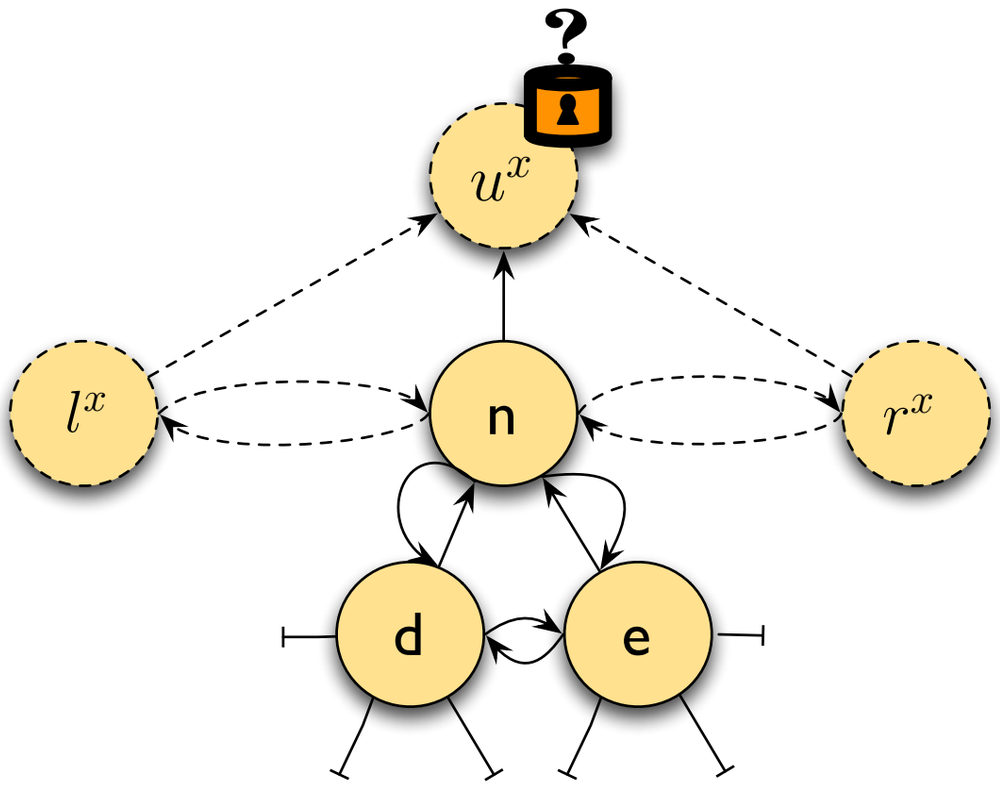
isParentLock(u^x)



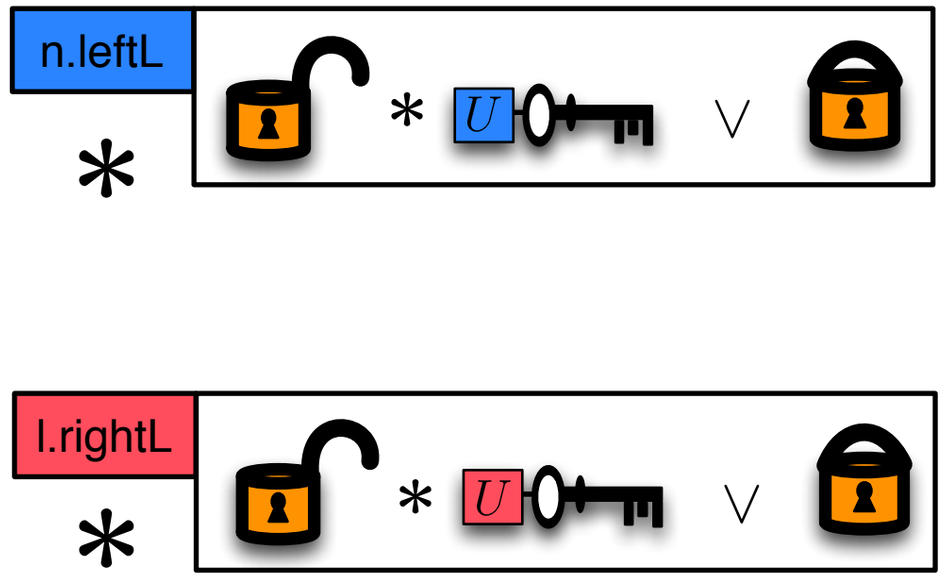


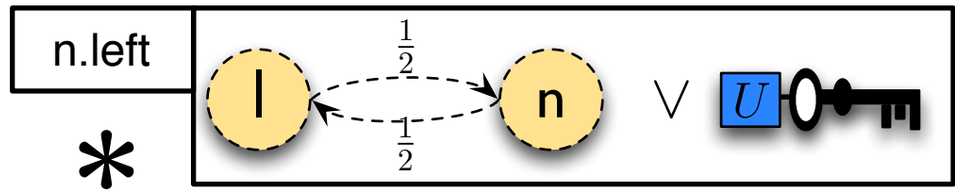
Refinement (Axiomatic Correctness)

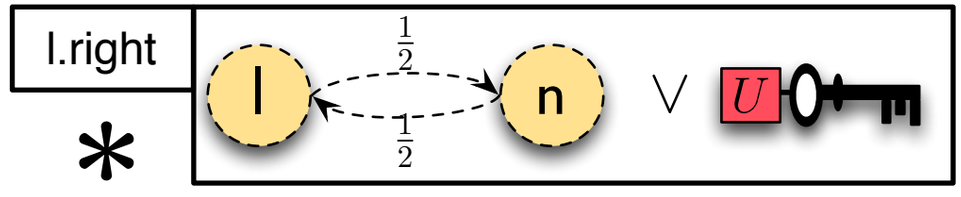
```
proc deleteTree(n){
  local l,u,d,r,ul in
    u := [n.up] ; d := [n.first] ; ul:= [n.upL] ;
    //Acquiring the necessary locks.
    lock(ul) ;
    lock(n.leftL) ; l:= [n.left] ;
    if l ≠ null then lock(l.rightL)
    else if u ≠ null then lock(u.firstL) ;
    lock(n.rightL) ; r:= [n.right] ;
    if r ≠ null then lock(r.leftL) ;
    else if u ≠ null then lock(u.lastL) ;
    unlock(ul) ;
    //Pointer Swinging.
    if l ≠ null then      [l.right] := r ;
    else if u ≠ null then [u.first] := r ;
    if r ≠ null then [r.left] := l ;
    else if u ≠ null then [u.last] := l ;
    //Unlocking the acquired locks.
    if l ≠ null then unlock(l.rightL) ;
    else if u ≠ null then unlock(u.firstL) ;
    if r ≠ null then unlock(r.leftL) ;
    else if u ≠ null then unlock(u.lastL) ;
    call disposeForest(d) ;
    disposeNode(n) ;
}
```



isParentLock(u^x)

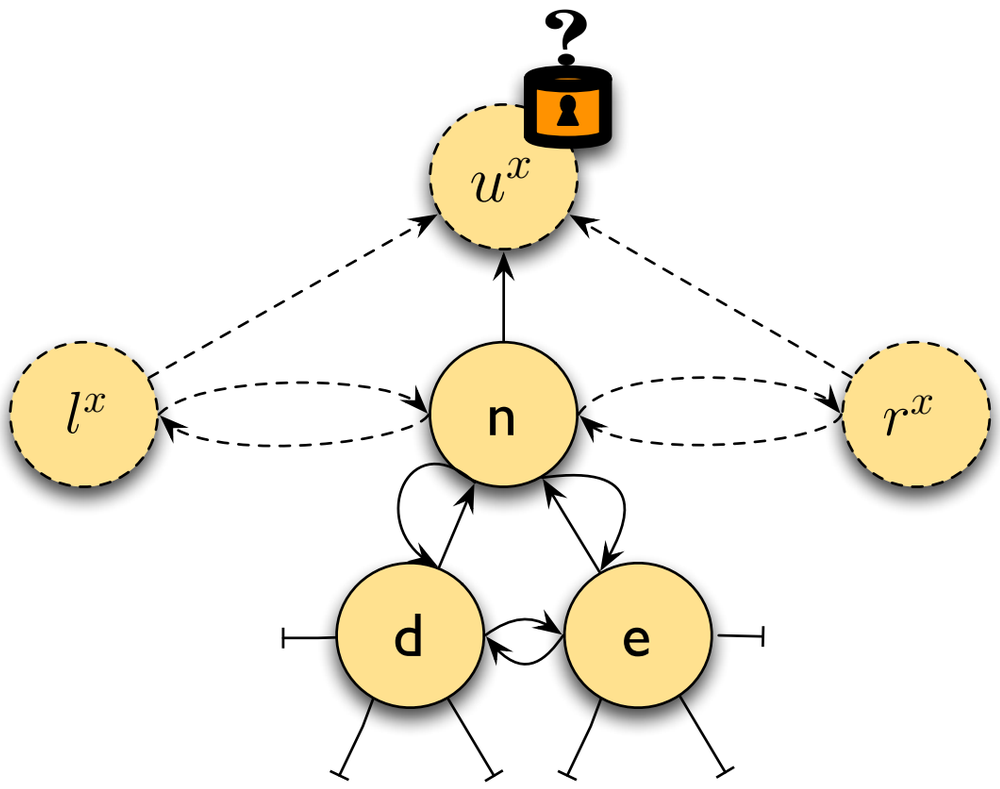




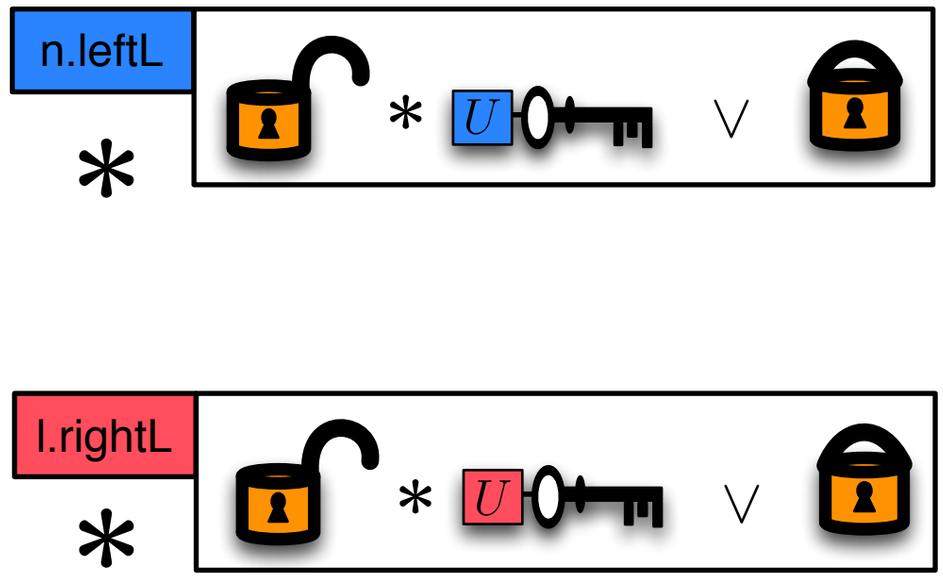


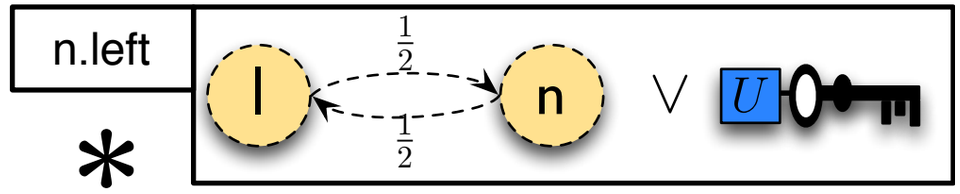
Refinement (Axiomatic Correctness)

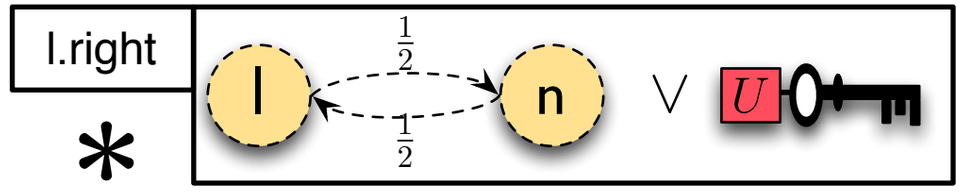
```
proc deleteTree(n){
  local l,u,d,r,ul in
    u := [n.up] ; d := [n.first] ; ul:= [n.upL] ;
    //Acquiring the necessary locks.
    lock(ul) ;
    lock(n.leftL) ; l:= [n.left] ;
    if l ≠ null then lock(l.rightL)
    else if u ≠ null then lock(u.firstL) ;
    if r ≠ null then lock(r.leftL) ;
    else if u ≠ null then lock(u.lastL) ;
    u:=n.up;
    lock(u);
    if l ≠ null then    [l.right] := r ;
    else if u ≠ null then [u.first] := r ;
    if r ≠ null then [r.left] := l ;
    else if u ≠ null then [u.last] := l ;
    //Unlocking the acquired locks.
    if l ≠ null then unlock(l.rightL) ;
    else if u ≠ null then unlock(u.firstL) ;
    if r ≠ null then unlock(r.leftL) ;
    else if u ≠ null then unlock(u.lastL) ;
    call disposeForest(d) ;
    disposeNode(n) ;
}
```



isParentLock(u^x)



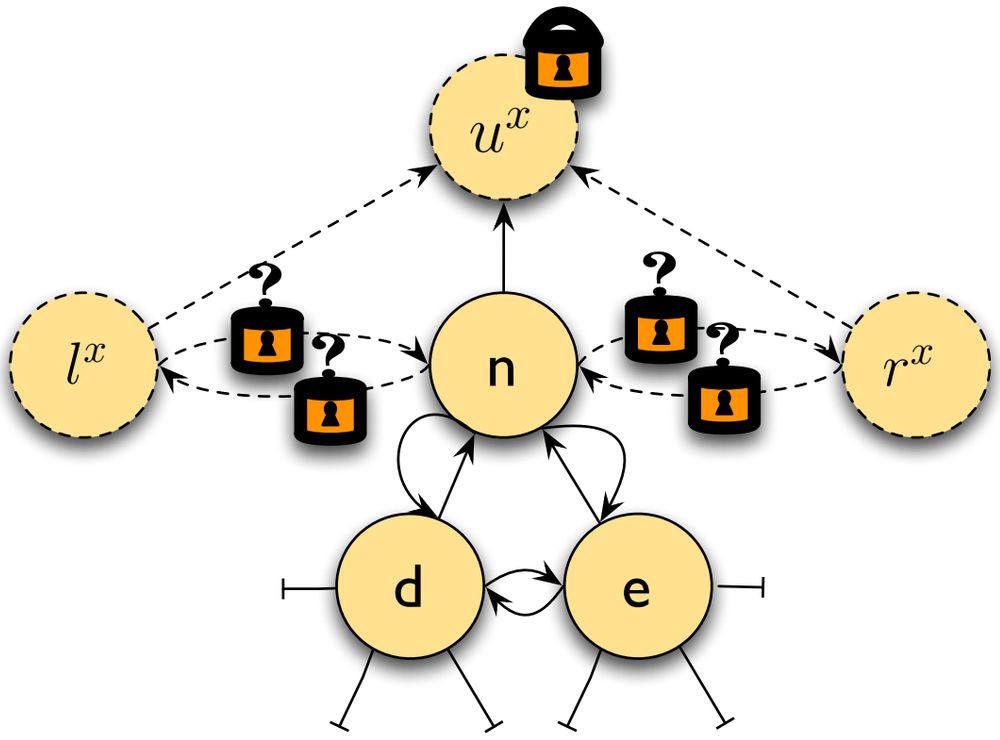




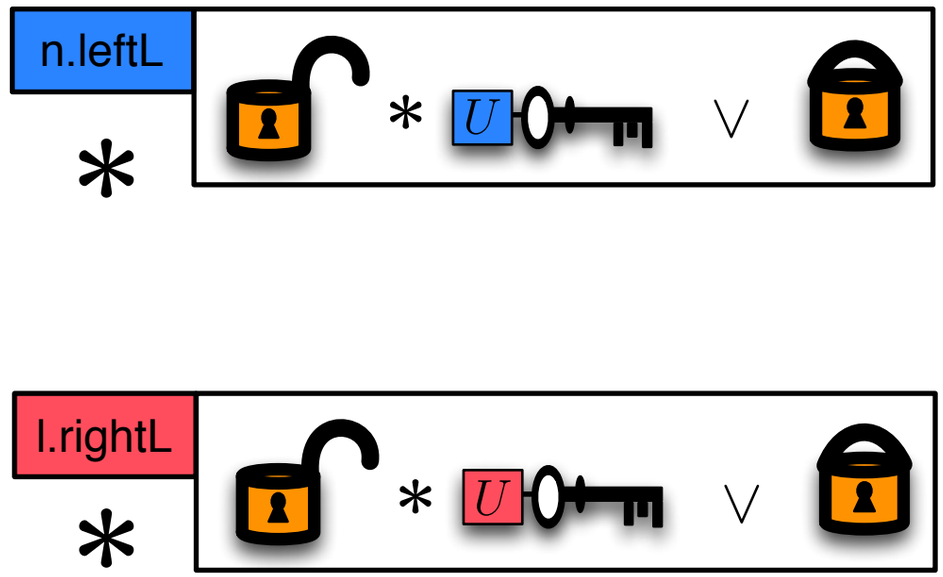
Refinement (Axiomatic Correctness)

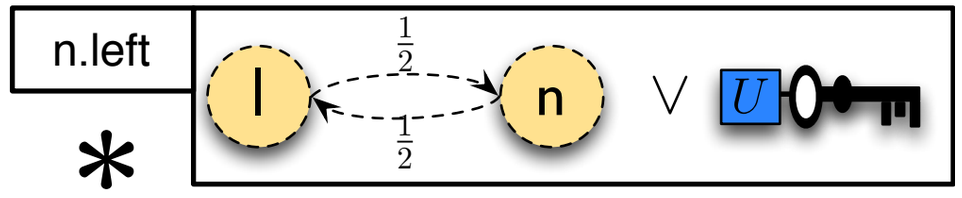
```
proc deleteTree(n){
  local l,u,d,r,ul in
    u := [n.up] ; d := [n.first] ; ul:= [n.upL] ;
    //Acquiring the necessary locks.
    lock(ul) ;
    lock(n.leftL) ; l:= [n.left] ;
    if l ≠ null then lock(l.rightL)
    else if u ≠ null then lock(u.firstL) ;
    if r ≠ null then lock(r.leftL) ;
    else if u ≠ null then lock(u.lastL) ;
    //Updating the forest.
    if l ≠ null then [l.right] := r ;
    else if u ≠ null then [u.first] := r ;
    if r ≠ null then [r.left] := l ;
    else if u ≠ null then [u.last] := l ;
    //Unlocking the acquired locks.
    if l ≠ null then unlock(l.rightL) ;
    else if u ≠ null then unlock(u.firstL) ;
    if r ≠ null then unlock(r.leftL) ;
    else if u ≠ null then unlock(u.lastL) ;
    call disposeForest(d) ;
    disposeNode(n) ;
}
```

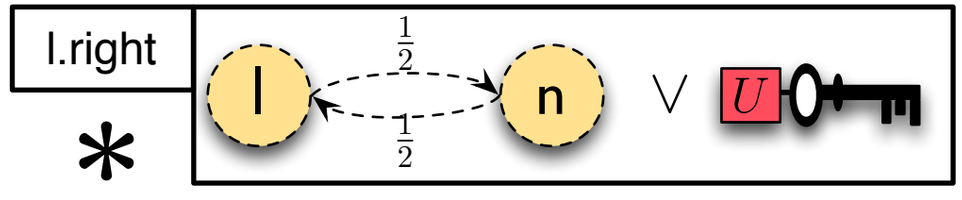
```
u:=n.up;
lock(u);
```



isParentLock(u^x)



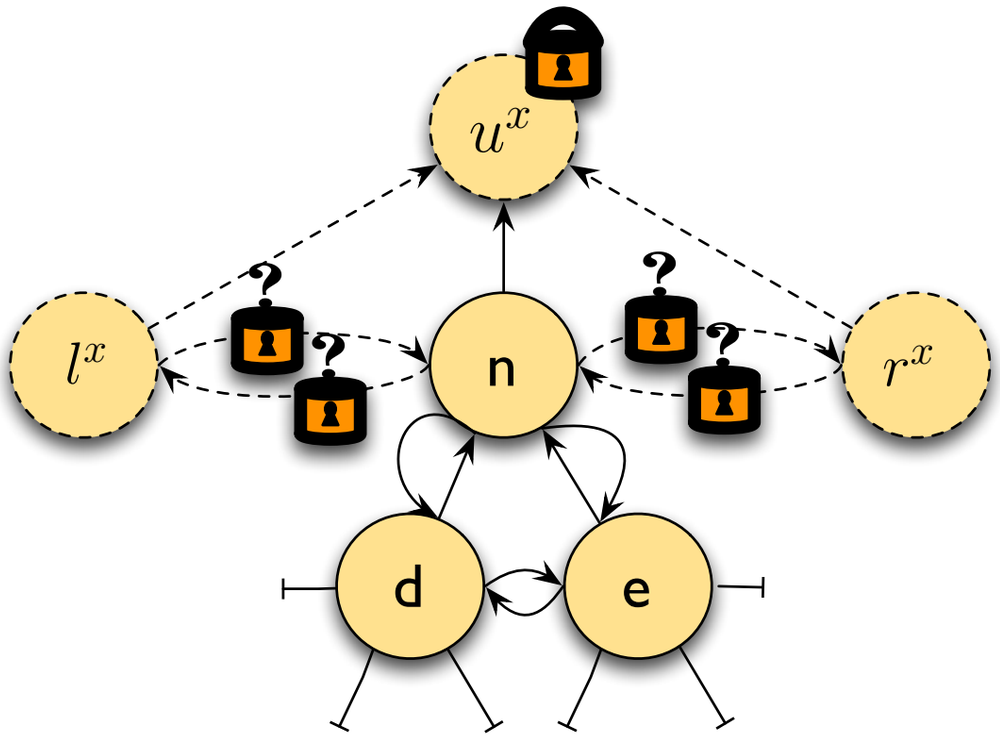




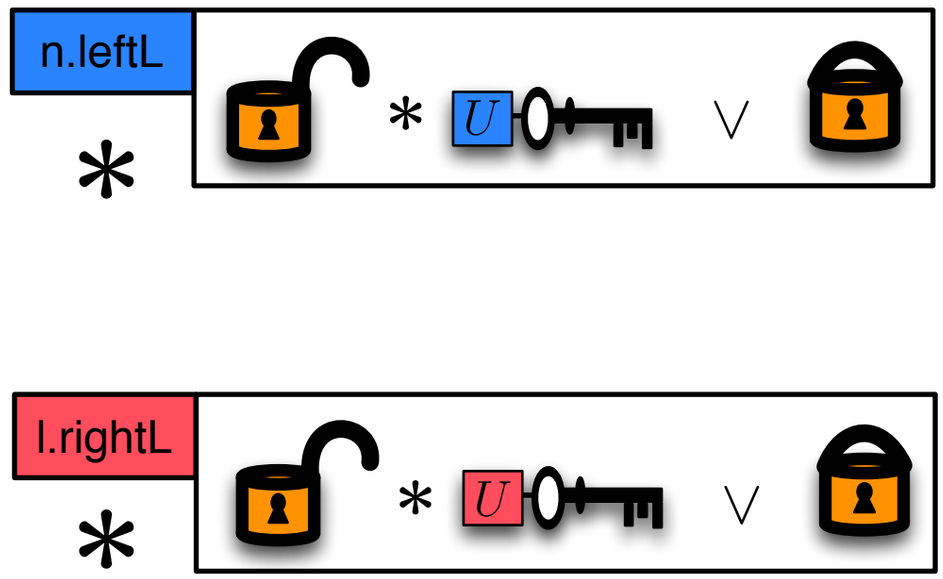
Refinement (Axiomatic Correctness)

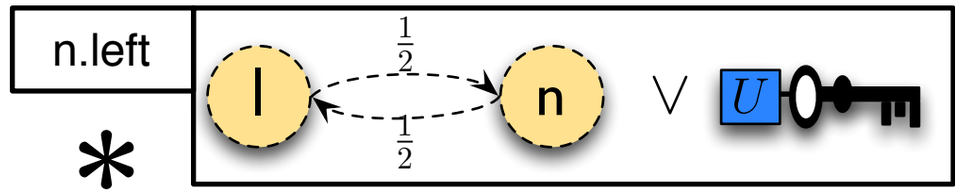
```
proc deleteTree(n){
  local l,u,d,r,ul in
    u := [n.up] ; d := [n.first] ; ul:= [n.upL] ;
    //Acquiring the necessary locks.
    lock(ul) ;
    lock(n.leftL) ; l:= [n.left] ;
    if l ≠ null then lock(l.rightL)
    else if u ≠ null then lock(u.firstL) ;
    if r ≠ null then lock(r.leftL) ;
    else if u ≠ null then lock(u.lastL) ;
    //Unlocking the acquired locks.
    if l ≠ null then unlock(l.rightL) ;
    else if u ≠ null then unlock(u.firstL) ;
    if r ≠ null then unlock(r.leftL) ;
    else if u ≠ null then unlock(u.lastL) ;
    call disposeForest(d) ;
    disposeNode(n) ;
}
```

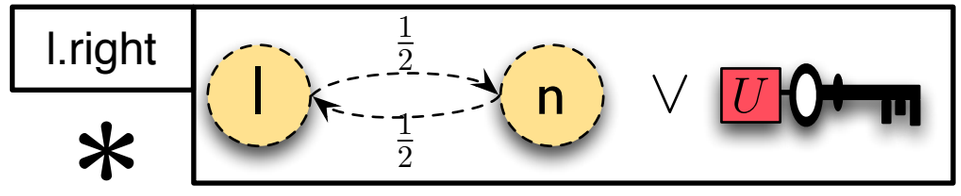
```
u:=n.up;
lock(u);
```



parentLocked(u^x)



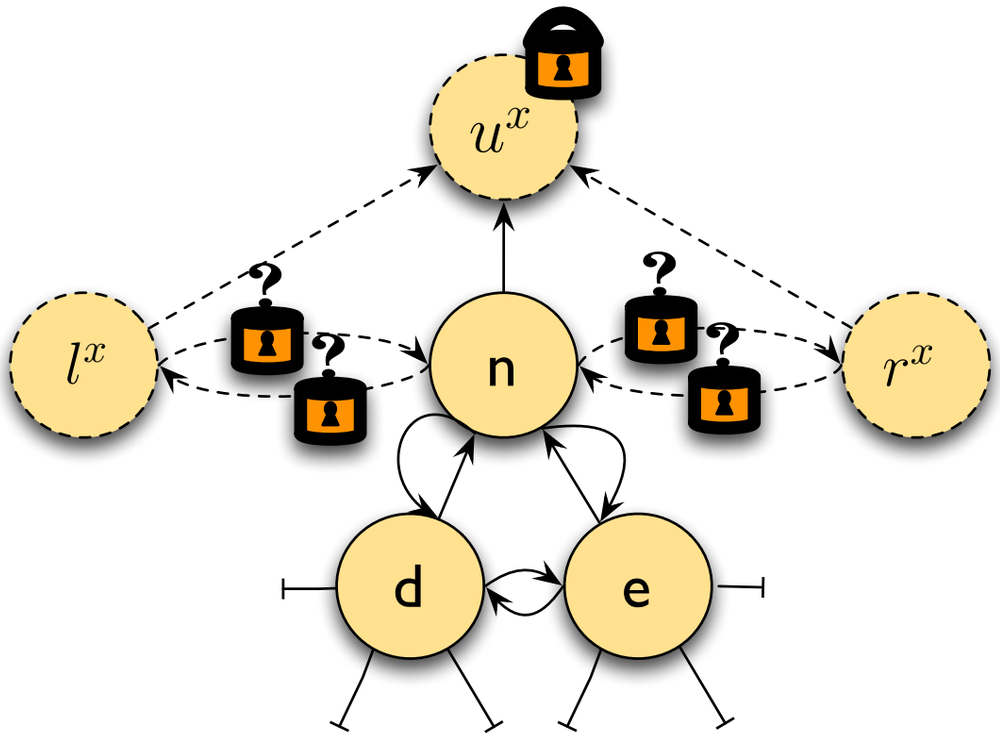




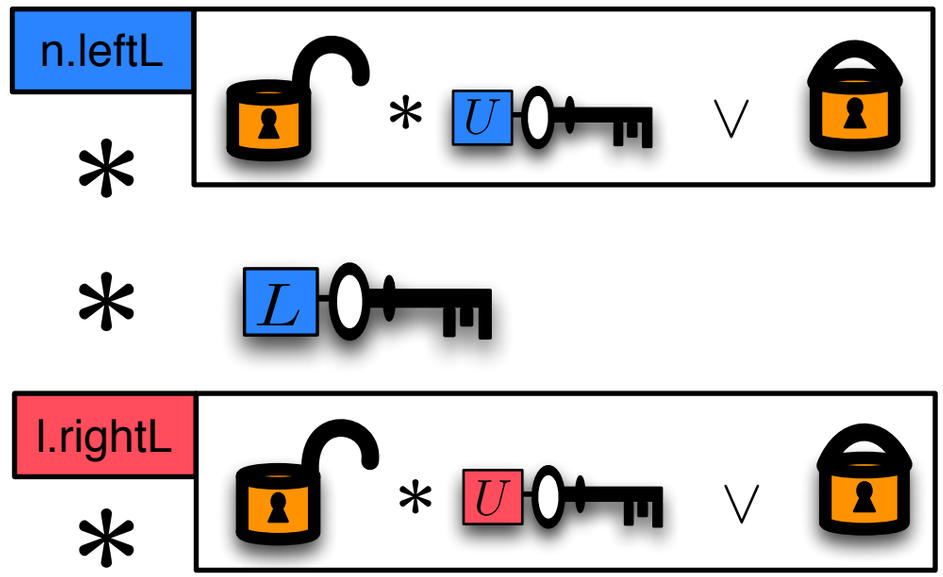
Refinement (Axiomatic Correctness)

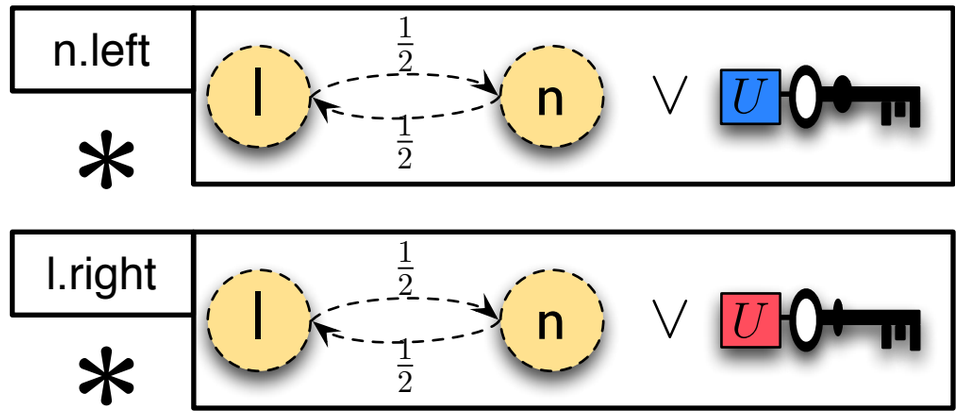
```
proc deleteTree(n){
  local l,u,d,r,ul in
    u := [n.up] ; d := [n.first] ; ul:= [n.upL] ;
    //Acquiring the necessary locks.
    lock(ul) ;
    lock(n.leftL) ; l:= [n.left] ;
    if l ≠ null then lock(l.rightL)
    else if u ≠ null then lock(u.firstL) ;
    r := [n.right] ;
    if r ≠ null then lock(r.leftL) ;
    if l ≠ null then [l.right] := r ;
    else if u ≠ null then [u.first] := r ;
    if r ≠ null then [r.left] := l ;
    else if u ≠ null then [u.last] := l ;
    //Unlocking the acquired locks.
    if l ≠ null then unlock(l.rightL) ;
    else if u ≠ null then unlock(u.firstL) ;
    if r ≠ null then unlock(r.leftL) ;
    else if u ≠ null then unlock(u.lastL) ;
    call disposeForest(d) ;
    disposeNode(n) ;
}
```

```
u:=n.up;
lock(u);
```



parentLocked(u^x)

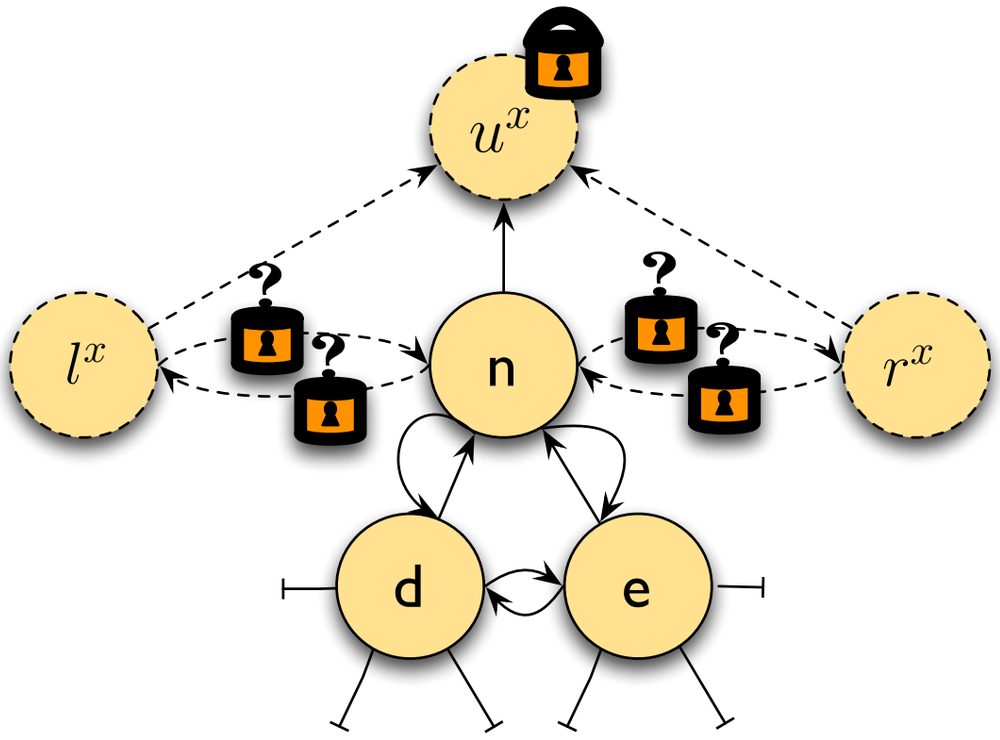




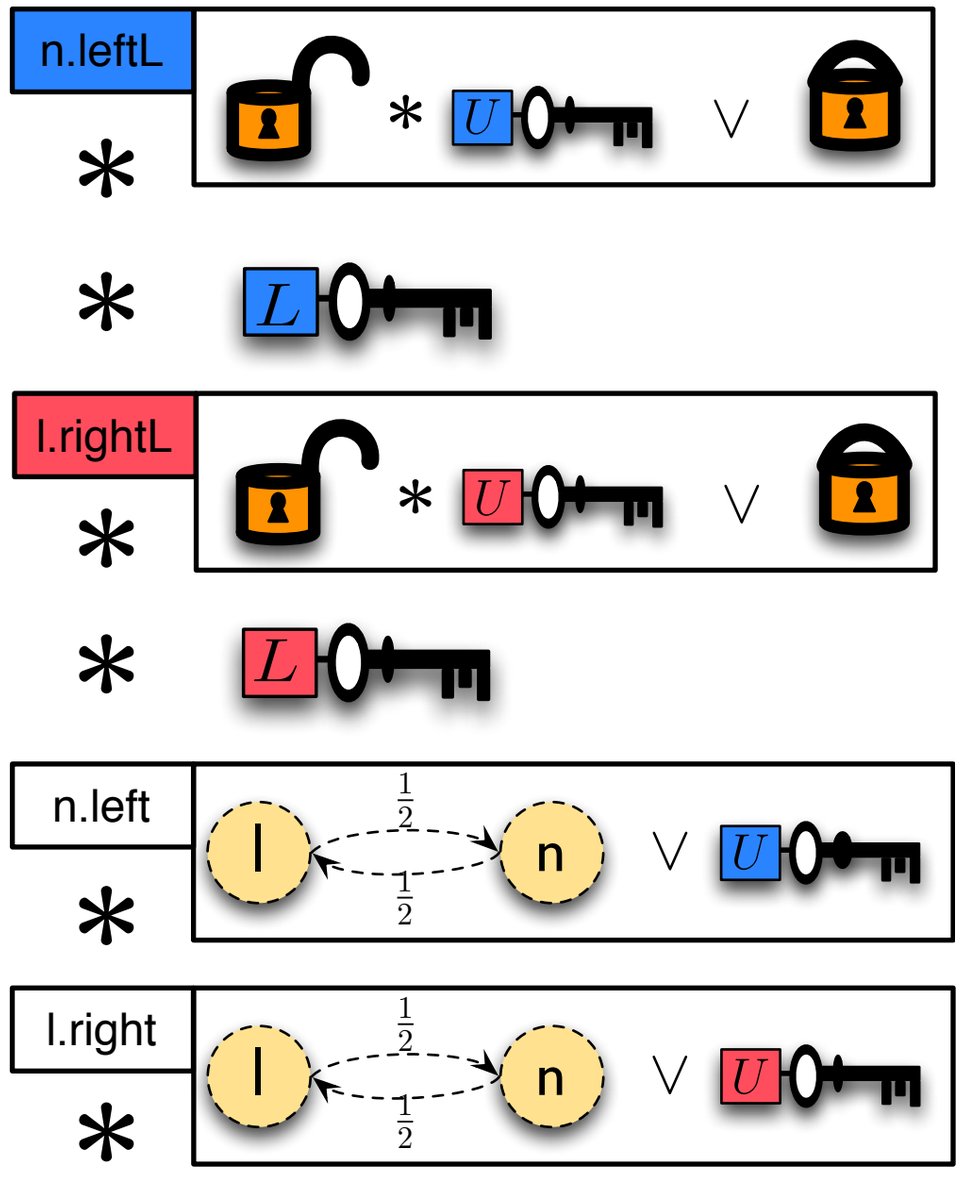
Refinement (Axiomatic Correctness)

```
proc deleteTree(n){
  local l,u,d,r,ul in
    u := [n.up] ; d := [n.first] ; ul:= [n.upL] ;
    //Acquiring the necessary locks.
    lock(ul) ;
    lock(n.leftL) ; l:= [n.left] ;
    if l ≠ null then lock(l.rightL)
    else if u ≠ null then lock(u.firstL) ;
    r := [n.right] ;
    if r ≠ null then lock(r.leftL) ;
    if l ≠ null then [l.right] := r ;
    else if u ≠ null then [u.first] := r ;
    if r ≠ null then [r.left] := l ;
    else if u ≠ null then [u.last] := l ;
    //Unlocking the acquired locks.
    if l ≠ null then unlock(l.rightL) ;
    else if u ≠ null then unlock(u.firstL) ;
    if r ≠ null then unlock(r.leftL) ;
    else if u ≠ null then unlock(u.lastL) ;
    call disposeForest(d) ;
    disposeNode(n) ;
}
```

```
u:=n.up;
lock(u);
```

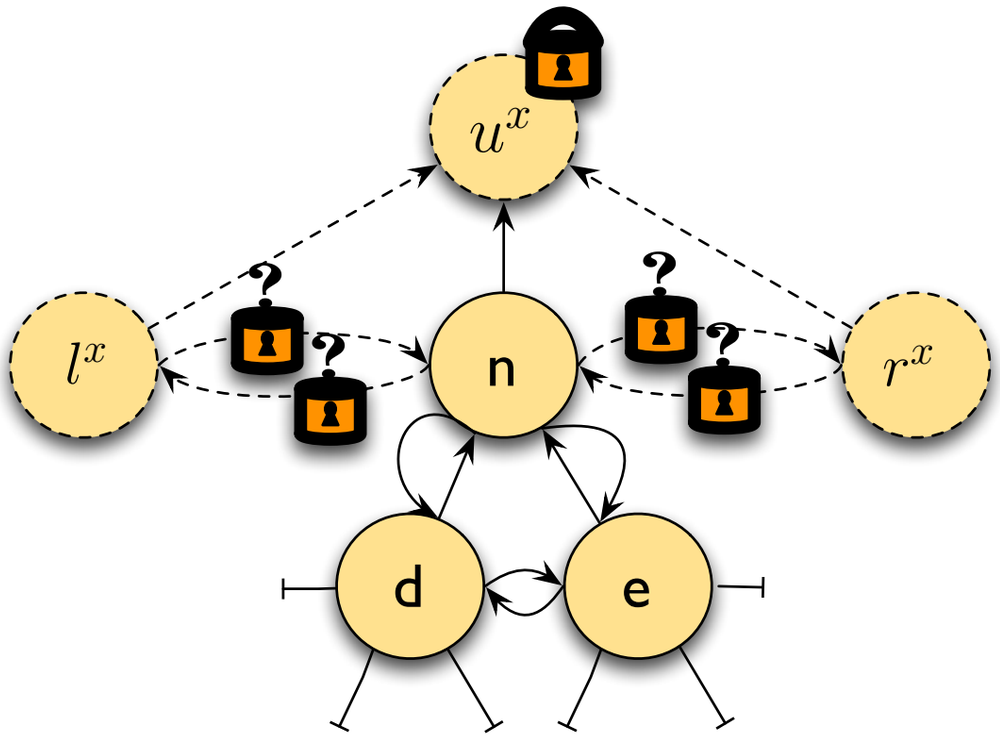


parentLocked(u^x)

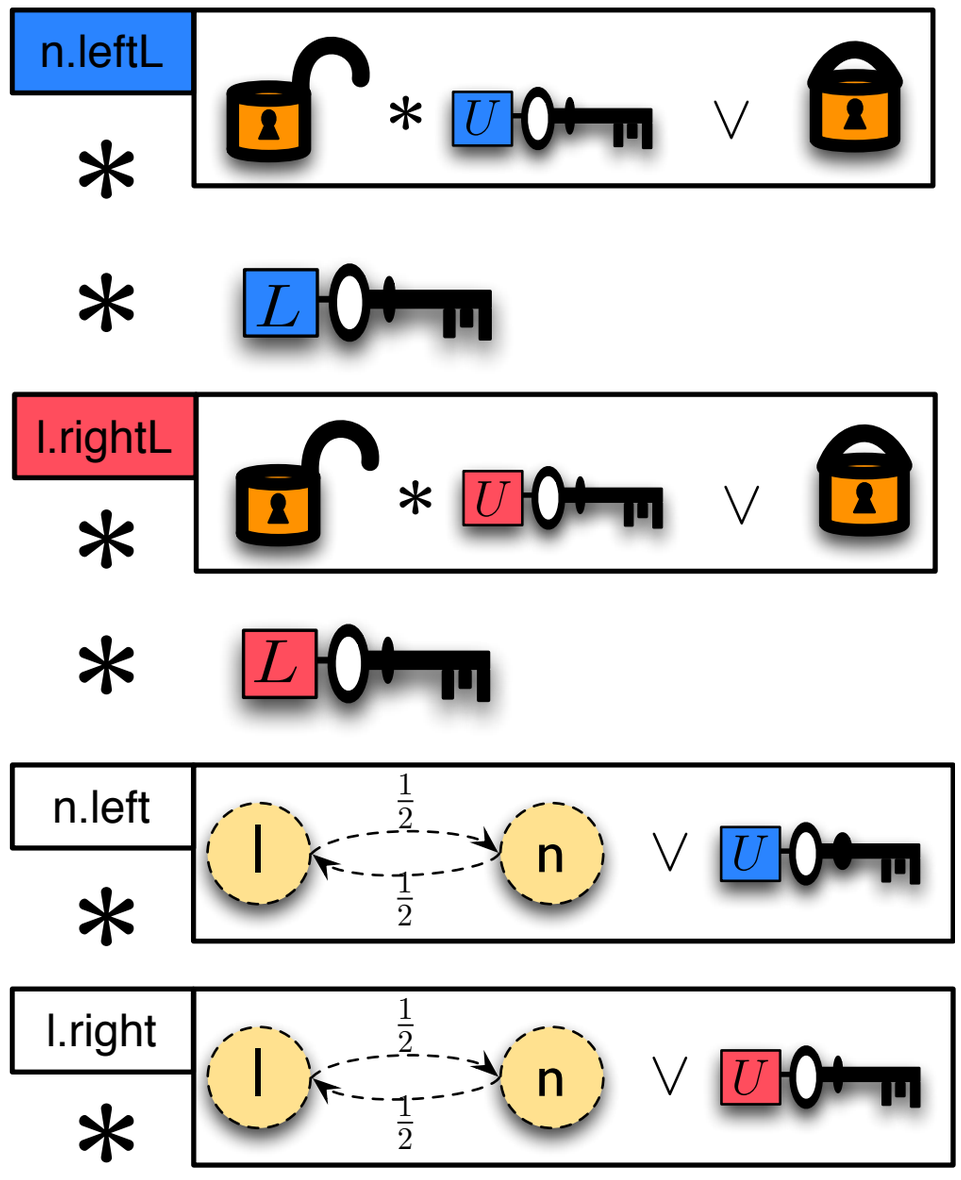


Refinement (Axiomatic Correctness)

```
proc deleteTree(n){
  local l,u,d,r,ul in
    u := [n.up] ; d := [n.first] ; ul:= [n.upL] ;
    //Acquiring the necessary locks.
    lock(ul) ;
    lock(n.leftL) ; l:= [n.left] ;
    if l ≠ null then lock(l.rightL)
lock(n.left);
    else if u ≠ null then lock(u.lastL) ;
    unlock(ul) ;
    //Pointer Swinging.
    if l ≠ null then      [l.right] := r ;
    else if u ≠ null then [u.first] := r ;
    if r ≠ null then [r.left] := l ;
    else if u ≠ null then [u.last] := l ;
    //Unlocking the acquired locks.
    if l ≠ null then unlock(l.rightL) ;
    else if u ≠ null then unlock(u.firstL) ;
    if r ≠ null then unlock(r.leftL) ;
    else if u ≠ null then unlock(u.lastL) ;
    call disposeForest(d) ;
    disposeNode(n) ;
}
```



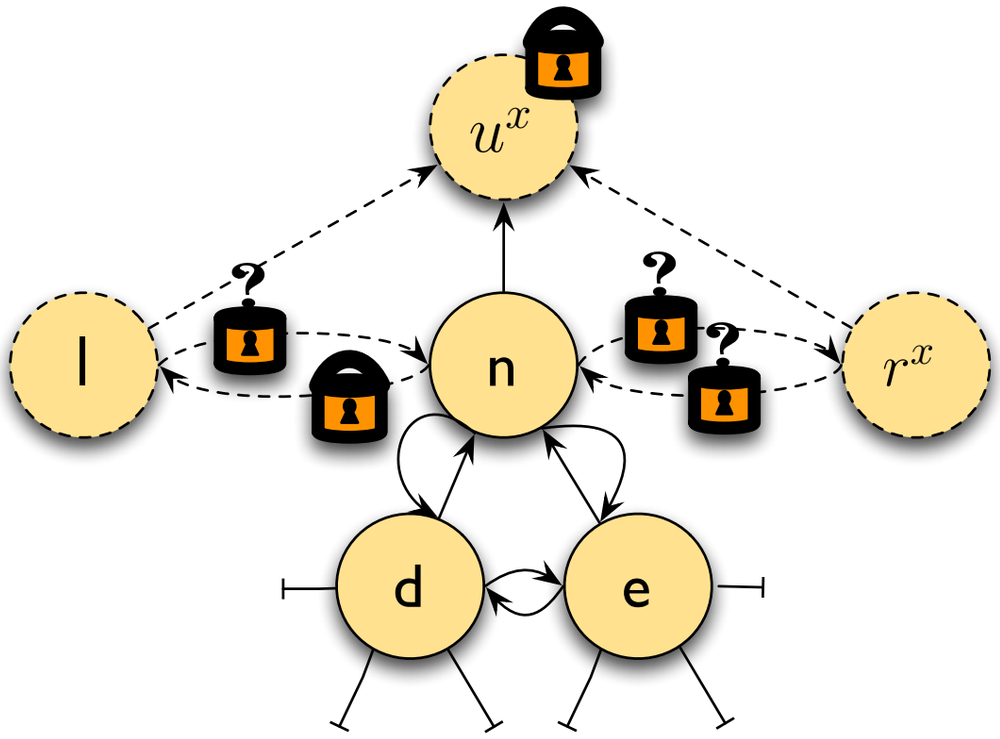
parentLocked(u^x)



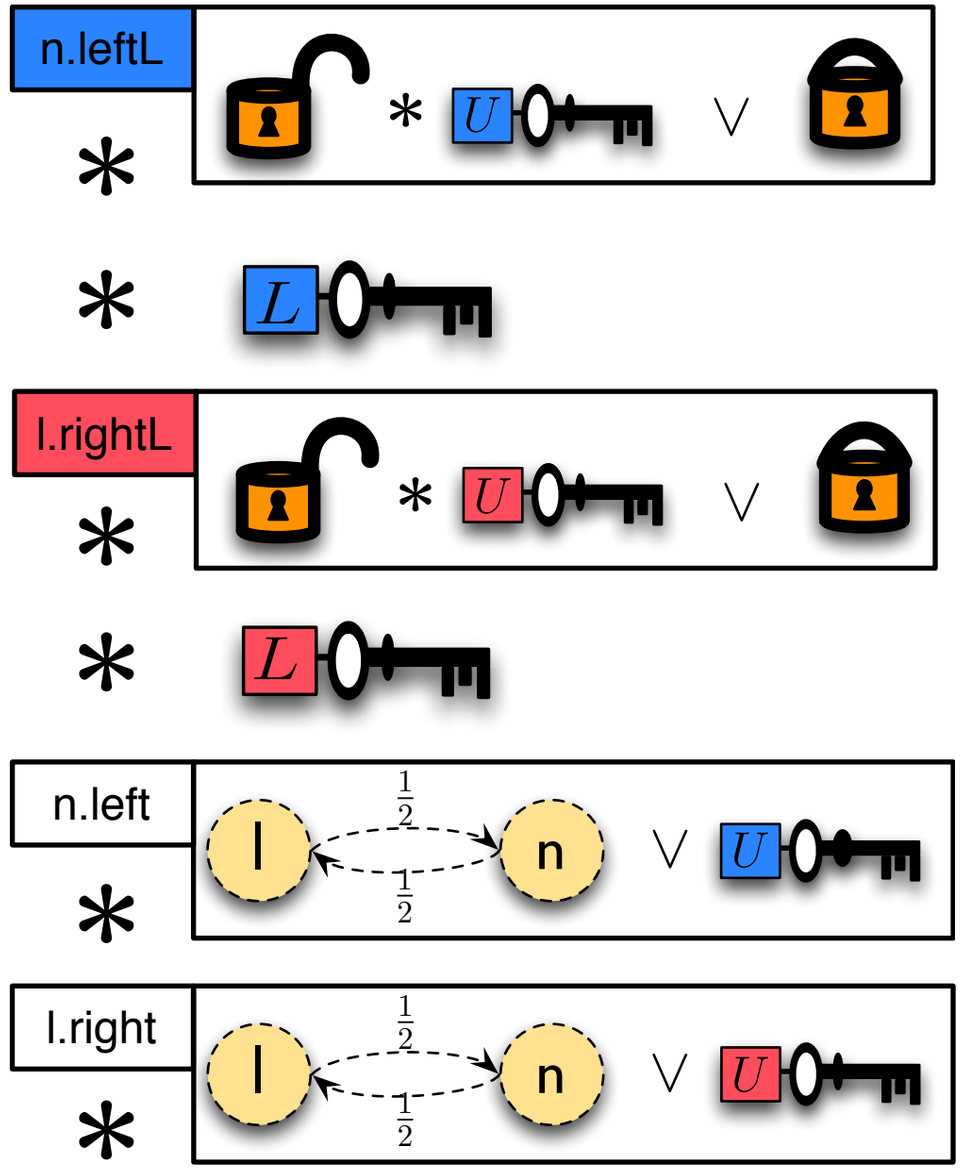
Refinement (Axiomatic Correctness)

```
proc deleteTree(n){
  local l,u,d,r,ul in
    u := [n.up] ; d := [n.first] ; ul:= [n.upL] ;
    //Acquiring the necessary locks.
    lock(ul) ;
    lock(n.leftL) ; l:= [n.left] ;
    if l ≠ null then lock(l.rightL)
    else if u ≠ null then lock(u.lastL) ;
    unlock(ul) ;
    //Pointer Swinging.
    if l ≠ null then      [l.right] := r ;
    else if u ≠ null then [u.first] := r ;
    if r ≠ null then [r.left] := l ;
    else if u ≠ null then [u.last] := l ;
    //Unlocking the acquired locks.
    if l ≠ null then unlock(l.rightL) ;
    else if u ≠ null then unlock(u.firstL) ;
    if r ≠ null then unlock(r.leftL) ;
    else if u ≠ null then unlock(u.lastL) ;
    call disposeForest(d) ;
    disposeNode(n) ;
}
```

lock(n.left);



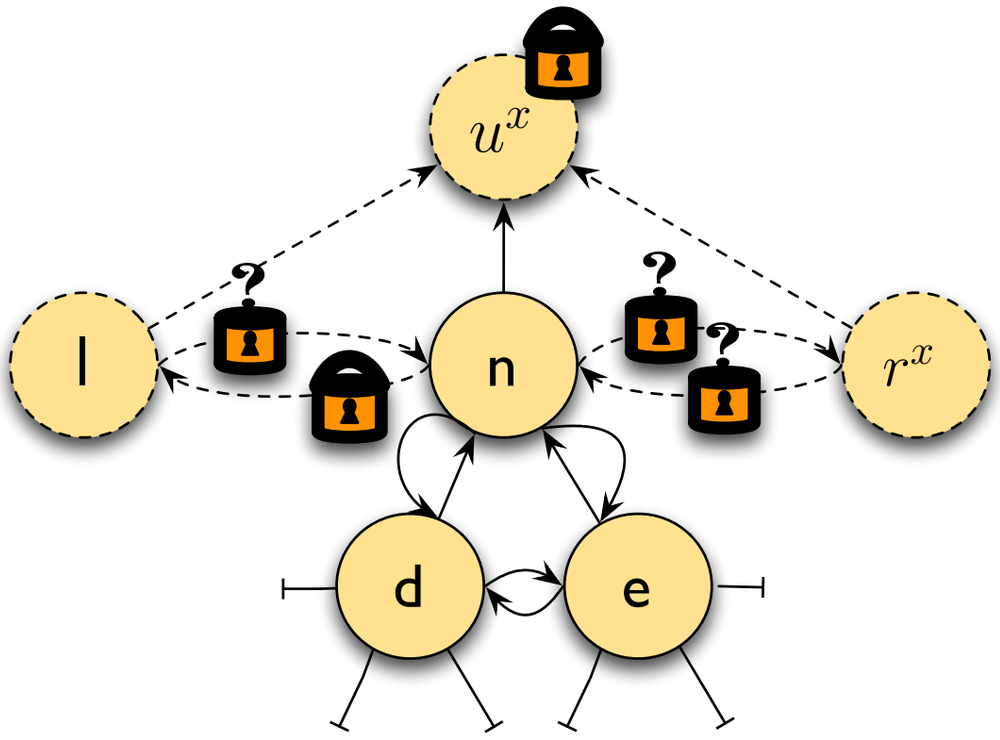
parentLocked(u^x)



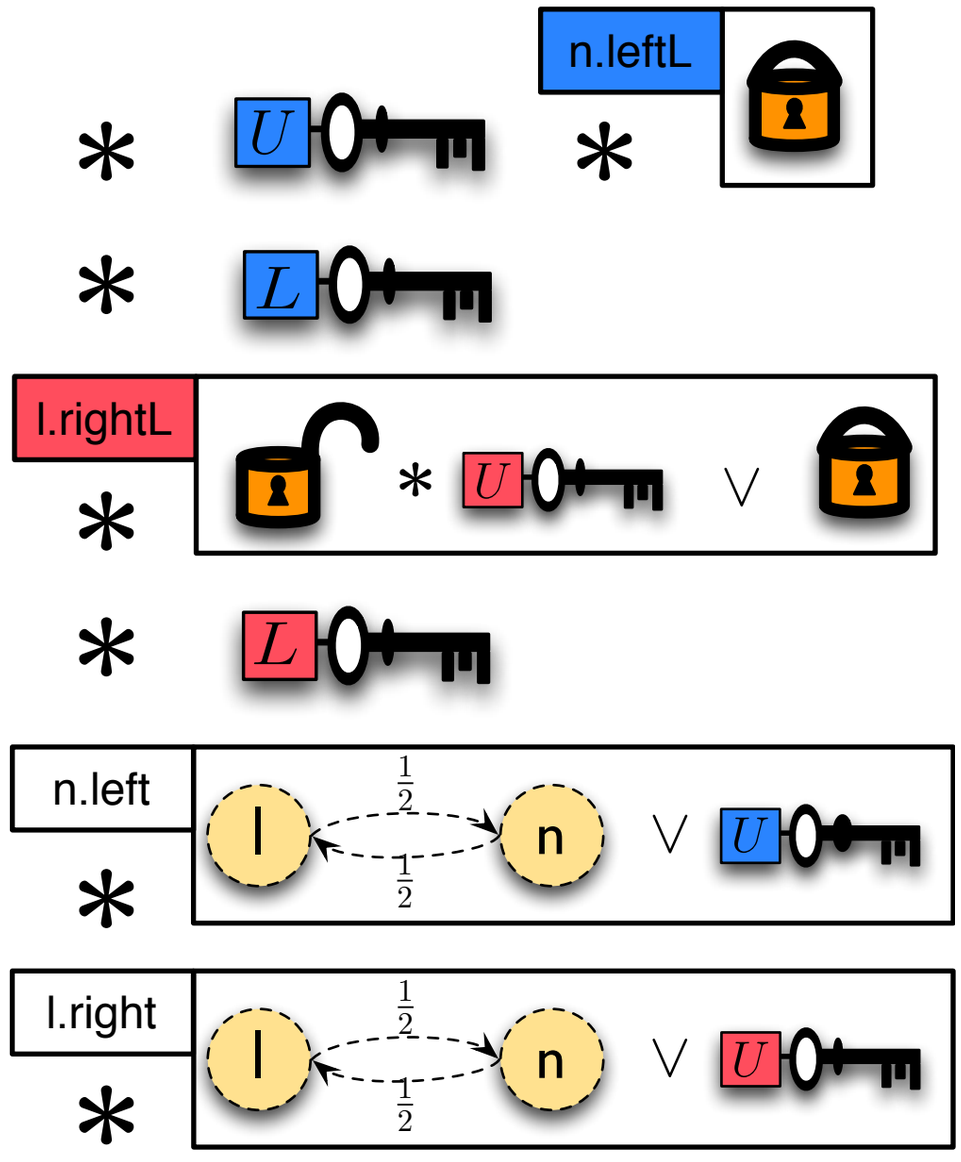
Refinement (Axiomatic Correctness)

```
proc deleteTree(n){
  local l,u,d,r,ul in
    u := [n.up] ; d := [n.first] ; ul:= [n.upL] ;
    //Acquiring the necessary locks.
    lock(ul) ;
    lock(n.leftL) ; l:= [n.left] ;
    if l ≠ null then lock(l.rightL)
    else if u ≠ null then lock(u.lastL) ;
    unlock(ul) ;
    //Pointer Swinging.
    if l ≠ null then      [l.right] := r ;
    else if u ≠ null then [u.first] := r ;
    if r ≠ null then [r.left] := l ;
    else if u ≠ null then [u.last] := l ;
    //Unlocking the acquired locks.
    if l ≠ null then unlock(l.rightL) ;
    else if u ≠ null then unlock(u.firstL) ;
    if r ≠ null then unlock(r.leftL) ;
    else if u ≠ null then unlock(u.lastL) ;
    call disposeForest(d) ;
    disposeNode(n) ;
}
```

lock(n.left);



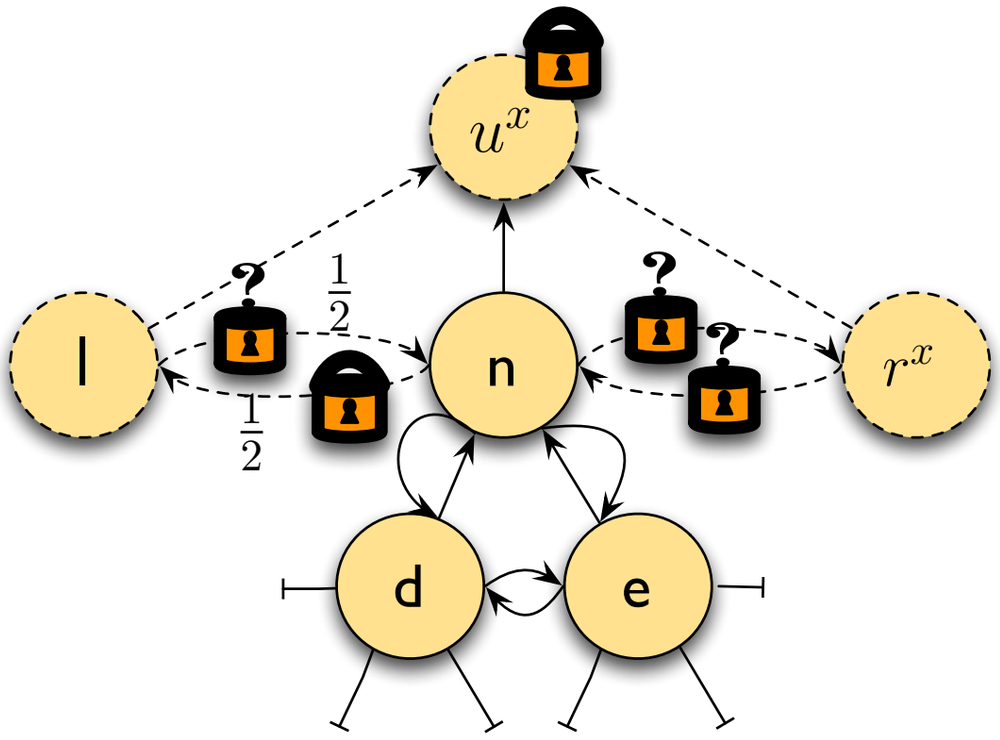
parentLocked(u^x)



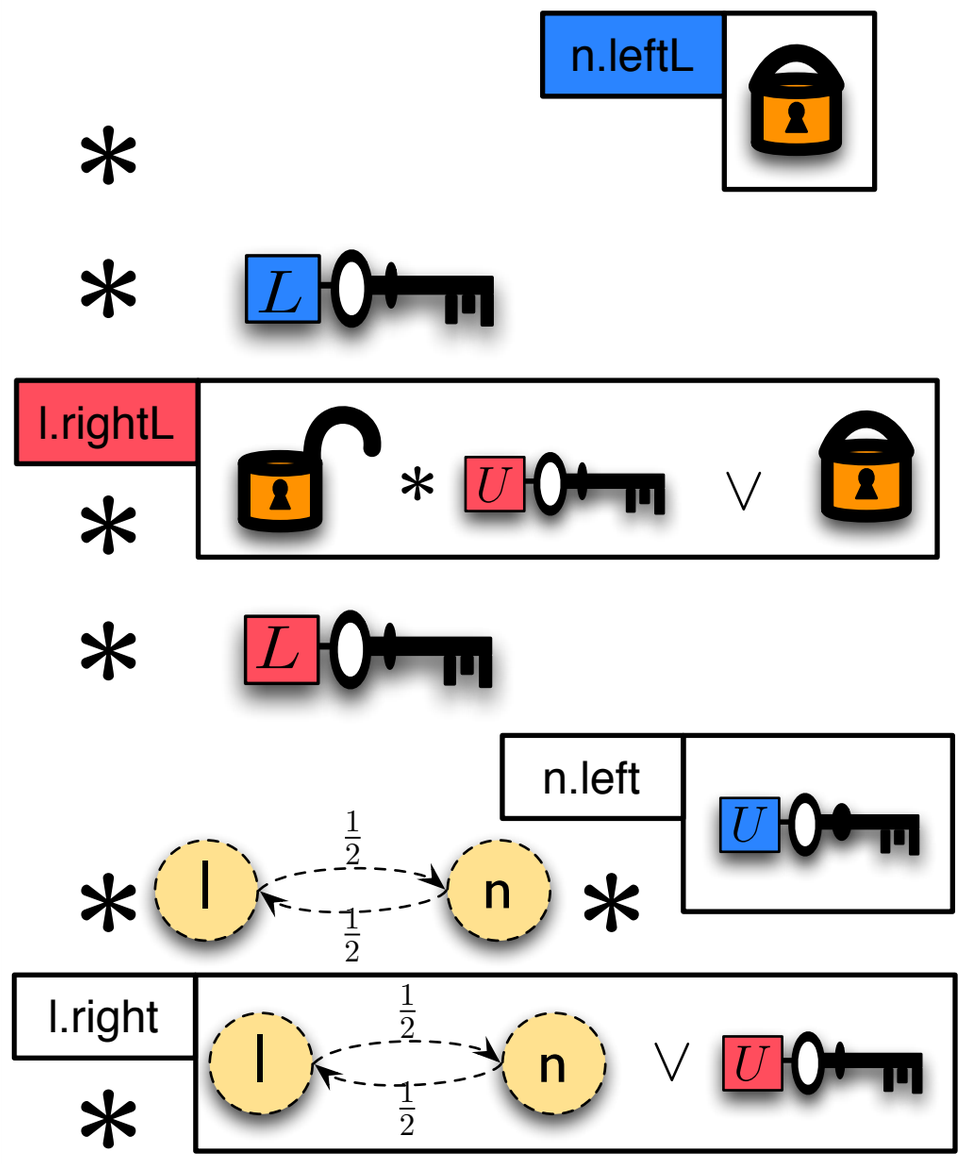
Refinement (Axiomatic Correctness)

```
proc deleteTree(n){
  local l,u,d,r,ul in
    u := [n.up] ; d := [n.first] ; ul:= [n.upL] ;
    //Acquiring the necessary locks.
    lock(ul) ;
    lock(n.leftL) ; l:= [n.left] ;
    if l ≠ null then lock(l.rightL)
    else if u ≠ null then lock(u.lastL) ;
    unlock(ul) ;
    //Pointer Swinging.
    if l ≠ null then      [l.right] := r ;
    else if u ≠ null then [u.first] := r ;
    if r ≠ null then [r.left] := l ;
    else if u ≠ null then [u.last] := l ;
    //Unlocking the acquired locks.
    if l ≠ null then unlock(l.rightL) ;
    else if u ≠ null then unlock(u.firstL) ;
    if r ≠ null then unlock(r.leftL) ;
    else if u ≠ null then unlock(u.lastL) ;
    call disposeForest(d) ;
    disposeNode(n) ;
}
```

lock(n.left);



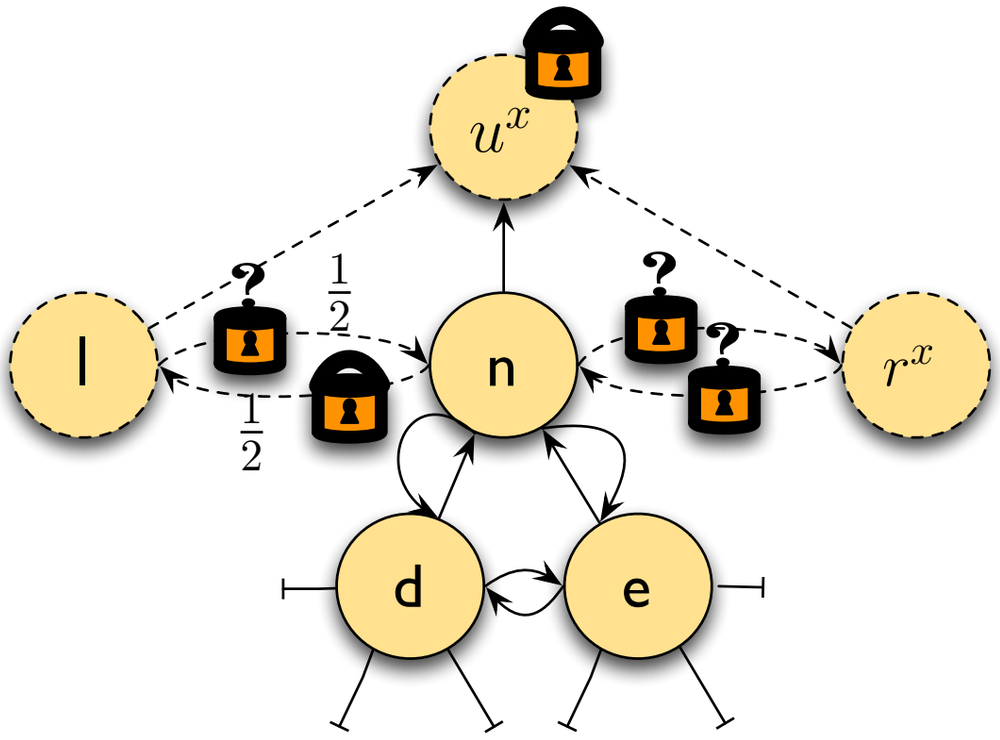
parentLocked(u^x)



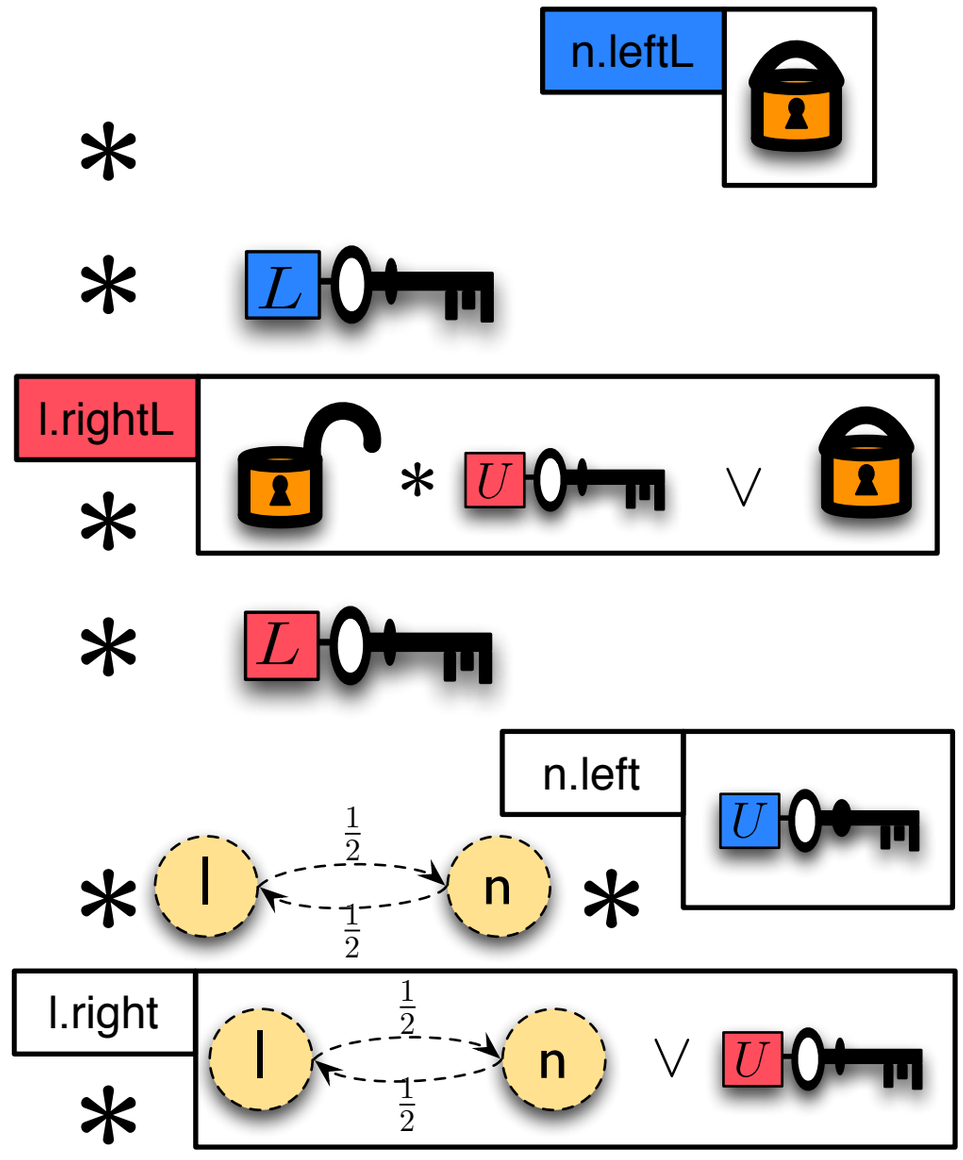
Refinement (Axiomatic Correctness)

```
proc deleteTree(n){
  local l,u,d,r,ul in
    u := [n.up] ; d := [n.first] ; ul:= [n.upL] ;
    //Acquiring the necessary locks.
    lock(ul) ;
    lock(n.leftL) ; l:= [n.left] ;
    if l ≠ null then lock(l.rightL)
    else if u ≠ null then lock(u.firstL) ;
    //Pointer Swinging.
    if l ≠ null then     [l.right] := r ;
    else if u ≠ null then [u.first] := r ;
    if r ≠ null then [r.left] := l ;
    else if u ≠ null then [u.last] := l ;
    //Unlocking the acquired locks.
    if l ≠ null then unlock(l.rightL) ;
    else if u ≠ null then unlock(u.firstL) ;
    if r ≠ null then unlock(r.leftL) ;
    else if u ≠ null then unlock(u.lastL) ;
    call disposeForest(d) ;
    disposeNode(n) ;
}
```

```
l:= n.left;
lock(l.right);
```



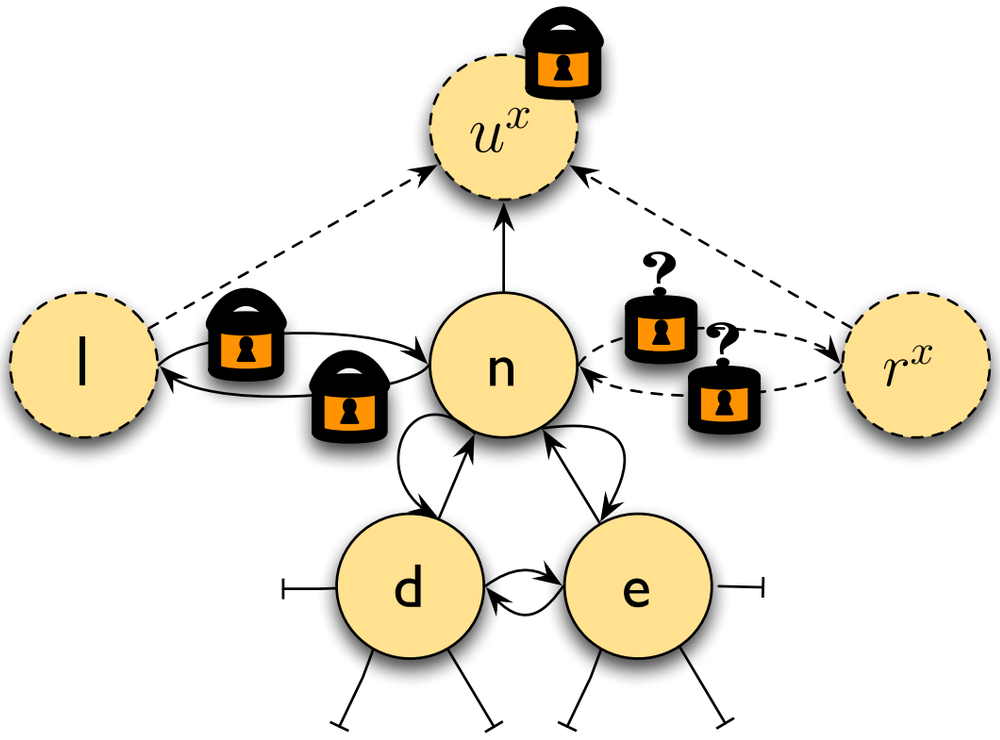
parentLocked(u^x)



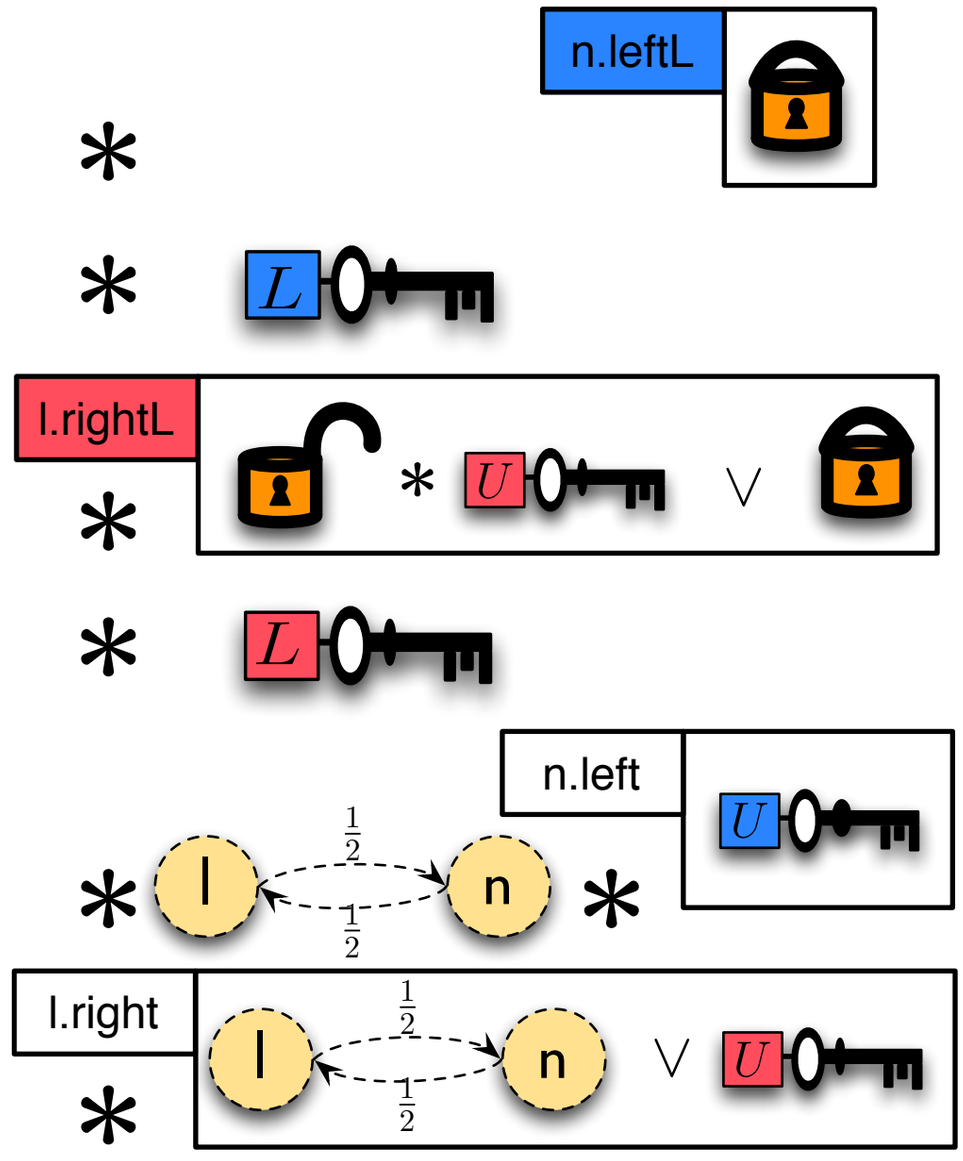
Refinement (Axiomatic Correctness)

```
proc deleteTree(n){
  local l,u,d,r,ul in
    u := [n.up] ; d := [n.first] ; ul:= [n.upL] ;
    //Acquiring the necessary locks.
    lock(ul) ;
    lock(n.leftL) ; l:= [n.left] ;
    if l ≠ null then lock(l.rightL)
    else if u ≠ null then lock(u.firstL) ;
    //Pointer Swinging.
    if l ≠ null then     [l.right] := r ;
    else if u ≠ null then [u.first] := r ;
    if r ≠ null then [r.left] := l ;
    else if u ≠ null then [u.last] := l ;
    //Unlocking the acquired locks.
    if l ≠ null then unlock(l.rightL) ;
    else if u ≠ null then unlock(u.firstL) ;
    if r ≠ null then unlock(r.leftL) ;
    else if u ≠ null then unlock(u.lastL) ;
    call disposeForest(d) ;
    disposeNode(n) ;
}
```

```
l:= n.left;
lock(l.right);
```



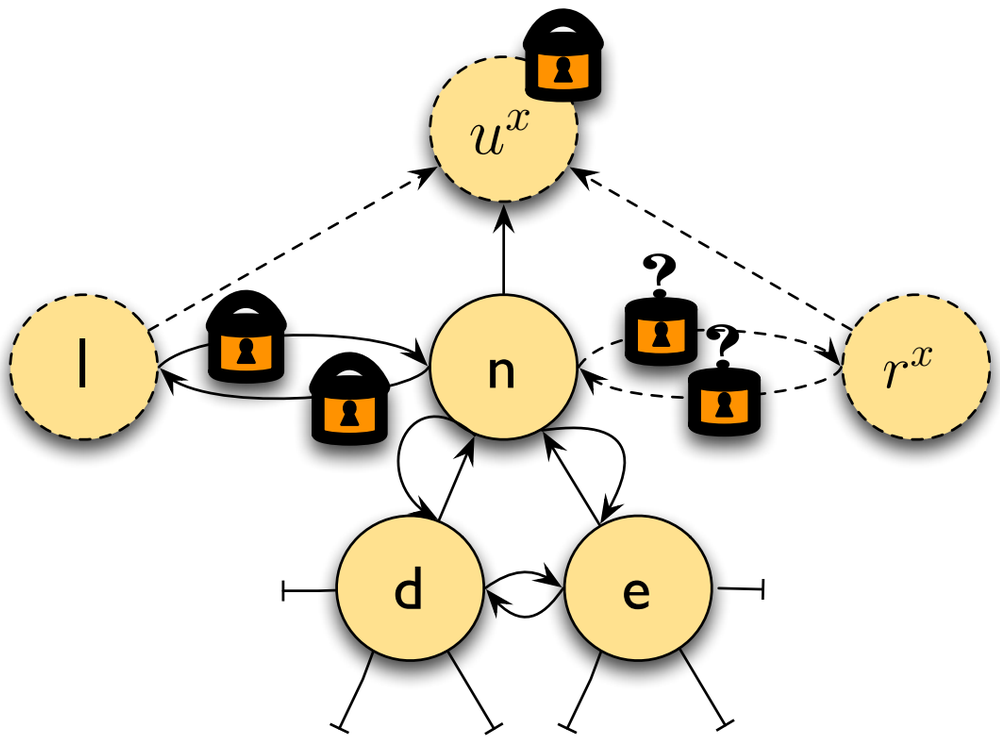
parentLocked(u^x)



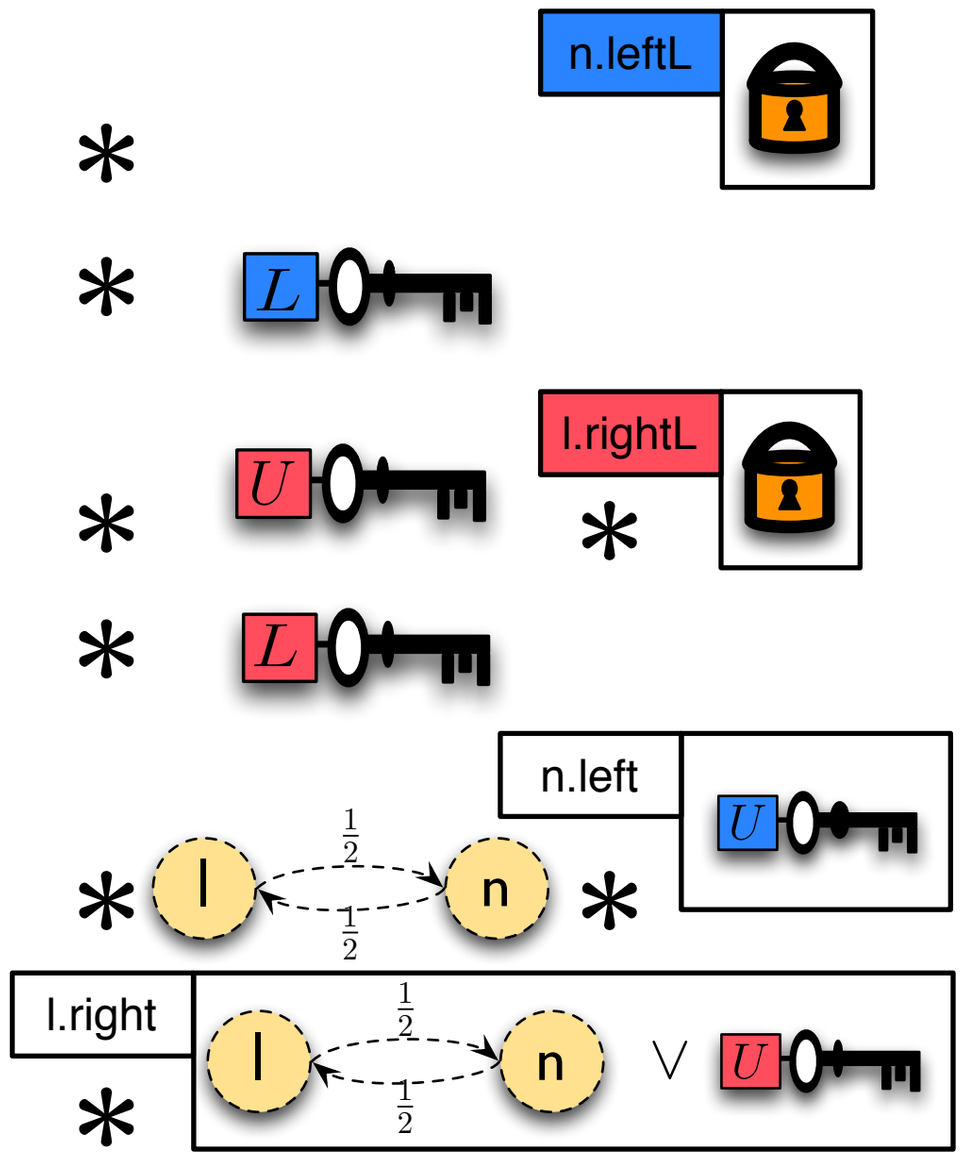
Refinement (Axiomatic Correctness)

```
proc deleteTree(n){
  local l,u,d,r,ul in
    u := [n.up] ; d := [n.first] ; ul:= [n.upL] ;
    //Acquiring the necessary locks.
    lock(ul) ;
    lock(n.leftL) ; l:= [n.left] ;
    if l ≠ null then lock(l.rightL)
    else if u ≠ null then lock(u.firstL) ;
    //Pointer Swinging.
    if l ≠ null then     [l.right] := r ;
    else if u ≠ null then [u.first] := r ;
    if r ≠ null then [r.left] := l ;
    else if u ≠ null then [u.last] := l ;
    //Unlocking the acquired locks.
    if l ≠ null then unlock(l.rightL) ;
    else if u ≠ null then unlock(u.firstL) ;
    if r ≠ null then unlock(r.leftL) ;
    else if u ≠ null then unlock(u.lastL) ;
    call disposeForest(d) ;
    disposeNode(n) ;
}
```

```
l:= n.left;
lock(l.right);
```



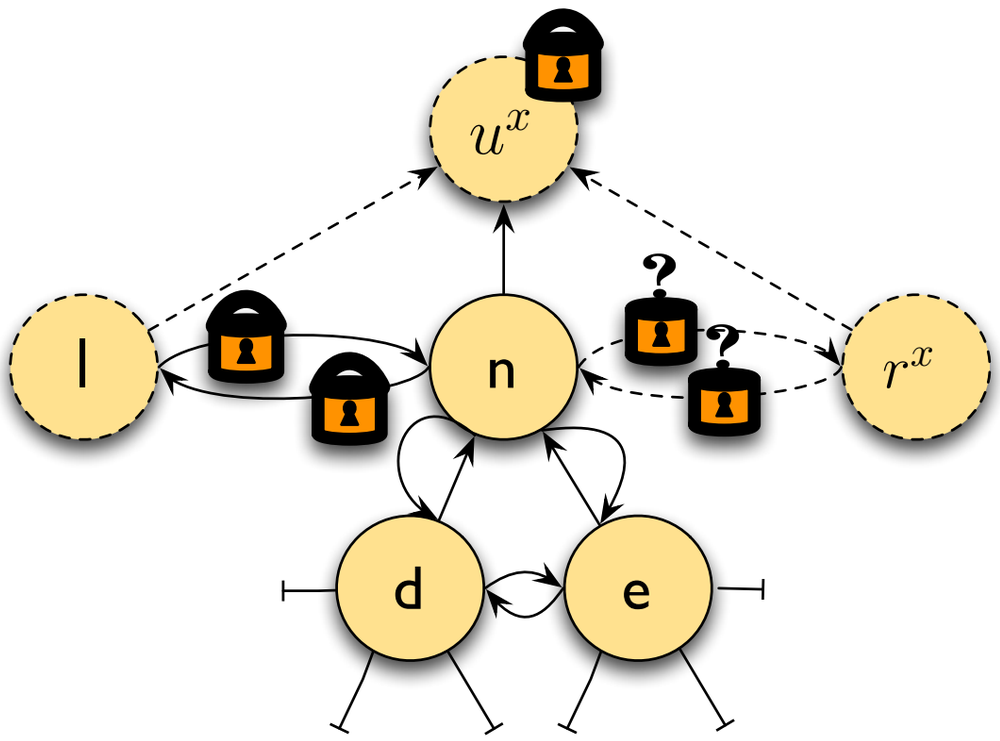
parentLocked(u^x)



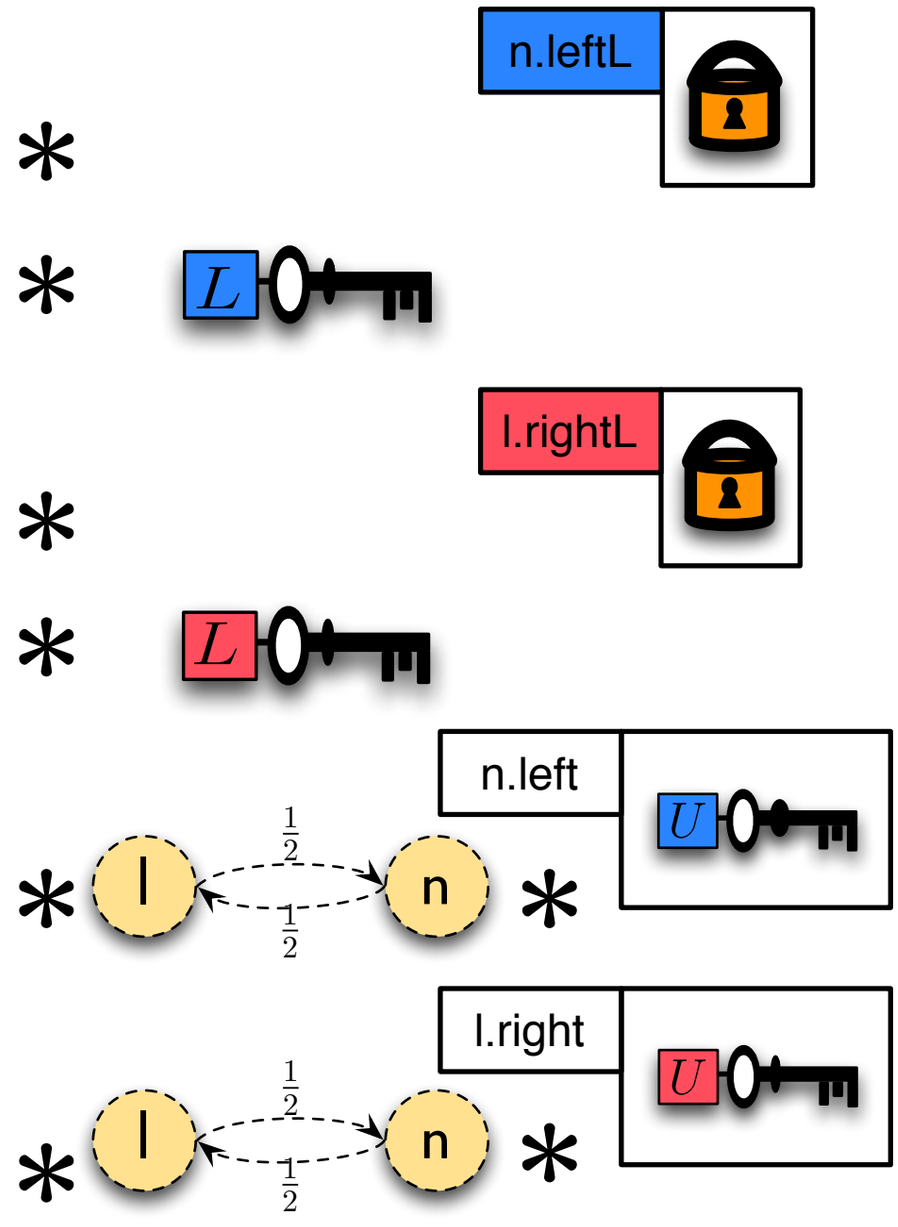
Refinement (Axiomatic Correctness)

```
proc deleteTree(n){
  local l,u,d,r,ul in
    u := [n.up] ; d := [n.first] ; ul:= [n.upL] ;
    //Acquiring the necessary locks.
    lock(ul) ;
    lock(n.leftL) ; l:= [n.left] ;
    if l ≠ null then lock(l.rightL)
    else if u ≠ null then lock(u.firstL) ;
    //Pointer Swinging.
    if l ≠ null then     [l.right] := r ;
    else if u ≠ null then [u.first] := r ;
    if r ≠ null then [r.left] := l ;
    else if u ≠ null then [u.last] := l ;
    //Unlocking the acquired locks.
    if l ≠ null then unlock(l.rightL) ;
    else if u ≠ null then unlock(u.firstL) ;
    if r ≠ null then unlock(r.leftL) ;
    else if u ≠ null then unlock(u.lastL) ;
    call disposeForest(d) ;
    disposeNode(n) ;
}
```

```
l:= n.left;
lock(l.right);
```

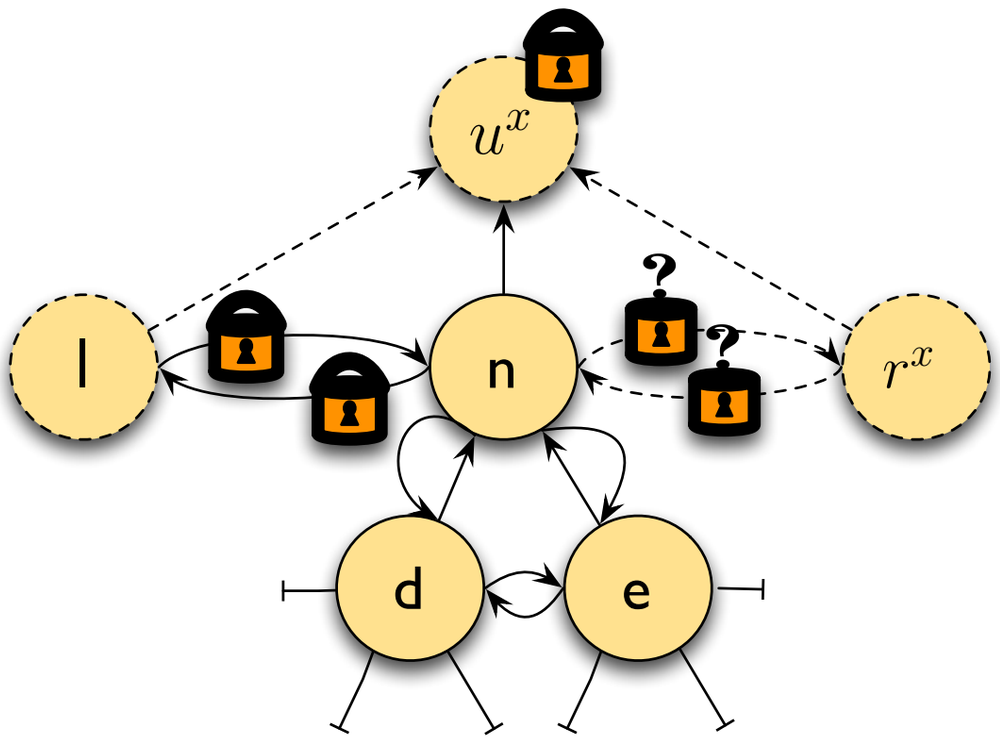


parentLocked(u^x)

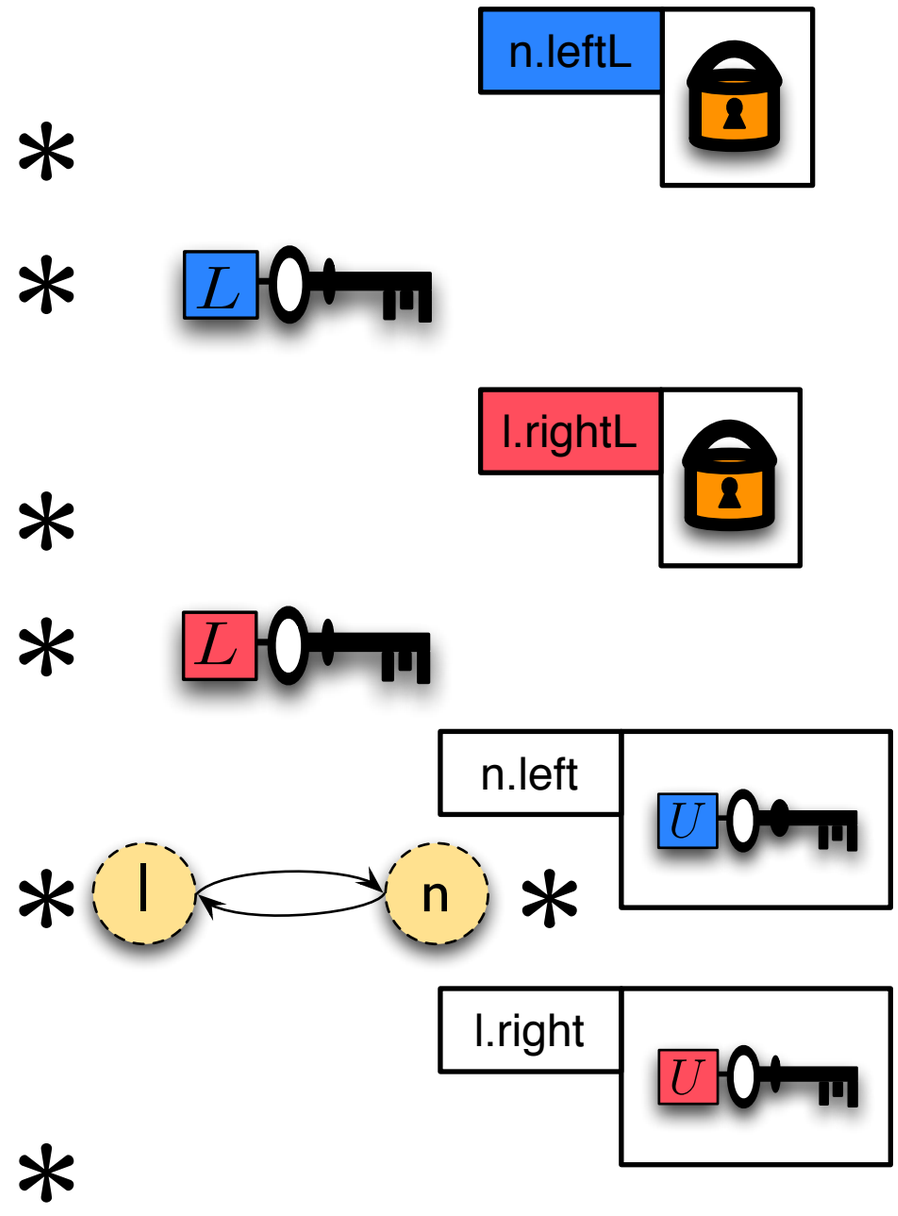


Refinement (Axiomatic Correctness)

```
proc deleteTree(n){
  local l,u,d,r,ul in
    u := [n.up] ; d := [n.first] ; ul:= [n.upL] ;
    //Acquiring the necessary locks.
    lock(ul) ;
    lock(n.leftL) ; l:= [n.left] ;
    if l ≠ null then lock(l.rightL)
    else if u ≠ null then lock(u.firstL) ;
    lock(n.rightL) ; r:= [n.right] ;
    if r ≠ null then lock(r.leftL) ;
    else if u ≠ null then lock(u.lastL) ;
    unlock(ul) ;
    //Pointer Swinging.
    if l ≠ null then      [l.right] := r ;
    else if u ≠ null then [u.first] := r ;
    if r ≠ null then [r.left] := l ;
    else if u ≠ null then [u.last] := l ;
    //Unlocking the acquired locks.
    if l ≠ null then unlock(l.rightL) ;
    else if u ≠ null then unlock(u.firstL) ;
    if r ≠ null then unlock(r.leftL) ;
    else if u ≠ null then unlock(u.lastL) ;
    call disposeForest(d) ;
    disposeNode(n) ;
}
```

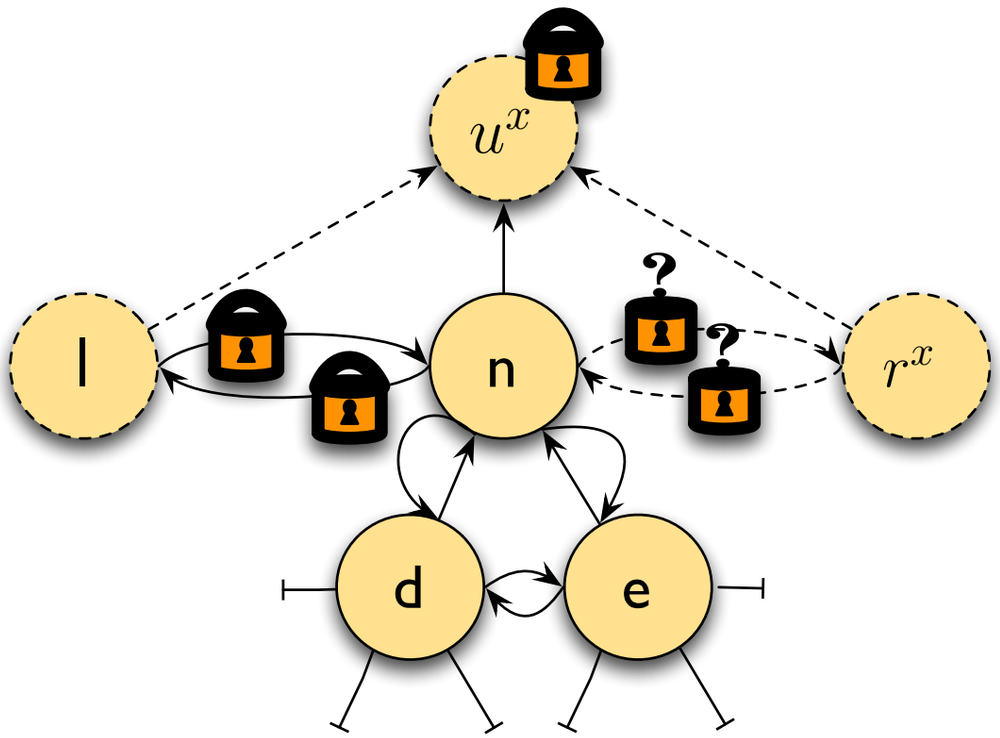


parentLocked(u^x)

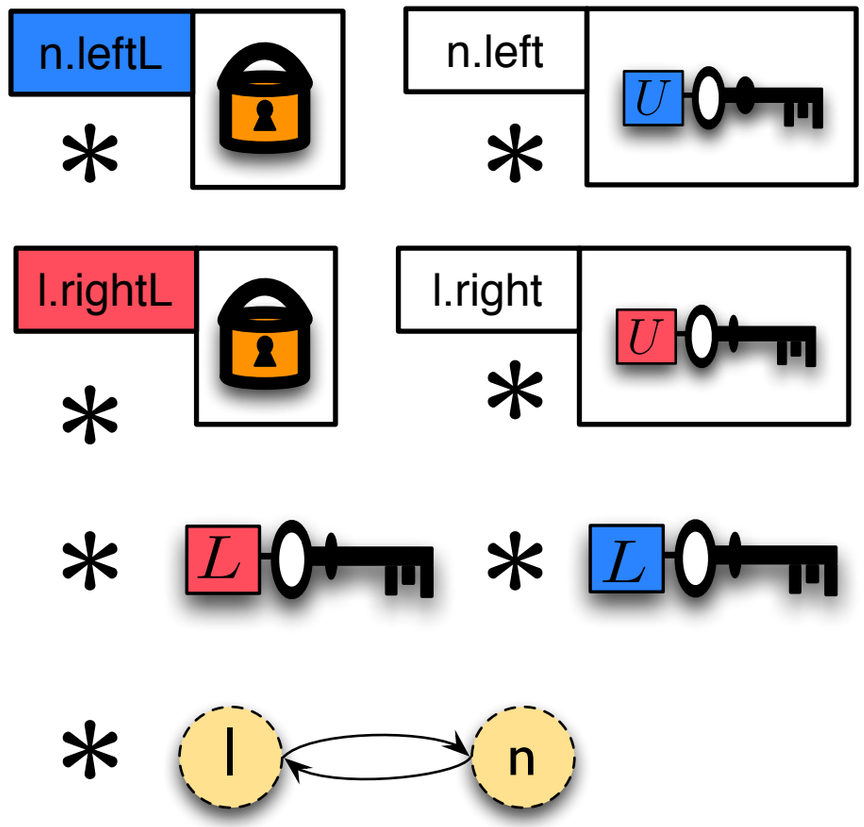


Refinement (Axiomatic Correctness)

```
proc deleteTree(n){
  local l,u,d,r,ul in
    u := [n.up] ; d := [n.first] ; ul:= [n.upL] ;
    //Acquiring the necessary locks.
    lock(ul) ;
    lock(n.leftL) ; l:= [n.left] ;
    if l ≠ null then lock(l.rightL)
    else if u ≠ null then lock(u.firstL) ;
    lock(n.rightL) ; r:= [n.right] ;
    if r ≠ null then lock(r.leftL) ;
    else if u ≠ null then lock(u.lastL) ;
    unlock(ul) ;
    //Pointer Swinging.
    if l ≠ null then      [l.right] := r ;
    else if u ≠ null then [u.first] := r ;
    if r ≠ null then [r.left] := l ;
    else if u ≠ null then [u.last] := l ;
    //Unlocking the acquired locks.
    if l ≠ null then unlock(l.rightL) ;
    else if u ≠ null then unlock(u.firstL) ;
    if r ≠ null then unlock(r.leftL) ;
    else if u ≠ null then unlock(u.lastL) ;
    call disposeForest(d) ;
    disposeNode(n) ;
}
```



parentLocked(u^x)



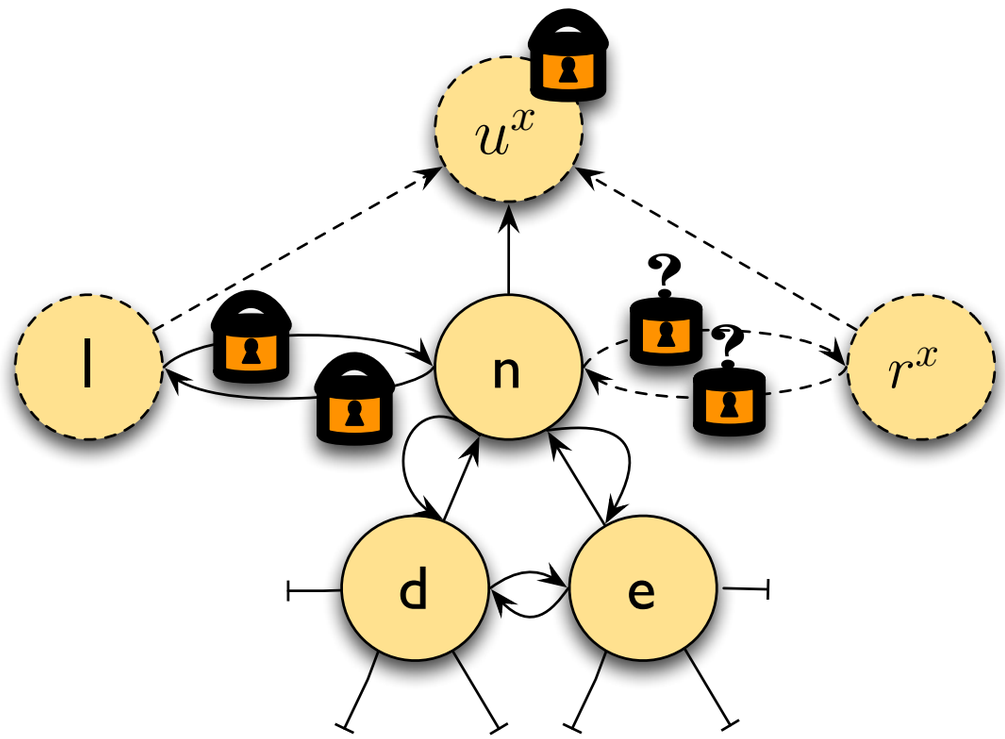
Refinement (Axiomatic Correctness)

```
proc deleteTree(n){
  local l,u,d,r,ul in
    u := [n.up] ; d := [n.first] ; ul:= [n.upL] ;
    //Acquiring the necessary locks.
    lock(ul) ;
    lock(n.leftL) ; l:= [n.left] ;
    if l ≠ null then lock(l.rightL)
    else if u ≠ null then lock(u.firstL) ;
```

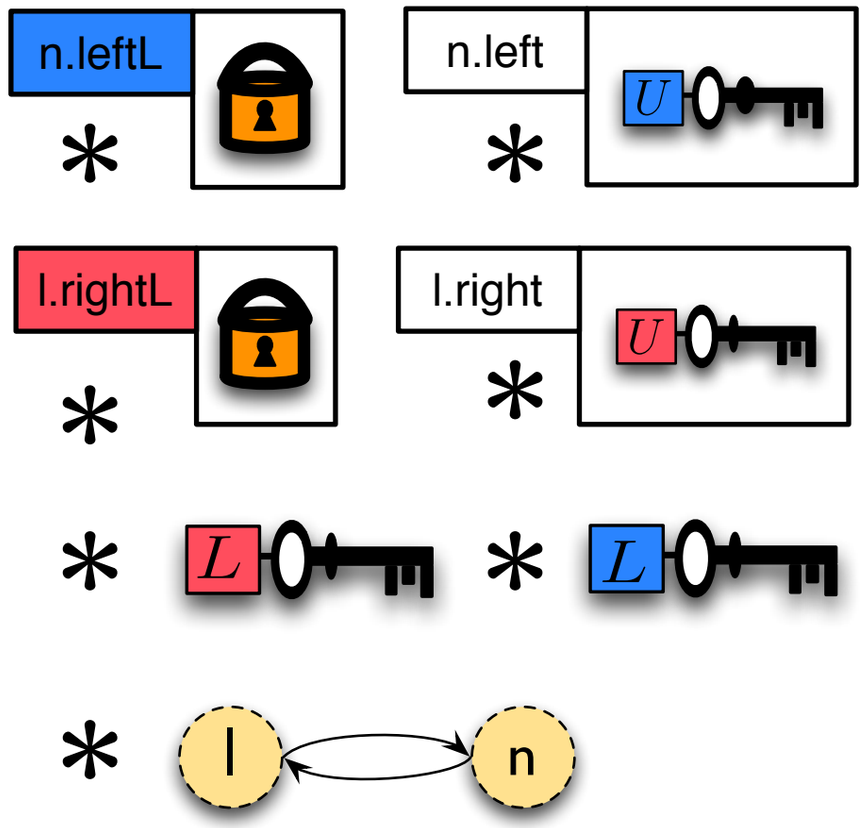
```
//Do the same for RHS

lock(n.right);
r:= n.right;
lock(r.left);
```

```
    if l ≠ null then unlock(l.rightL) ;
    else if u ≠ null then unlock(u.firstL) ;
    if r ≠ null then unlock(r.leftL) ;
    else if u ≠ null then unlock(u.lastL) ;
    call disposeForest(d) ;
    disposeNode(n) ;
}
```



parentLocked(u^x)



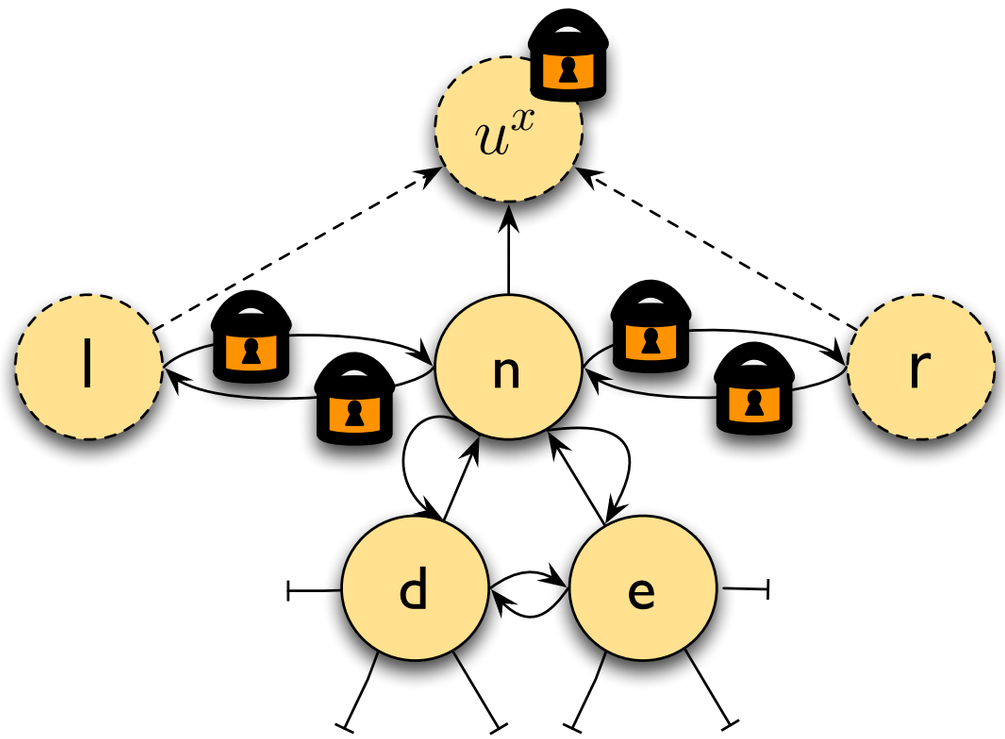
Refinement (Axiomatic Correctness)

```
proc deleteTree(n){
  local l,u,d,r,ul in
    u := [n.up] ; d := [n.first] ; ul:= [n.upL] ;
    //Acquiring the necessary locks.
    lock(ul) ;
    lock(n.leftL) ; l:= [n.left] ;
    if l ≠ null then lock(l.rightL)
    else if u ≠ null then lock(u.firstL) ;
```

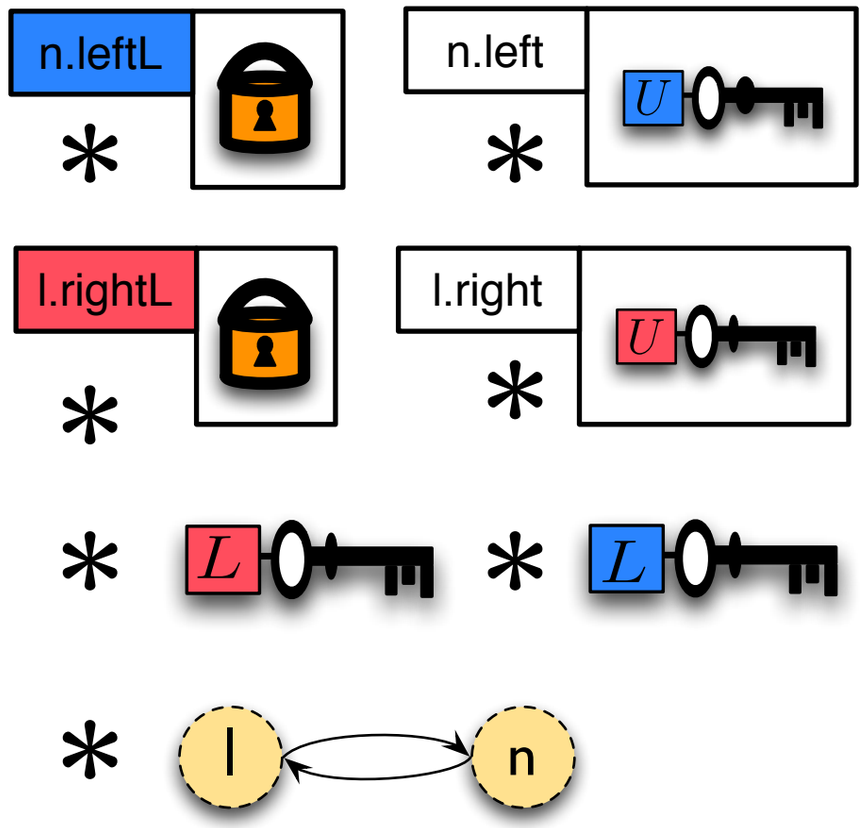
```
//Do the same for RHS

lock(n.right);
r:= n.right;
lock(r.left);
```

```
    if l ≠ null then unlock(l.rightL) ;
    else if u ≠ null then unlock(u.firstL) ;
    if r ≠ null then unlock(r.leftL) ;
    else if u ≠ null then unlock(u.lastL) ;
    call disposeForest(d) ;
    disposeNode(n) ;
}
```



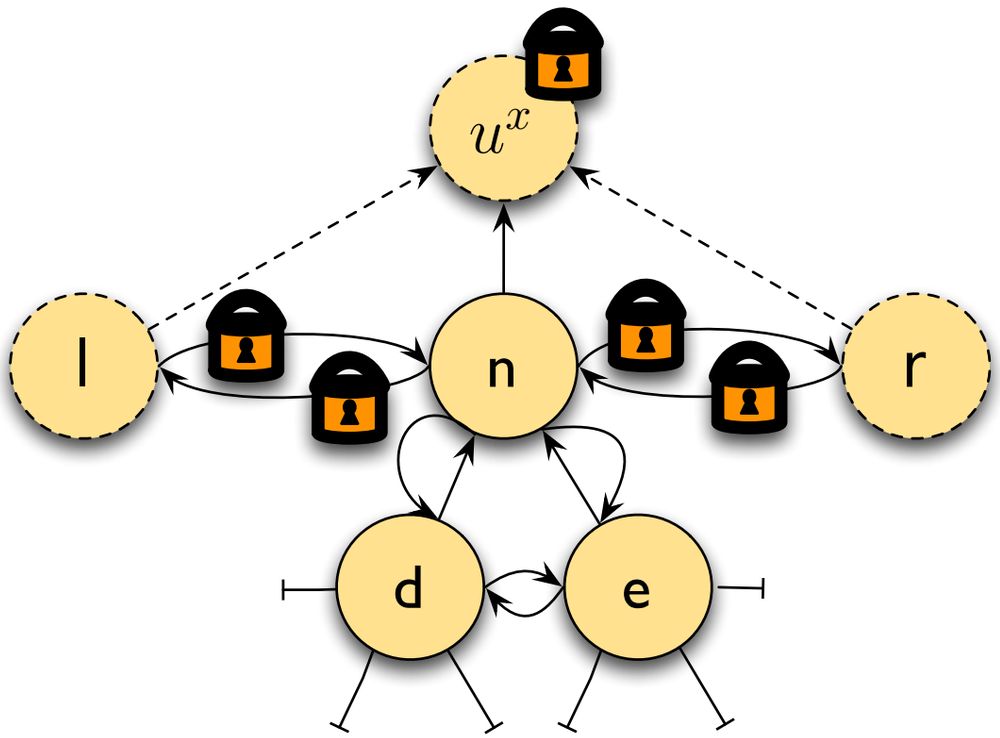
parentLocked(u^x)



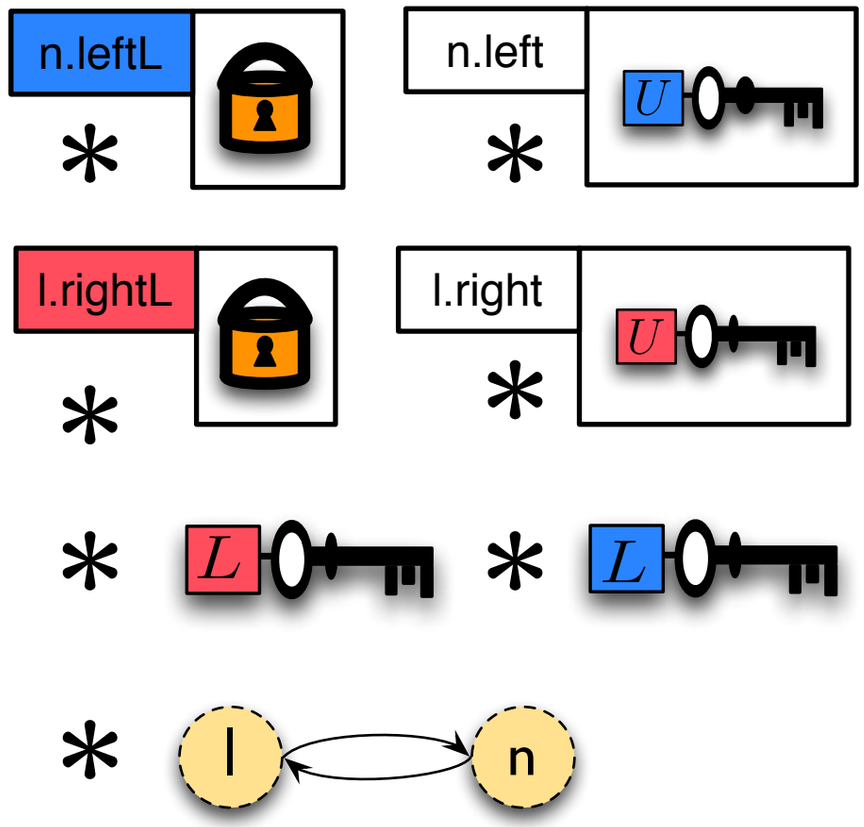
Refinement (Axiomatic Correctness)

```
proc deleteTree(n){
  local l,u,d,r,ul in
    u := [n.up] ; d := [n.first] ; ul:= [n.upL] ;
    //Acquiring the necessary locks.
    lock(ul) ;
    lock(n.leftL) ; l:= [n.left] ;
    if l ≠ null then lock(l.leftL) ;
    else if u ≠ null then lock(u.lastL) ;
    unlock(ul) ;
    //Pointer Swinging.
    if l ≠ null then      [l.right] := r ;
    else if u ≠ null then [u.first] := r ;
    if r ≠ null then [r.left] := l ;
    else if u ≠ null then [u.last] := l ;
    //Unlocking the acquired locks.
    if l ≠ null then unlock(l.rightL) ;
    else if u ≠ null then unlock(u.firstL) ;
    if r ≠ null then unlock(r.leftL) ;
    else if u ≠ null then unlock(u.lastL) ;
    call disposeForest(d) ;
    disposeNode(n) ;
}
```

unlock(u);



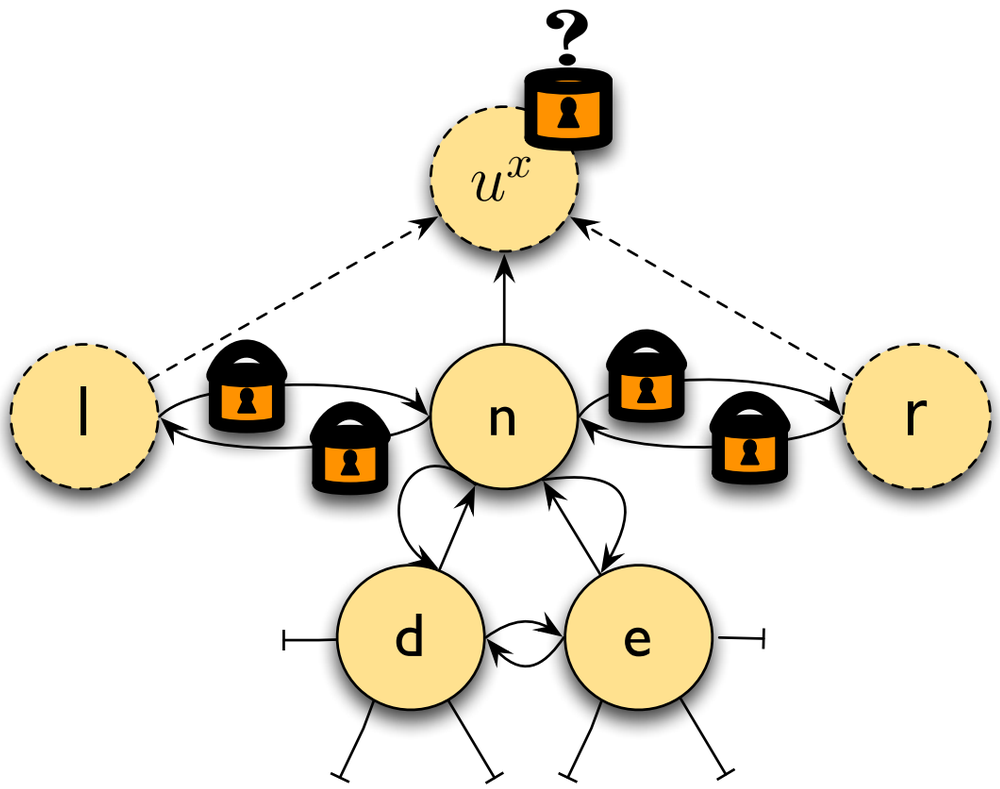
parentLocked(u^x)



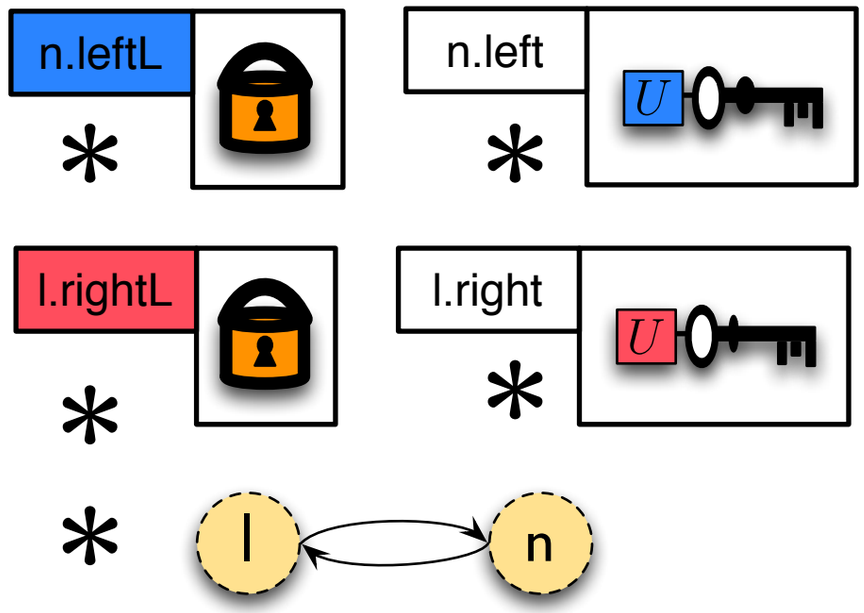
Refinement (Axiomatic Correctness)

```
proc deleteTree(n){
  local l,u,d,r,ul in
    u := [n.up] ; d := [n.first] ; ul:= [n.upL] ;
    //Acquiring the necessary locks.
    lock(ul) ;
    lock(n.leftL) ; l:= [n.left] ;
    if l ≠ null then lock(l.leftL) ;
    else if u ≠ null then lock(u.lastL) ;
    unlock(ul) ;
    //Pointer Swinging.
    if l ≠ null then      [l.right] := r ;
    else if u ≠ null then [u.first] := r ;
    if r ≠ null then [r.left] := l ;
    else if u ≠ null then [u.last] := l ;
    //Unlocking the acquired locks.
    if l ≠ null then unlock(l.rightL) ;
    else if u ≠ null then unlock(u.firstL) ;
    if r ≠ null then unlock(r.leftL) ;
    else if u ≠ null then unlock(u.lastL) ;
    call disposeForest(d) ;
    disposeNode(n) ;
}
```

unlock(u);

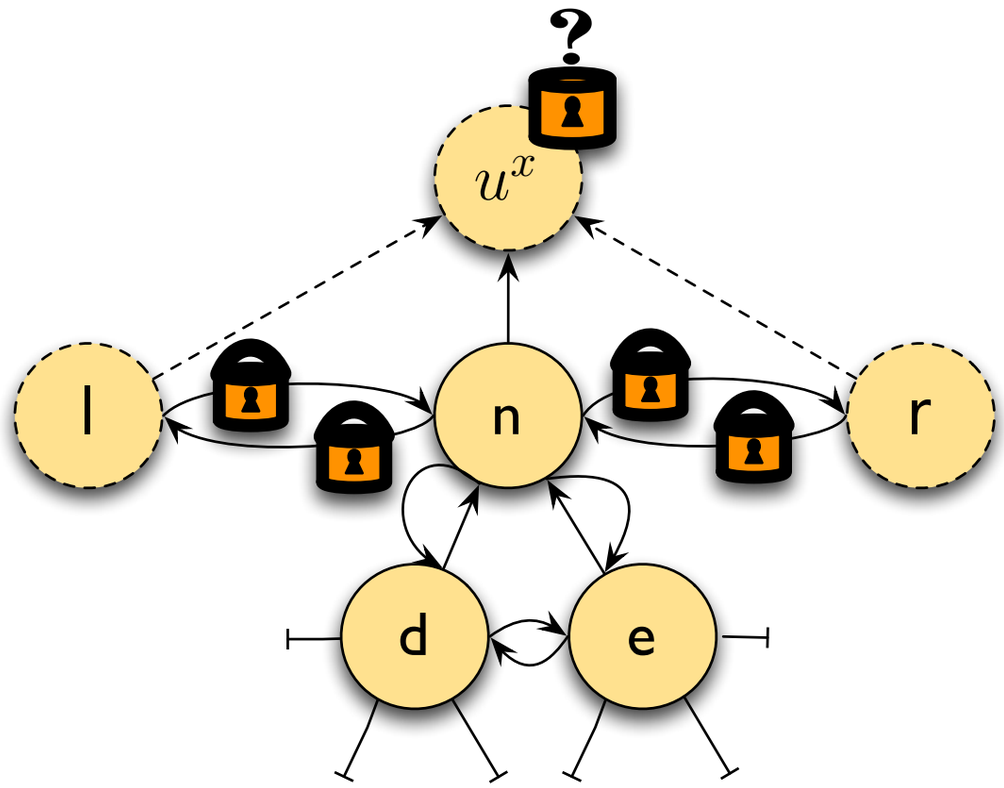


isParentLock(u^x)

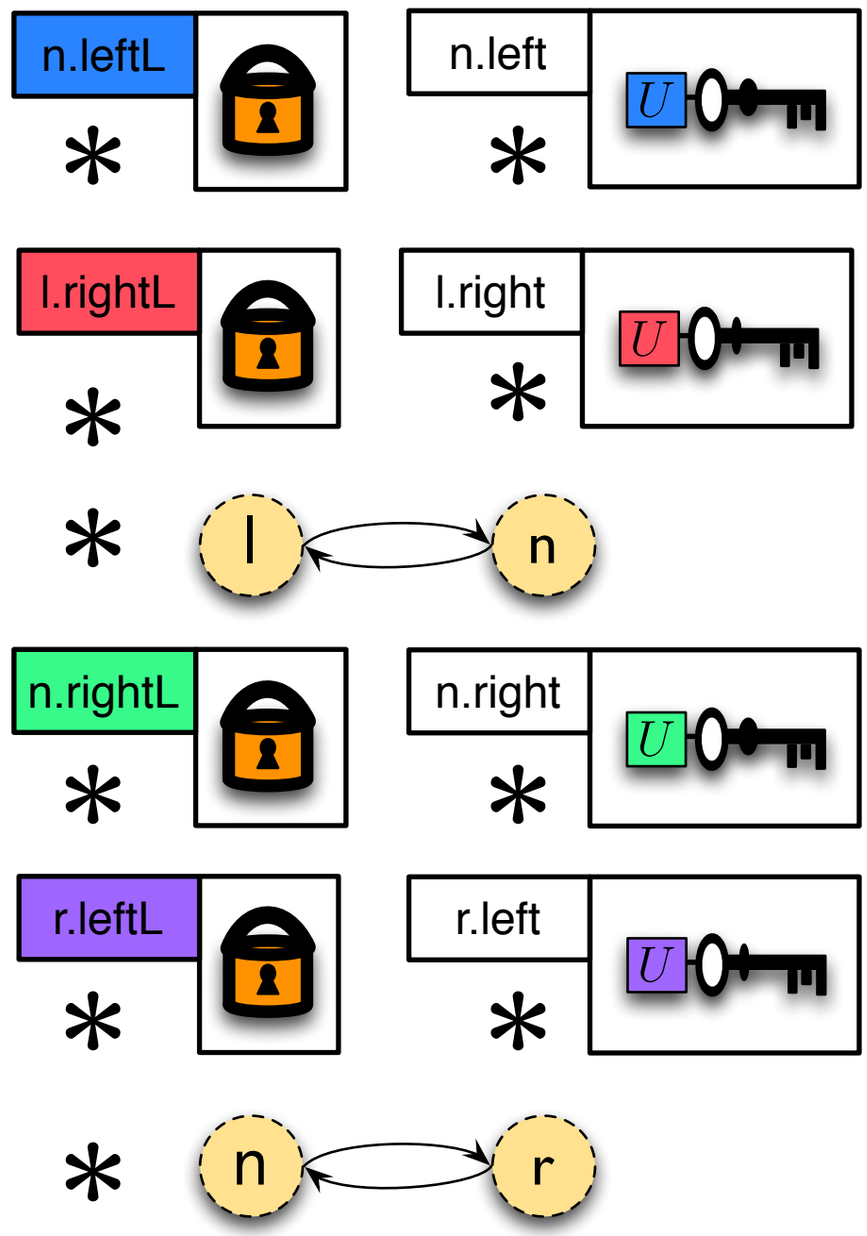


Refinement (Axiomatic Correctness)

```
proc deleteTree(n){
  local l,u,d,r,ul in
    u := [n.up] ; d := [n.first] ; ul:= [n.upL] ;
    //Acquiring the necessary locks.
    lock(ul) ;
    lock(n.leftL) ; l:= [n.left] ;
    if l ≠ null then lock(l.rightL)
    else if u ≠ null then lock(u.firstL) ;
    lock(n.rightL) ; r:= [n.right] ;
    if r ≠ null then lock(r.leftL) ;
    else if u ≠ null then lock(u.lastL) ;
    unlock(ul) ;
    //Pointer Swinging.
    if l ≠ null then      [l.right] := r ;
    else if u ≠ null then [u.first] := r ;
    if r ≠ null then [r.left] := l ;
    else if u ≠ null then [u.last] := l ;
    //Unlocking the acquired locks.
    if l ≠ null then unlock(l.rightL) ;
    else if u ≠ null then unlock(u.firstL) ;
    if r ≠ null then unlock(r.leftL) ;
    else if u ≠ null then unlock(u.lastL) ;
    call disposeForest(d) ;
    disposeNode(n) ;
}
```

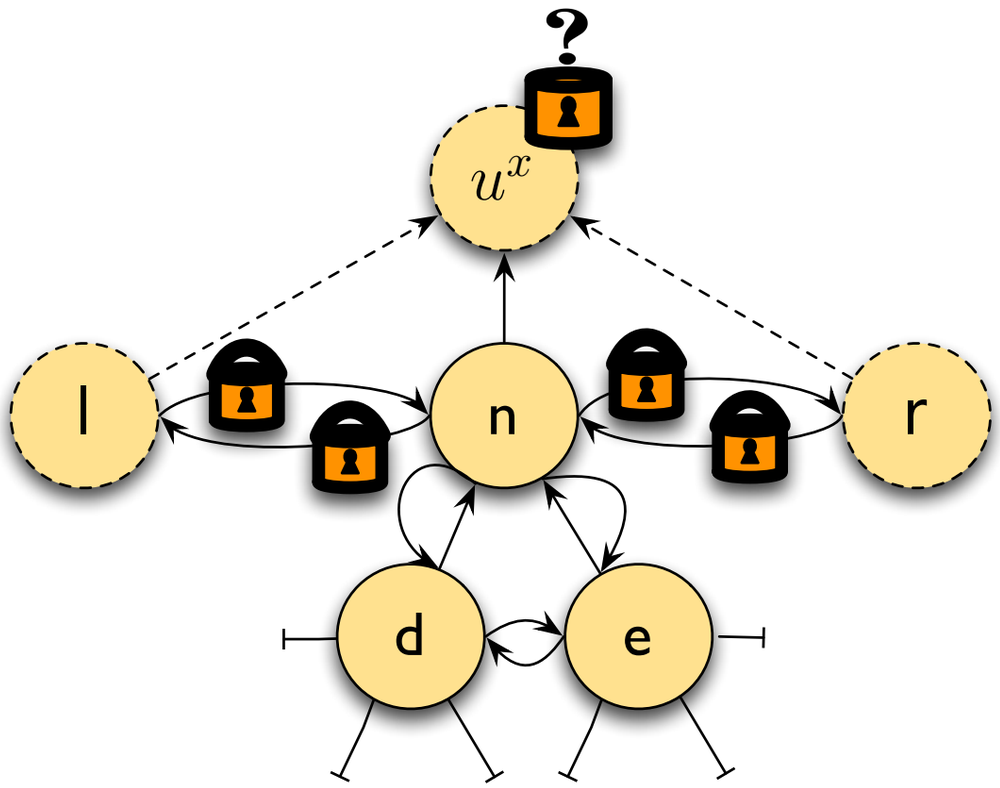


isParentLock(u^x)

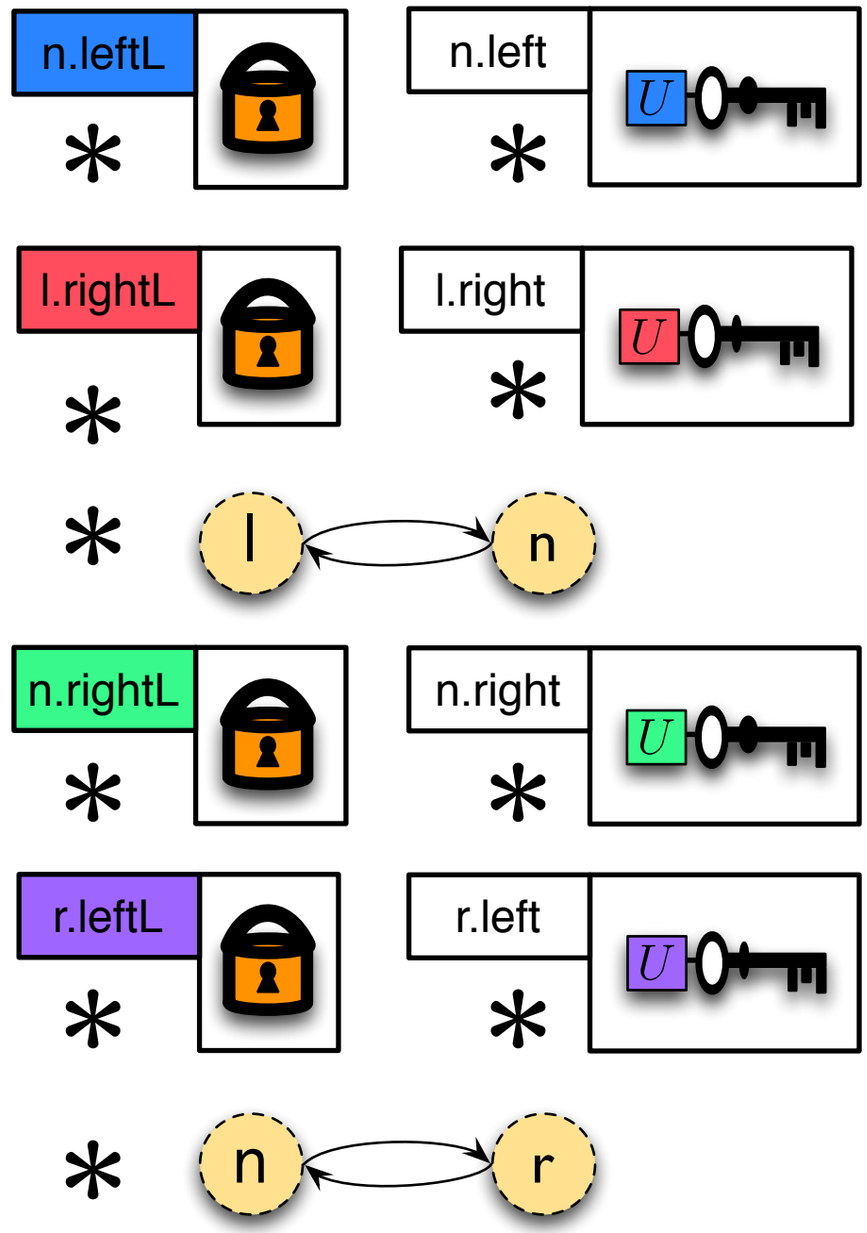


Refinement (Axiomatic Correctness)

```
proc deleteTree(n){
  local l,u,d,r,ul in
    u := [n.up] ; d := [n.first] ; ul:= [n.upL] ;
    //Acquiring the necessary locks.
    lock(ul) ;
    lock(n.leftL) ; l:= [n.left] ;
    [l.right]:=r;
    [r.left]:=l;
    unlock(ul) ;
    //Pointer Swinging.
    if l ≠ null then      [l.right] := r ;
    else if u ≠ null then [u.first] := r ;
    if r ≠ null then [r.left] := l ;
    else if u ≠ null then [u.last] := l ;
    //Unlocking the acquired locks.
    if l ≠ null then unlock(l.rightL) ;
    else if u ≠ null then unlock(u.firstL) ;
    if r ≠ null then unlock(r.leftL) ;
    else if u ≠ null then unlock(u.lastL) ;
    call disposeForest(d) ;
    disposeNode(n) ;
}
```

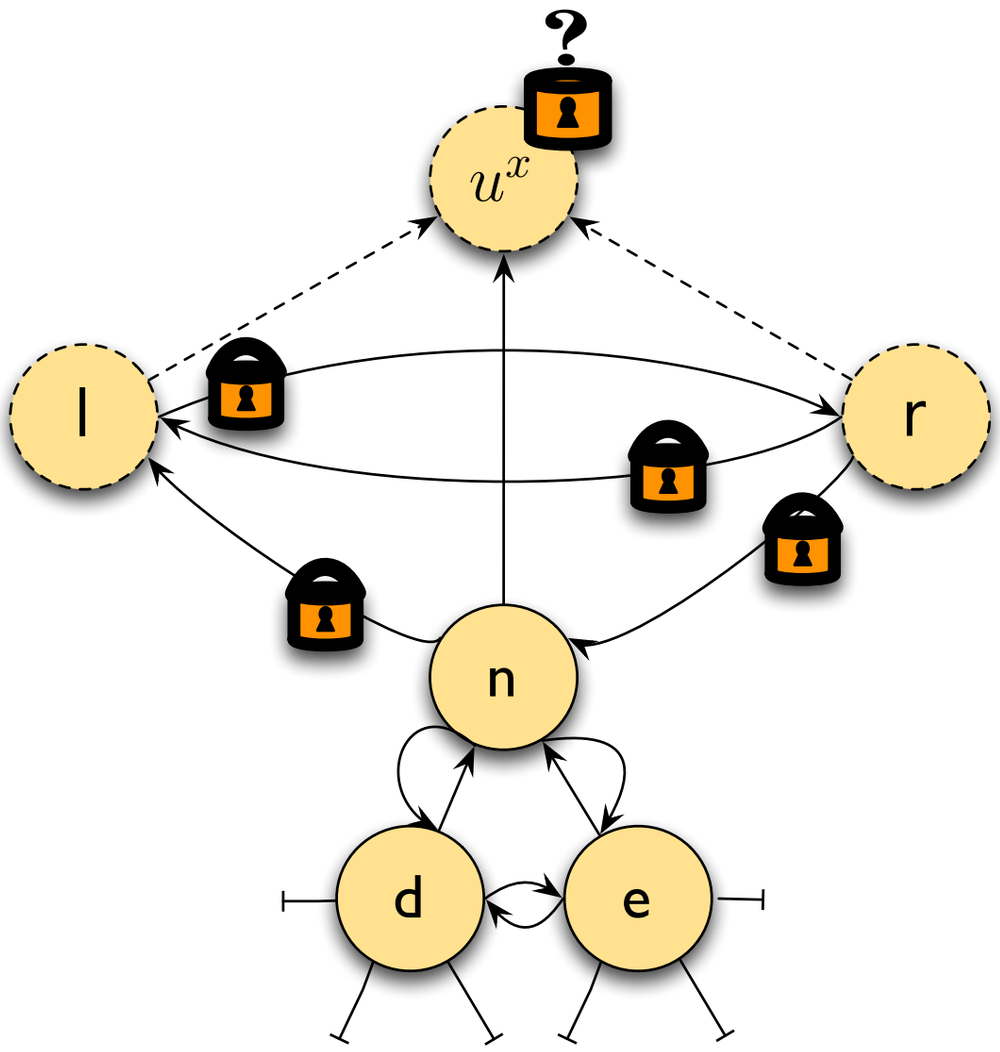


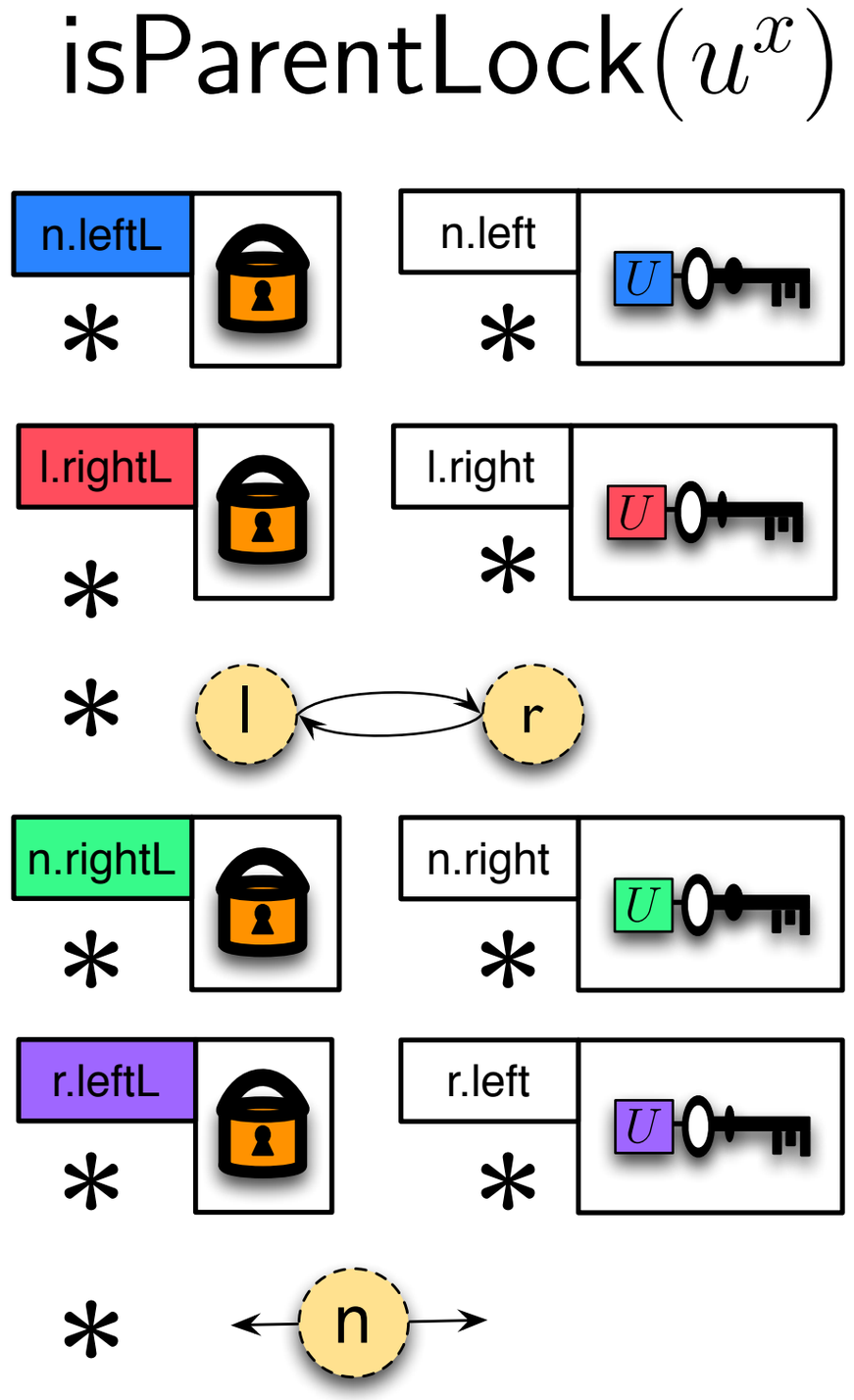
isParentLock(u^x)



Refinement (Axiomatic Correctness)

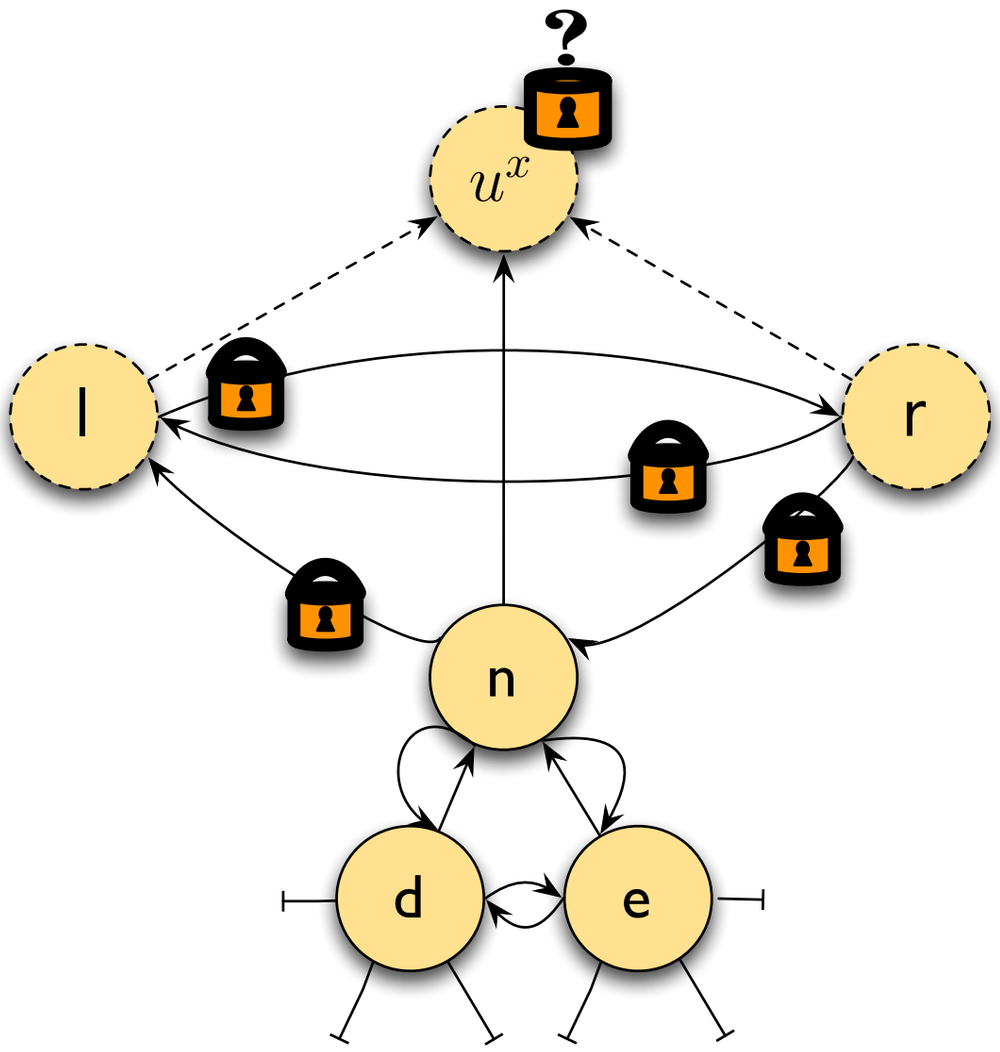
```
proc deleteTree(n){
  local l,u,d,r,ul in
    u := [n.up] ; d := [n.first] ; ul:= [n.upL] ;
    //Acquiring the necessary locks.
    lock(ul) ;
    lock(n.leftL) ; l:= [n.left] ;
    [l.right]:=r;
    [r.left]:=l;
    unlock(ul) ;
    //Pointer Swinging.
    if l ≠ null then      [l.right] := r ;
    else if u ≠ null then [u.first] := r ;
    if r ≠ null then [r.left] := l ;
    else if u ≠ null then [u.last] := l ;
    //Unlocking the acquired locks.
    if l ≠ null then unlock(l.rightL) ;
    else if u ≠ null then unlock(u.firstL) ;
    if r ≠ null then unlock(r.leftL) ;
    else if u ≠ null then unlock(u.lastL) ;
    call disposeForest(d) ;
    disposeNode(n) ;
}
```

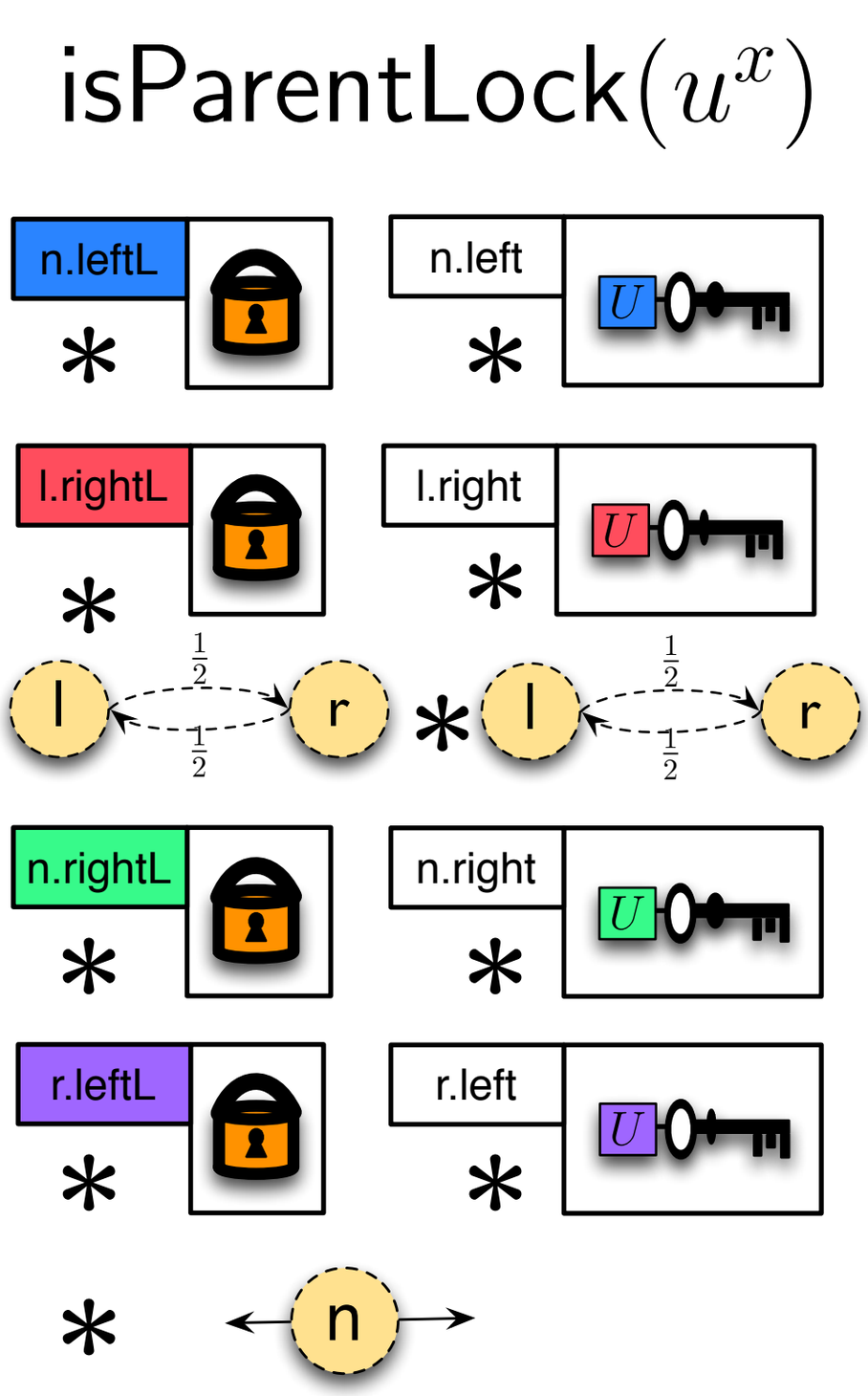




Refinement (Axiomatic Correctness)

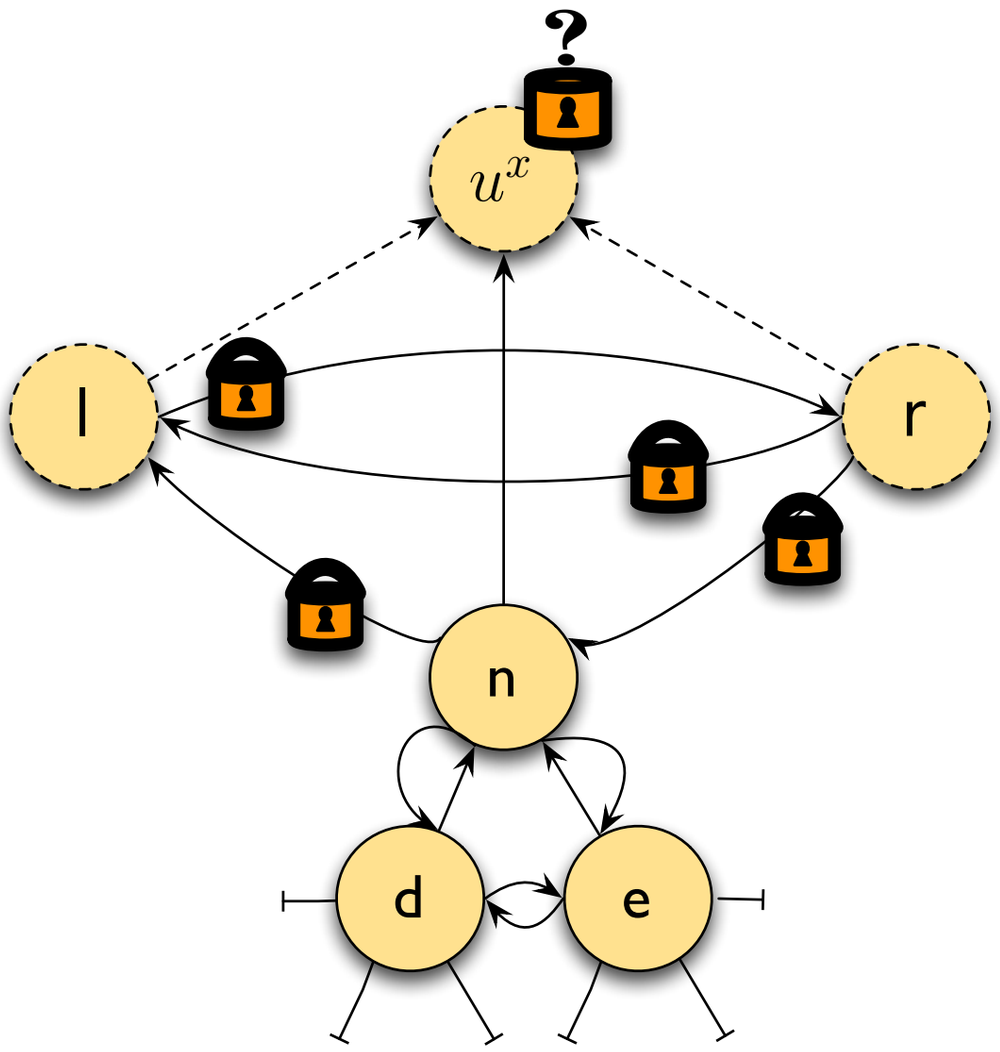
```
proc deleteTree(n){
  local l,u,d,r,ul in
    u := [n.up] ; d := [n.first] ; ul:= [n.upL] ;
    //Acquiring the necessary locks.
    lock(ul) ;
    lock(n.leftL) ; l:= [n.left] ;
    if l ≠ null then lock(l.rightL)
    else if u ≠ null then lock(u.firstL) ;
    lock(n.rightL) ; r:= [n.right] ;
    if r ≠ null then lock(r.leftL) ;
    else if u ≠ null then lock(u.lastL) ;
    unlock(ul) ;
    //Pointer Swinging.
    if l ≠ null then      [l.right] := r ;
    else if u ≠ null then [u.first] := r ;
    if r ≠ null then [r.left] := l ;
    else if u ≠ null then [u.last] := l ;
    //Unlocking the acquired locks.
    if l ≠ null then unlock(l.rightL) ;
    else if u ≠ null then unlock(u.firstL) ;
    if r ≠ null then unlock(r.leftL) ;
    else if u ≠ null then unlock(u.lastL) ;
    call disposeForest(d) ;
    disposeNode(n) ;
}
```



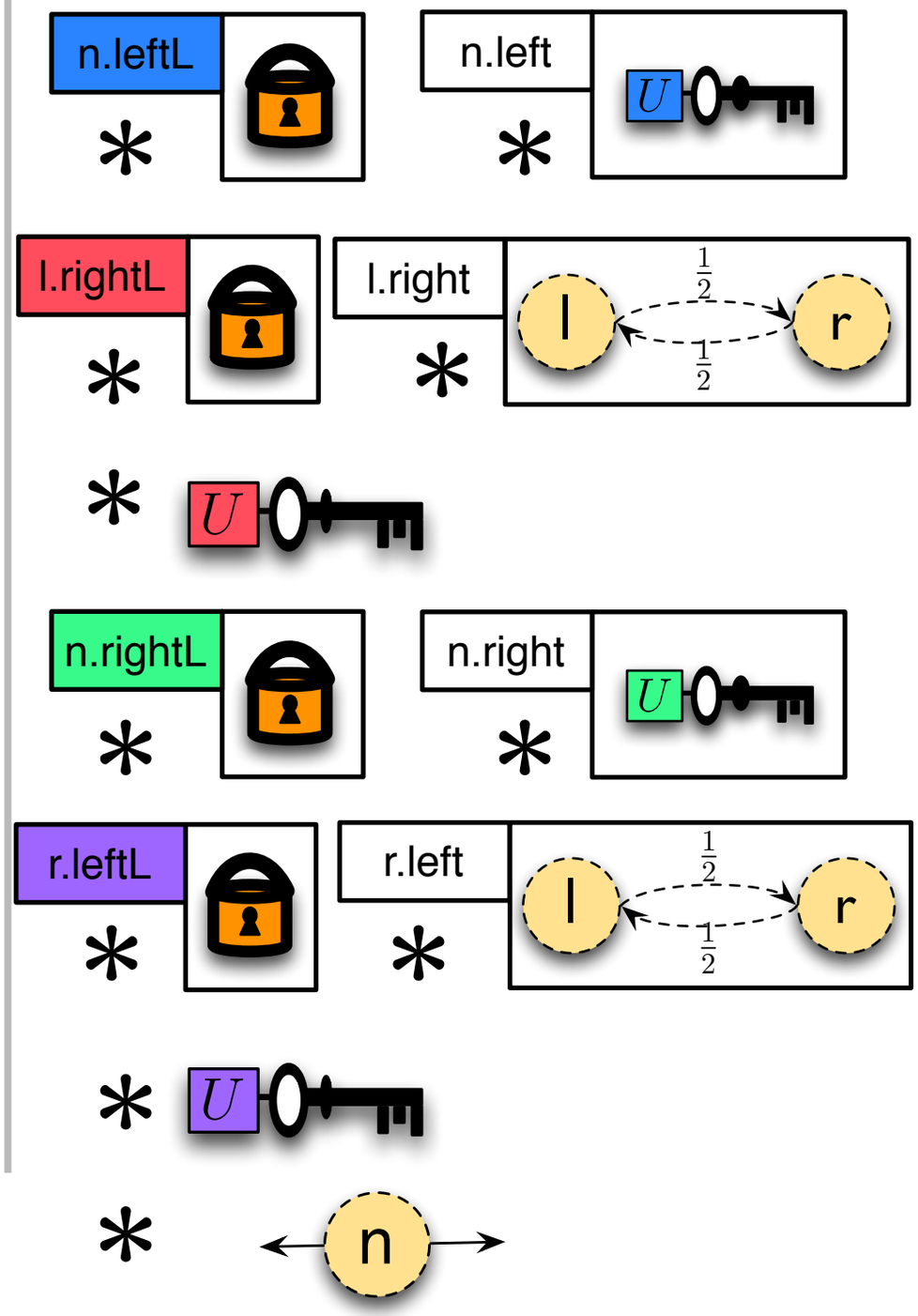


Refinement (Axiomatic Correctness)

```
proc deleteTree(n){
  local l,u,d,r,ul in
    u := [n.up] ; d := [n.first] ; ul:= [n.upL] ;
    //Acquiring the necessary locks.
    lock(ul) ;
    lock(n.leftL) ; l:= [n.left] ;
    if l ≠ null then lock(l.rightL)
    else if u ≠ null then lock(u.firstL) ;
    lock(n.rightL) ; r:= [n.right] ;
    if r ≠ null then lock(r.leftL) ;
    else if u ≠ null then lock(u.lastL) ;
    unlock(ul) ;
    //Pointer Swinging.
    if l ≠ null then      [l.right] := r ;
    else if u ≠ null then [u.first] := r ;
    if r ≠ null then [r.left] := l ;
    else if u ≠ null then [u.last] := l ;
    //Unlocking the acquired locks.
    if l ≠ null then unlock(l.rightL) ;
    else if u ≠ null then unlock(u.firstL) ;
    if r ≠ null then unlock(r.leftL) ;
    else if u ≠ null then unlock(u.lastL) ;
    call disposeForest(d) ;
    disposeNode(n) ;
}
```



isParentLock(u^x)

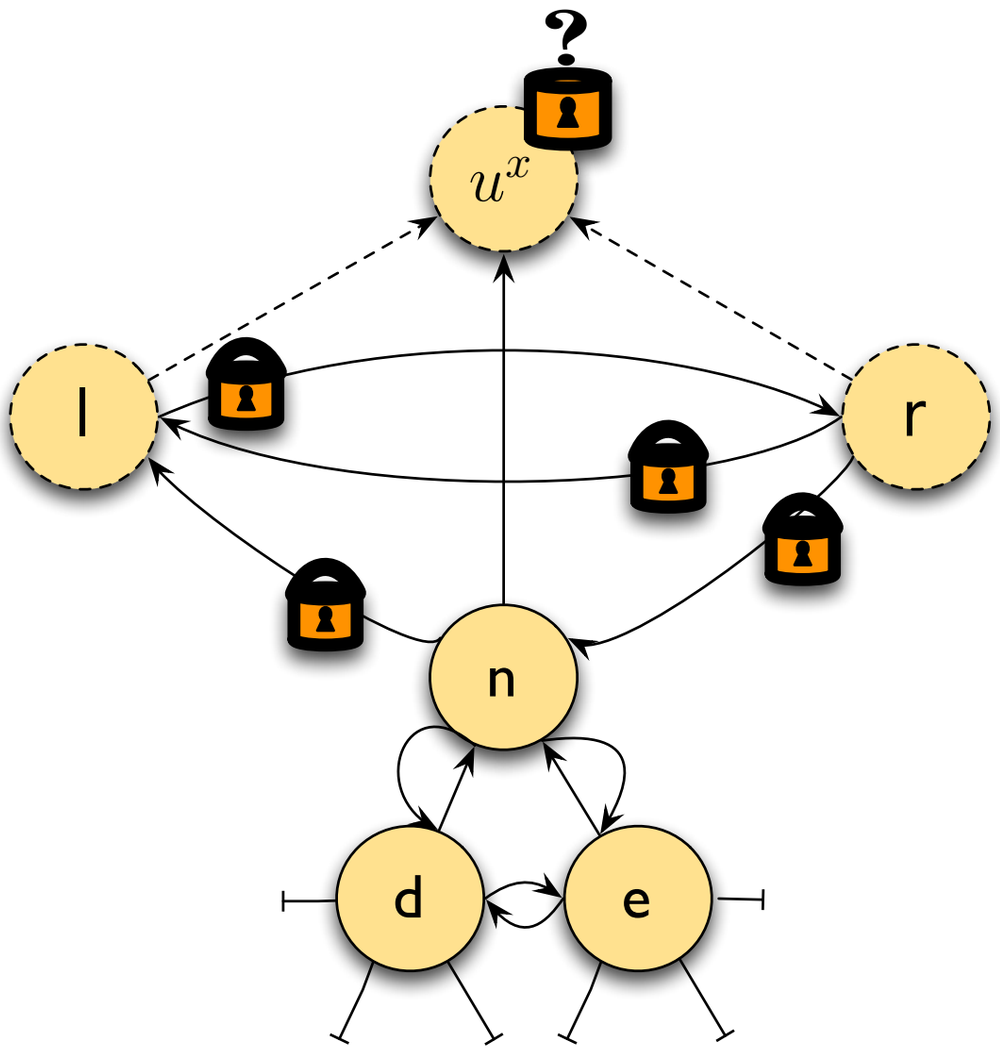


Refinement (Axiomatic Correctness)

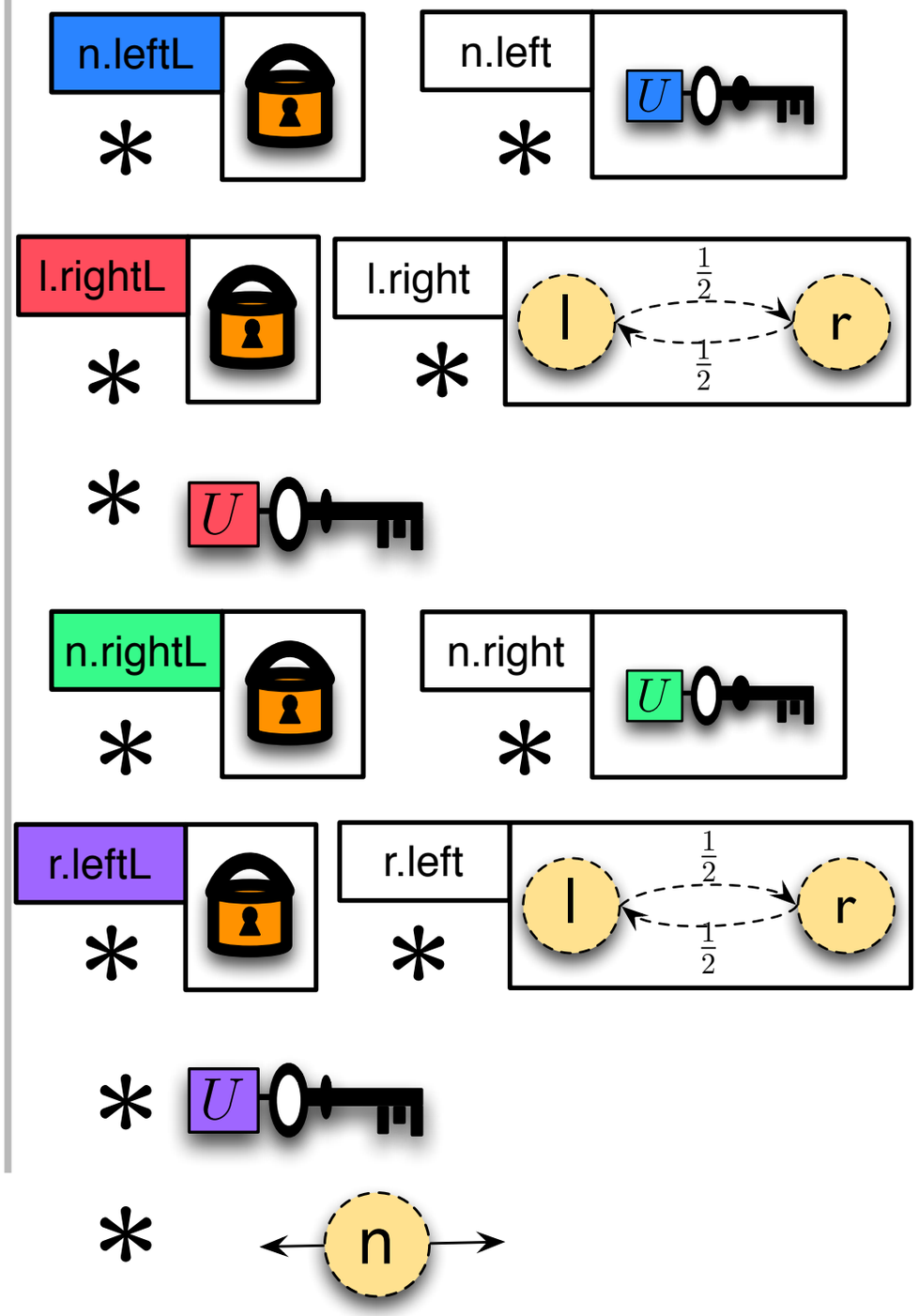
```
proc deleteTree(n){
  local l,u,d,r,ul in
    u := [n.up] ; d := [n.first] ; ul:= [n.upL] ;
    //Acquiring the necessary locks.
    lock(ul) ;
    lock(n.leftL) ; l:= [n.left] ;
    if l ≠ null then lock(l.rightL)
    else if u ≠ null then lock(u.firstL) ;
```

```
unlock(l.right);
unlock(r.left);
```

```
    //Pointer Swinging.
    if l ≠ null then      [l.right] := r ;
    else if u ≠ null then [u.first] := r ;
    if r ≠ null then [r.left] := l ;
    else if u ≠ null then [u.last] := l ;
    //Unlocking the acquired locks.
    if l ≠ null then unlock(l.rightL) ;
    else if u ≠ null then unlock(u.firstL) ;
    if r ≠ null then unlock(r.leftL) ;
    else if u ≠ null then unlock(u.lastL) ;
    call disposeForest(d) ;
    disposeNode(n) ;
}
```



isParentLock(u^x)

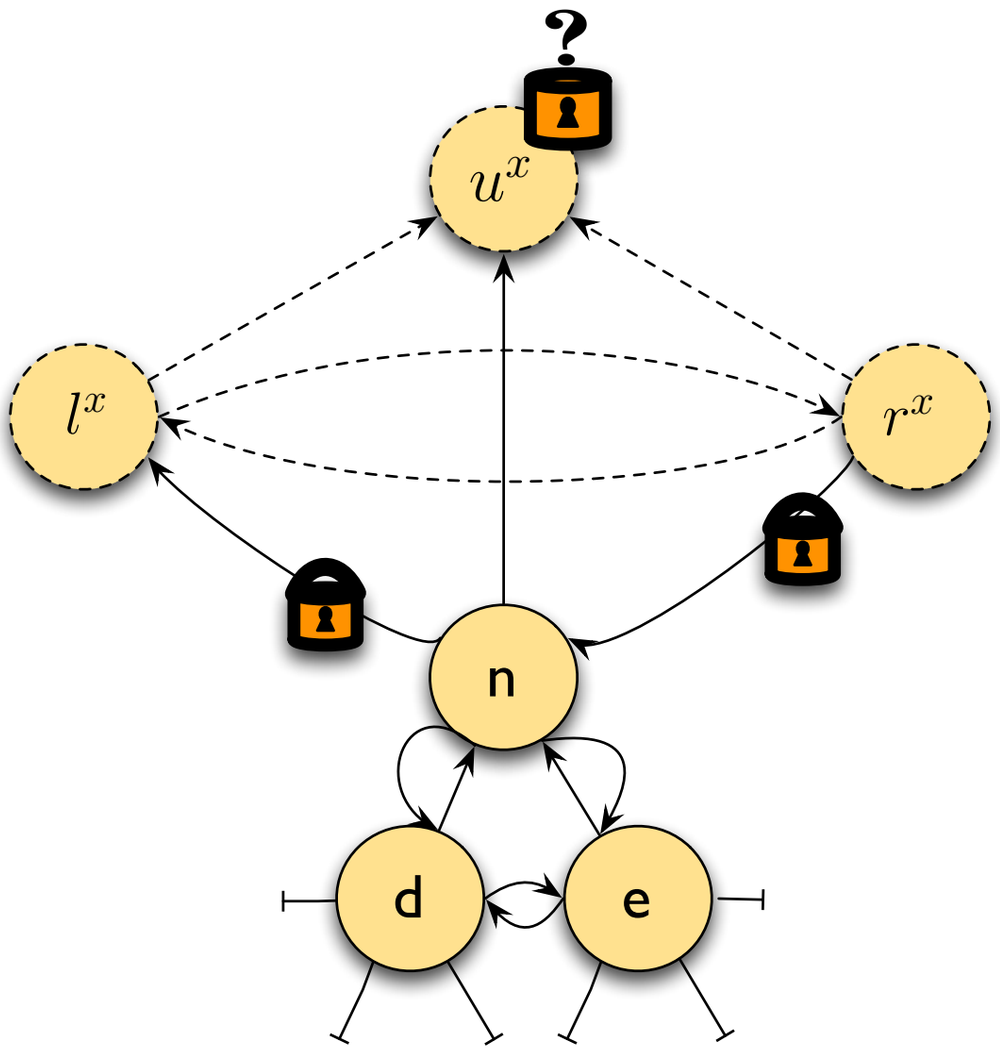


Refinement (Axiomatic Correctness)

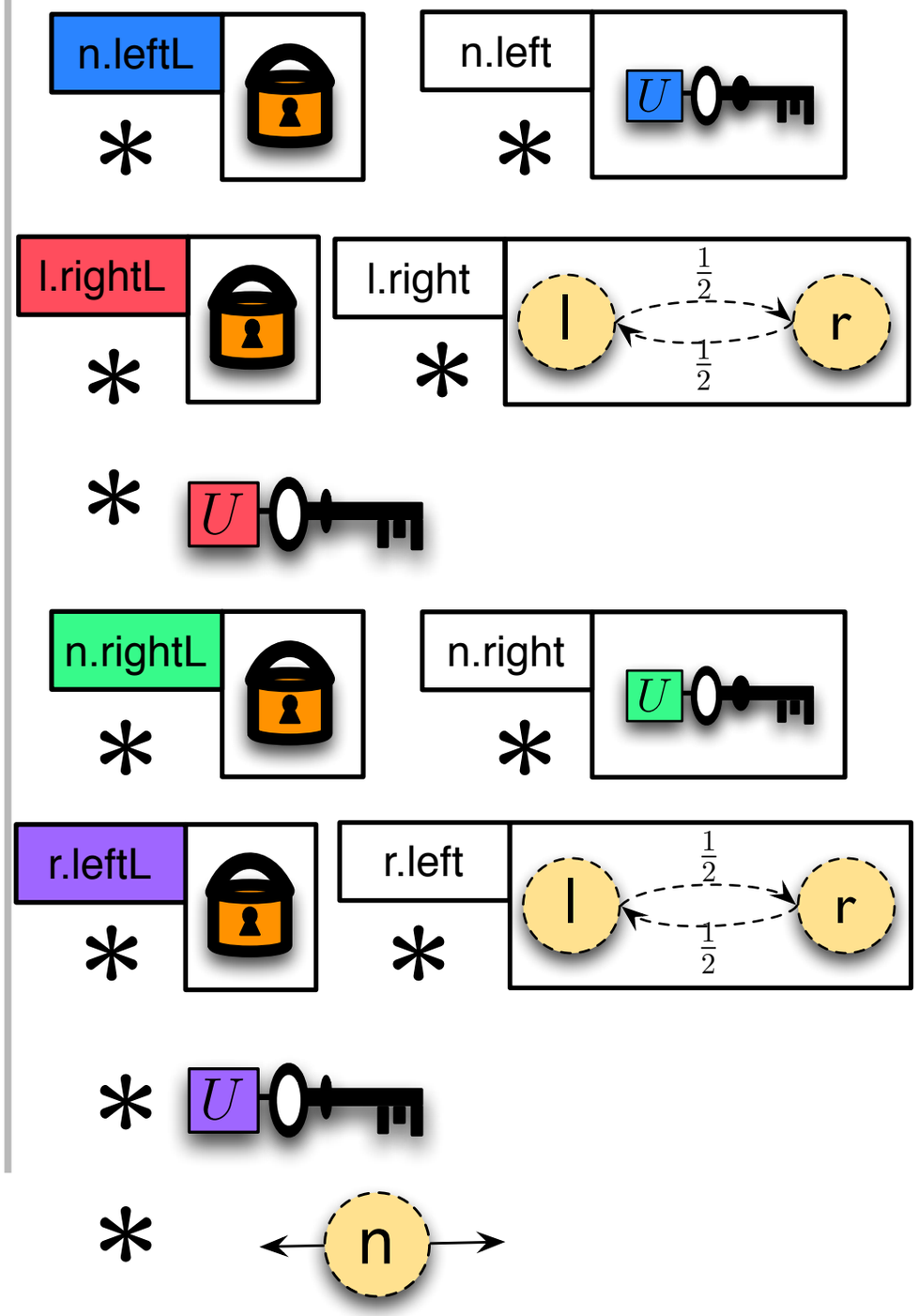
```
proc deleteTree(n){
  local l,u,d,r,ul in
    u := [n.up] ; d := [n.first] ; ul:= [n.upL] ;
    //Acquiring the necessary locks.
    lock(ul) ;
    lock(n.leftL) ; l:= [n.left] ;
    if l ≠ null then lock(l.rightL)
    else if u ≠ null then lock(u.firstL) ;
```

```
unlock(l.right);
unlock(r.left);
```

```
    //Pointer Swinging.
    if l ≠ null then      [l.right] := r ;
    else if u ≠ null then [u.first] := r ;
    if r ≠ null then [r.left] := l ;
    else if u ≠ null then [u.last] := l ;
    //Unlocking the acquired locks.
    if l ≠ null then unlock(l.rightL) ;
    else if u ≠ null then unlock(u.firstL) ;
    if r ≠ null then unlock(r.leftL) ;
    else if u ≠ null then unlock(u.lastL) ;
    call disposeForest(d) ;
    disposeNode(n) ;
}
```



isParentLock(u^x)

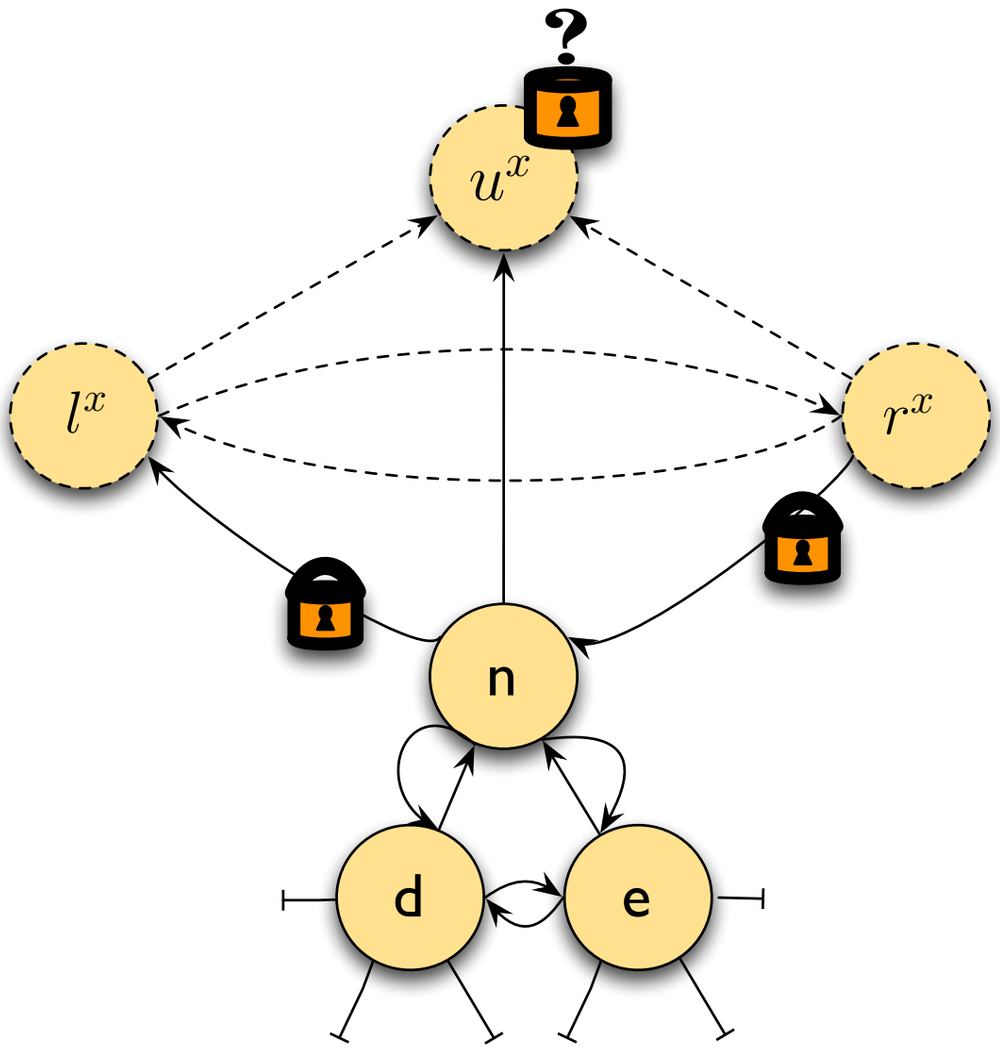


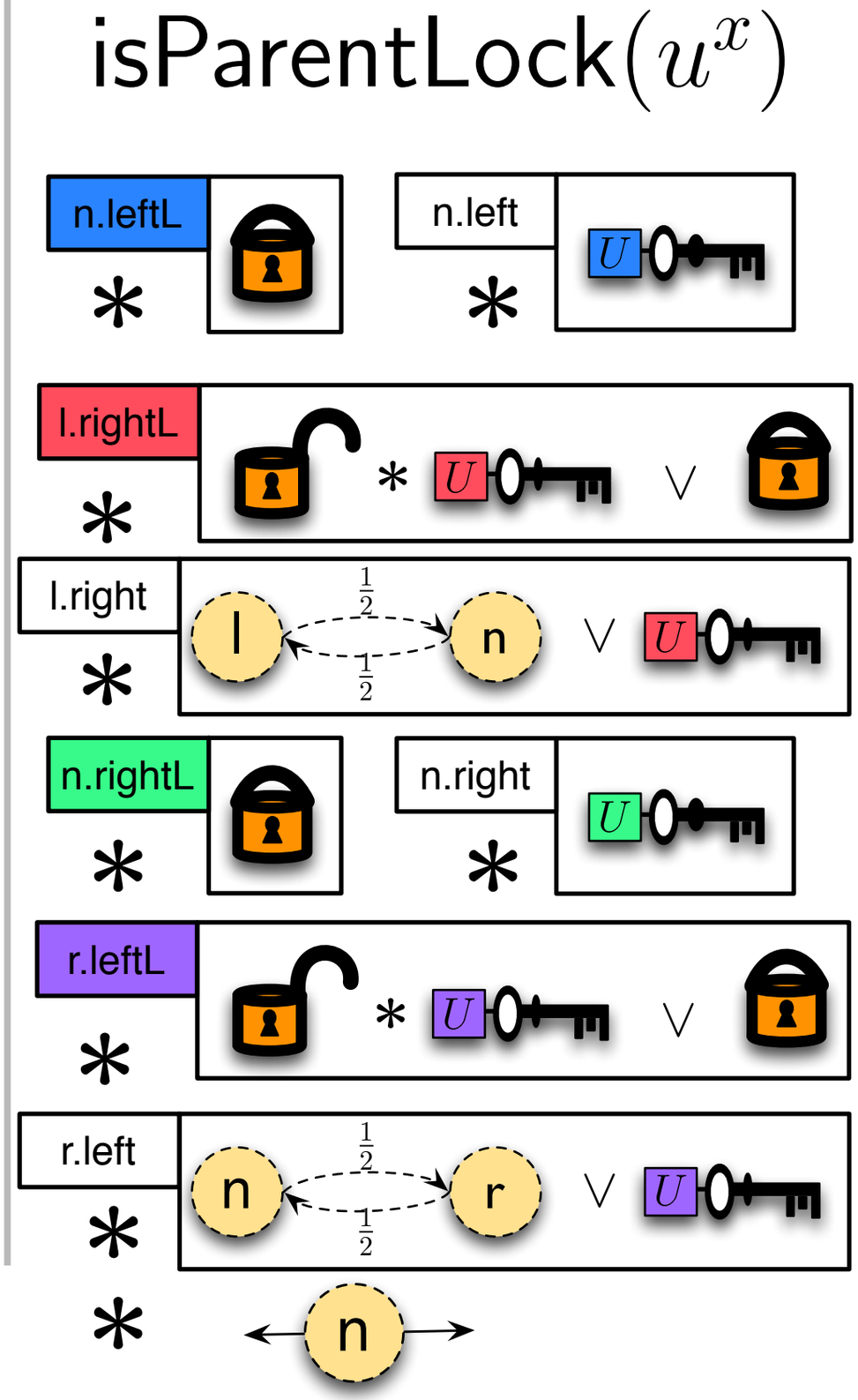
Refinement (Axiomatic Correctness)

```
proc deleteTree(n){
  local l,u,d,r,ul in
    u := [n.up] ; d := [n.first] ; ul:= [n.upL] ;
    //Acquiring the necessary locks.
    lock(ul) ;
    lock(n.leftL) ; l:= [n.left] ;
    if l ≠ null then lock(l.rightL)
    else if u ≠ null then lock(u.firstL) ;
```

```
unlock(l.right);
unlock(r.left);
```

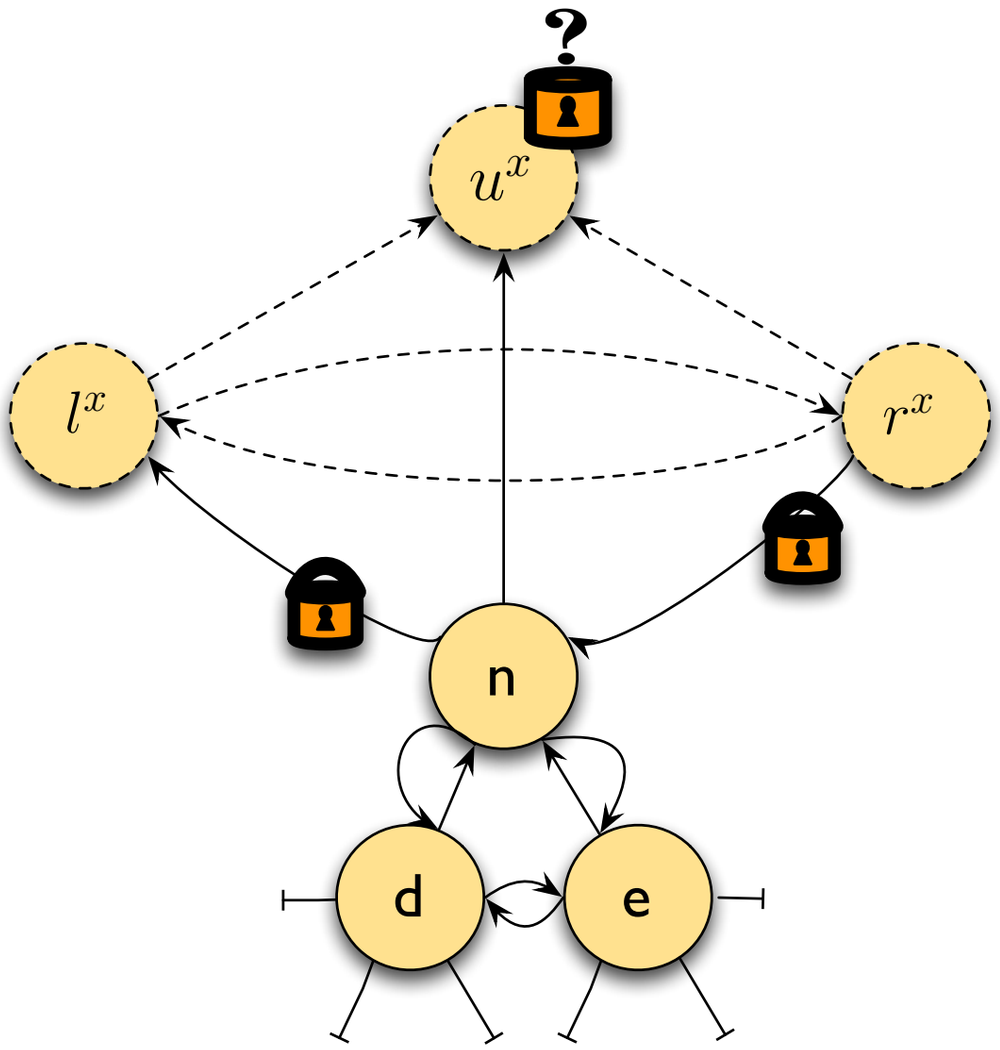
```
    //Pointer Swinging.
    if l ≠ null then      [l.right] := r ;
    else if u ≠ null then [u.first] := r ;
    if r ≠ null then [r.left] := l ;
    else if u ≠ null then [u.last] := l ;
    //Unlocking the acquired locks.
    if l ≠ null then unlock(l.rightL) ;
    else if u ≠ null then unlock(u.firstL) ;
    if r ≠ null then unlock(r.leftL) ;
    else if u ≠ null then unlock(u.lastL) ;
    call disposeForest(d) ;
    disposeNode(n) ;
}
```



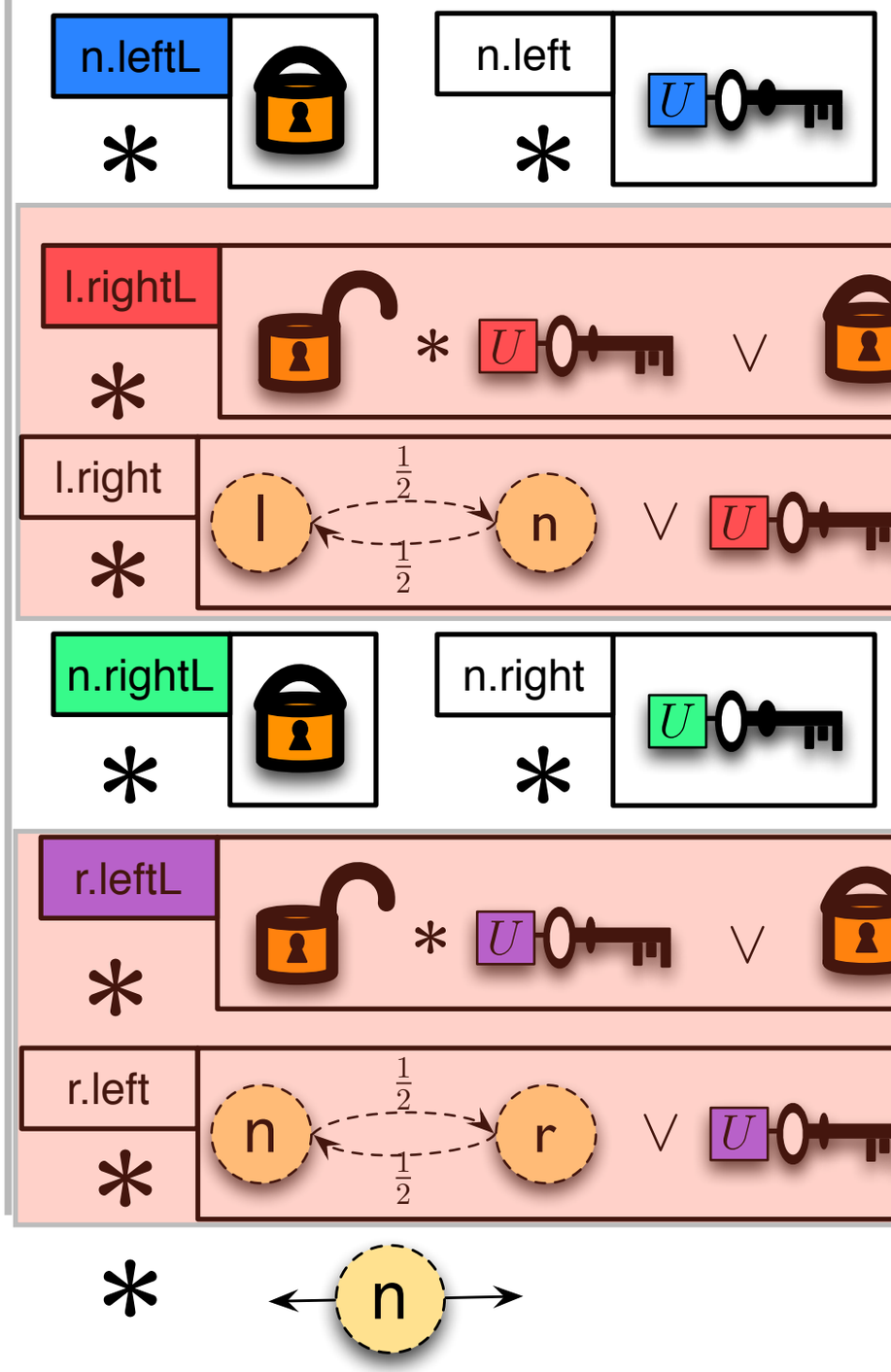


Refinement (Axiomatic Correctness)

```
proc deleteTree(n){
  local l,u,d,r,ul in
    u := [n.up] ; d := [n.first] ; ul:= [n.upL] ;
    //Acquiring the necessary locks.
    lock(ul) ;
    lock(n.leftL) ; l:= [n.left] ;
    if l ≠ null then lock(l.rightL)
    else if u ≠ null then lock(u.firstL) ;
    lock(n.rightL) ; r:= [n.right] ;
    if r ≠ null then lock(r.leftL) ;
    else if u ≠ null then lock(u.lastL) ;
    unlock(ul) ;
    //Pointer Swinging.
    if l ≠ null then      [l.right] := r ;
    else if u ≠ null then [u.first] := r ;
    if r ≠ null then [r.left] := l ;
    else if u ≠ null then [u.last] := l ;
    //Unlocking the acquired locks.
    if l ≠ null then unlock(l.rightL) ;
    else if u ≠ null then unlock(u.firstL) ;
    if r ≠ null then unlock(r.leftL) ;
    else if u ≠ null then unlock(u.lastL) ;
    call disposeForest(d) ;
    disposeNode(n) ;
}
```

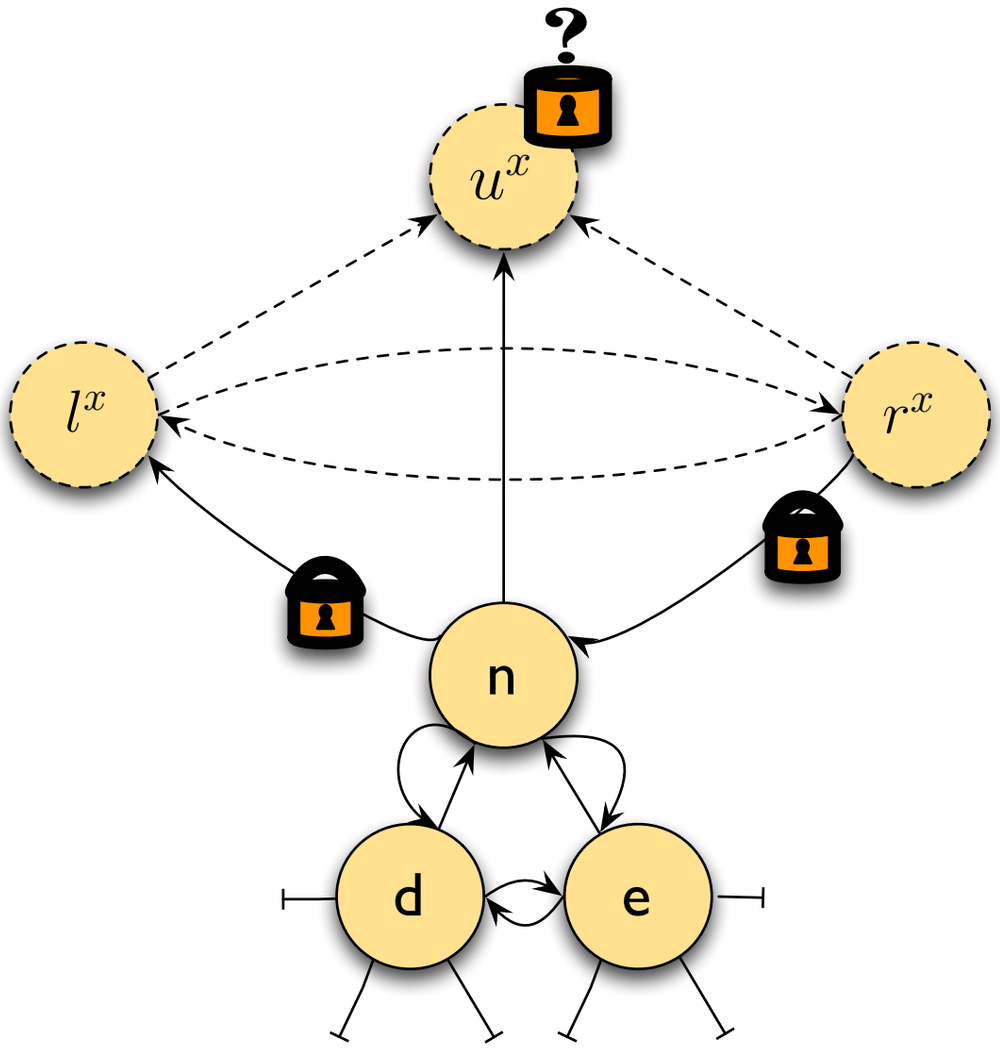


isParentLock(u^x)

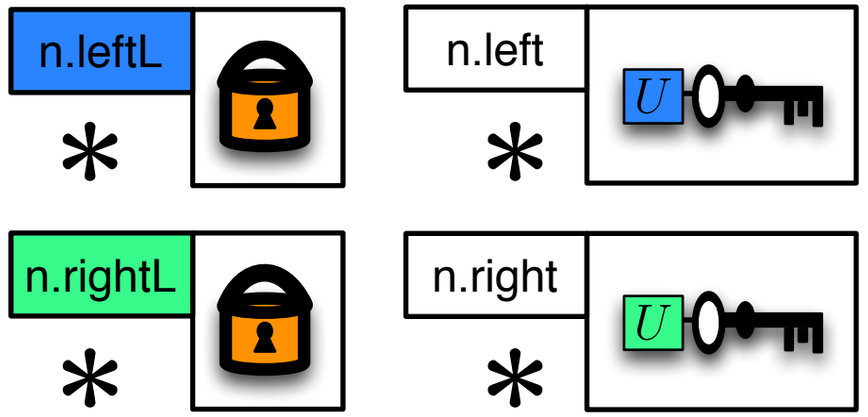


Refinement (Axiomatic Correctness)

```
proc deleteTree(n){
  local l,u,d,r,ul in
    u := [n.up] ; d := [n.first] ; ul:= [n.upL] ;
    //Acquiring the necessary locks.
    lock(ul) ;
    lock(n.leftL) ; l:= [n.left] ;
    if l ≠ null then lock(l.rightL)
    else if u ≠ null then lock(u.firstL) ;
    lock(n.rightL) ; r:= [n.right] ;
    if r ≠ null then lock(r.leftL) ;
    else if u ≠ null then lock(u.lastL) ;
    unlock(ul) ;
    //Pointer Swinging.
    if l ≠ null then      [l.right] := r ;
    else if u ≠ null then [u.first] := r ;
    if r ≠ null then [r.left] := l ;
    else if u ≠ null then [u.last] := l ;
    //Unlocking the acquired locks.
    if l ≠ null then unlock(l.rightL) ;
    else if u ≠ null then unlock(u.firstL) ;
    if r ≠ null then unlock(r.leftL) ;
    else if u ≠ null then unlock(u.lastL) ;
    call disposeForest(d) ;
    disposeNode(n) ;
}
```



isParentLock(u^x)

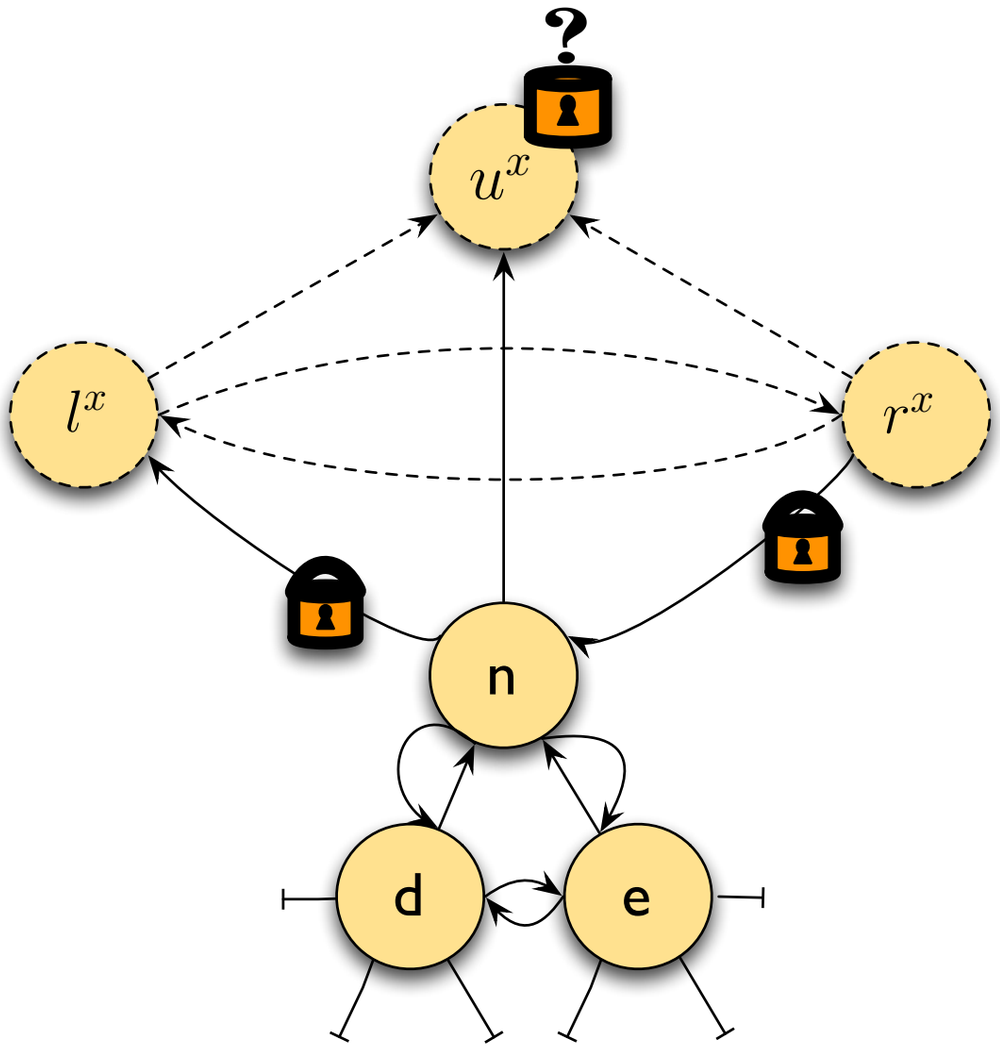


$*$ Crust $(r^x, l^x)\,(l^x, u^x, r^x)$

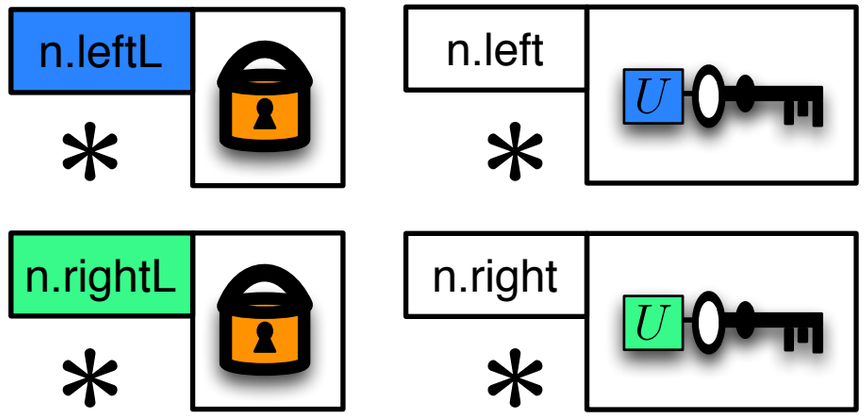
$*$ n

Refinement (Axiomatic Correctness)

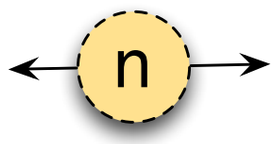
```
proc deleteTree(n){
  local l,u,d,r,ul in
    u := [n.up] ; d := [n.first] ; ul:= [n.upL] ;
    //Acquiring the necessary locks.
    lock(ul) ;
    lock(n.leftL) ; l:= [n.left] ;
    if l ≠ null then lock(l.rightL)
    else if u ≠ null then lock(u.firstL) ;
    lock(n.rightL) ; r:= [n.right] ;
    if r ≠ null then lock(r.leftL) ;
    else if u ≠ null then lock(u.lastL) ;
    unlock(ul) ;
    //Pointer Swinging.
    if l ≠ null then      [l.right] := r ;
    else if u ≠ null then [u.first] := r ;
    if r ≠ null then [r.left] := l ;
    else if u ≠ null then [u.last] := l ;
    //Unlocking the acquired locks.
    if l ≠ null then unlock(l.rightL) ;
    else if u ≠ null then unlock(u.firstL) ;
    if r ≠ null then unlock(r.leftL) ;
    else if u ≠ null then unlock(u.lastL) ;
    call disposeForest(d) ;
    disposeNode(n) ;
}
```



isParentLock(u^x)



$*$ Crust (r^x, l^x) (l^x, u^x, r^x)

$*$ 

Refinement (Axiomatic Correctness)

```
proc deleteTree(n){
  local l,u,d,r,ul in
    u := [n.up] ; d := [n.first] ; ul:= [n.upL] ;
    //Acquiring the necessary locks.
    lock(ul) ;
    lock(n.leftL) ; l:= [n.left] ;
    if l ≠ null then lock(l.rightL)
    else if u ≠ null then lock(u.firstL) ;
    lock(n.rightL) ; r:= [n.right] ;
    if r ≠ null then lock(r.leftL) ;
    else if u ≠ null then lock(u.lastL) ;
    unlock(ul) ;
    //Pointer Swinging.
    if l ≠ null then      [l.right] := r ;
    else if u ≠ null then [u.first] := r ;
    if r ≠ null then [r.left] := l ;
    else if u ≠ null then [u.last] := l ;
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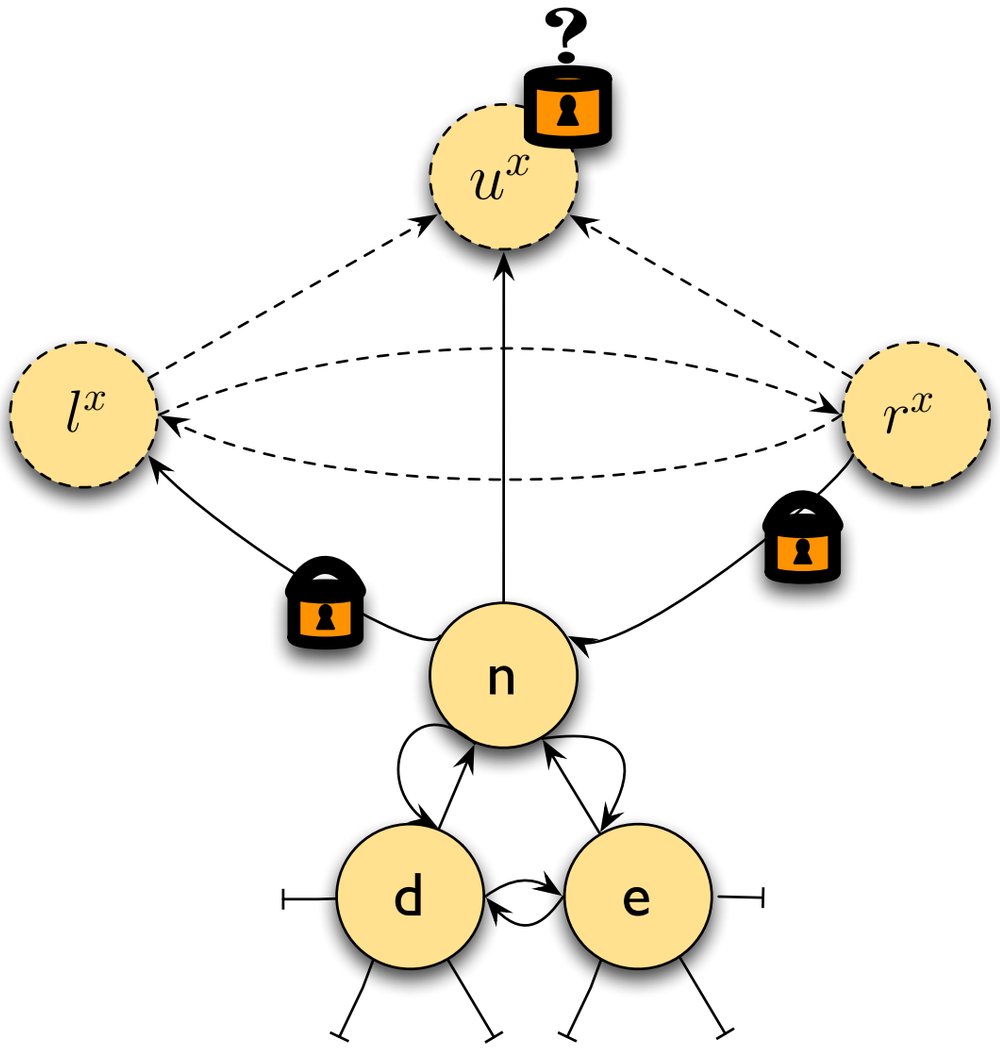
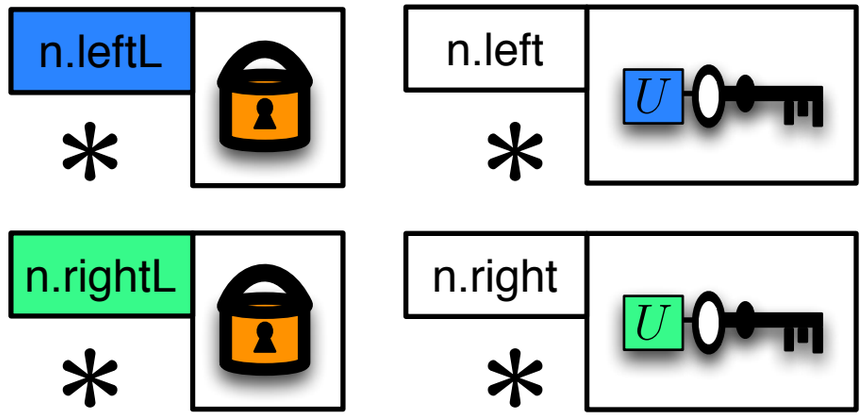
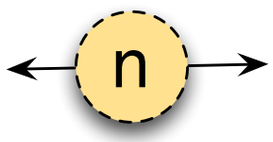
    else if u ≠ null then unlock(u.firstL) ;
    if r ≠ null then unlock(r.leftL) ;
    else if u ≠ null then unlock(u.lastL) ;
    call disposeForest(d) ;
    disposeNode(n) ;
}
```



isParentLock(u^x)



* Crust $(r^x, l^x)\,(l^x, u^x, r^x)$

* 

Refinement (Axiomatic Correctness)

```
proc deleteTree(n){
  local l,u,d,r,ul in
    u := [n.up] ; d := [n.first] ; ul:= [n.upL] ;
    //Acquiring the necessary locks.
    lock(ul) ;
    lock(n.leftL) ; l:= [n.left] ;
    if l ≠ null then lock(l.rightL)
    else if u ≠ null then lock(u.firstL) ;
    lock(n.rightL) ; r:= [n.right] ;
    if r ≠ null then lock(r.leftL) ;
    else if u ≠ null then lock(u.lastL) ;
    unlock(ul) ;
    //Pointer Swinging.
    if l ≠ null then      [l.right] := r ;
    else if u ≠ null then [u.first] := r ;
    if r ≠ null then [r.left] := l ;
    else if u ≠ null then [u.last] := l ;
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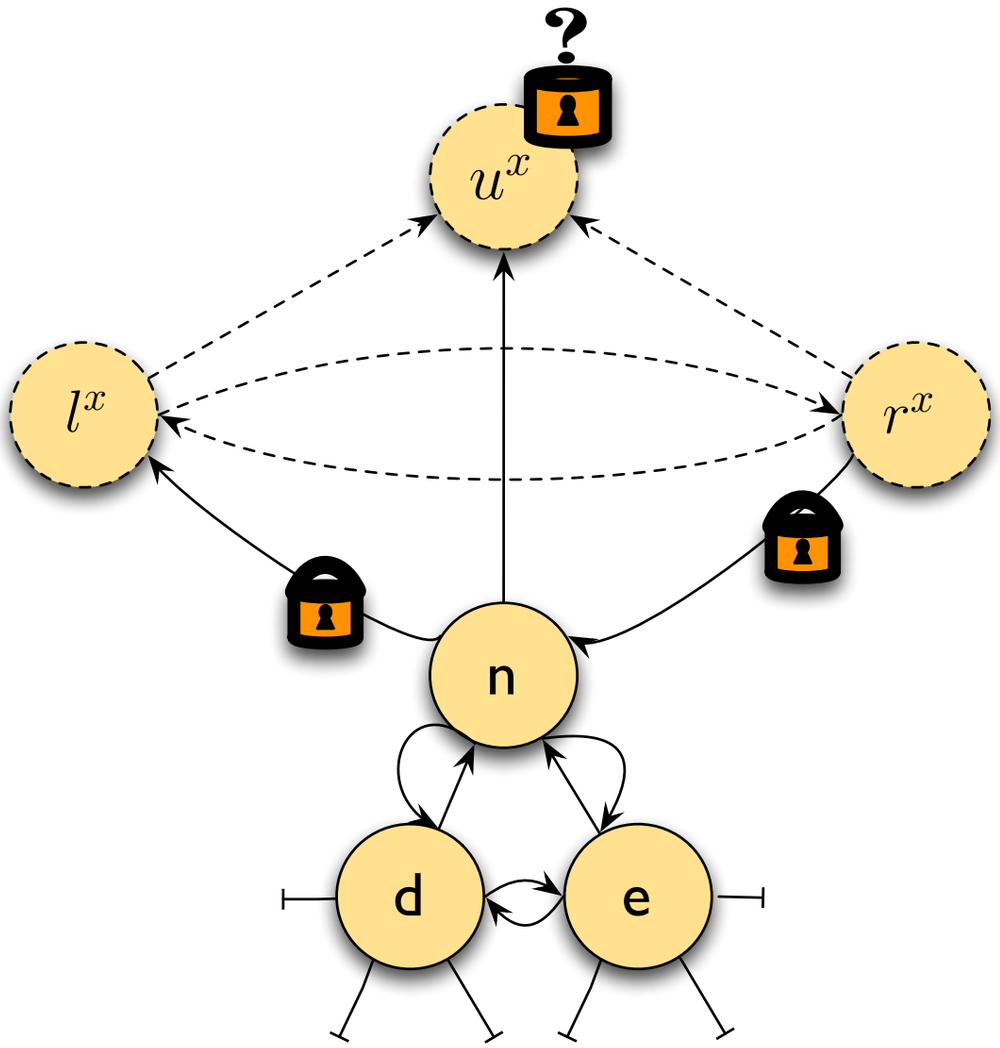
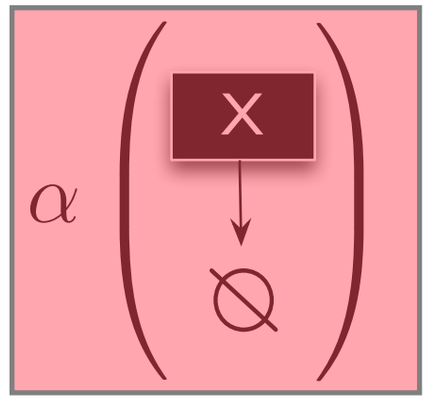
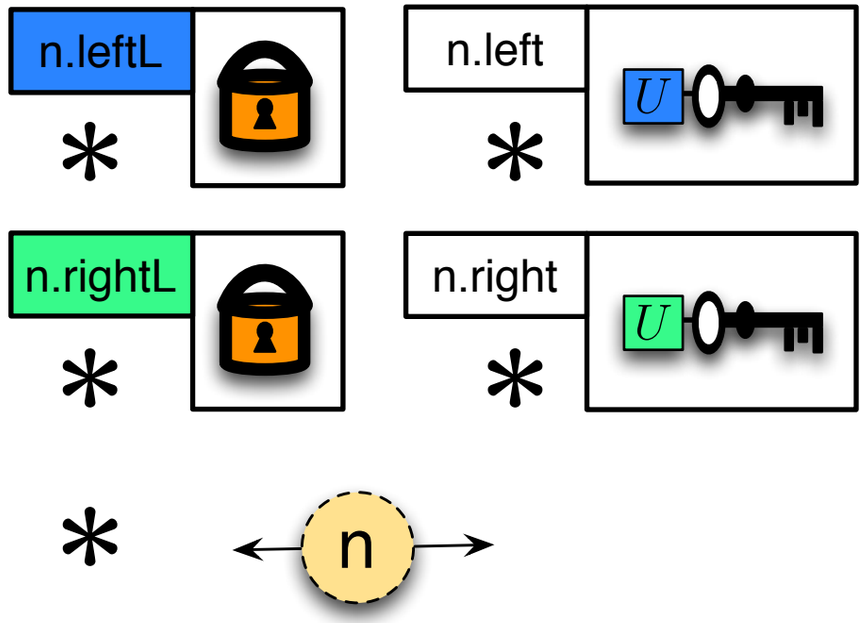
    else if u ≠ null then unlock(u.firstL) ;
    if r ≠ null then unlock(r.leftL) ;
    else if u ≠ null then unlock(u.lastL) ;
    call disposeForest(d) ;
    disposeNode(n) ;
}
```

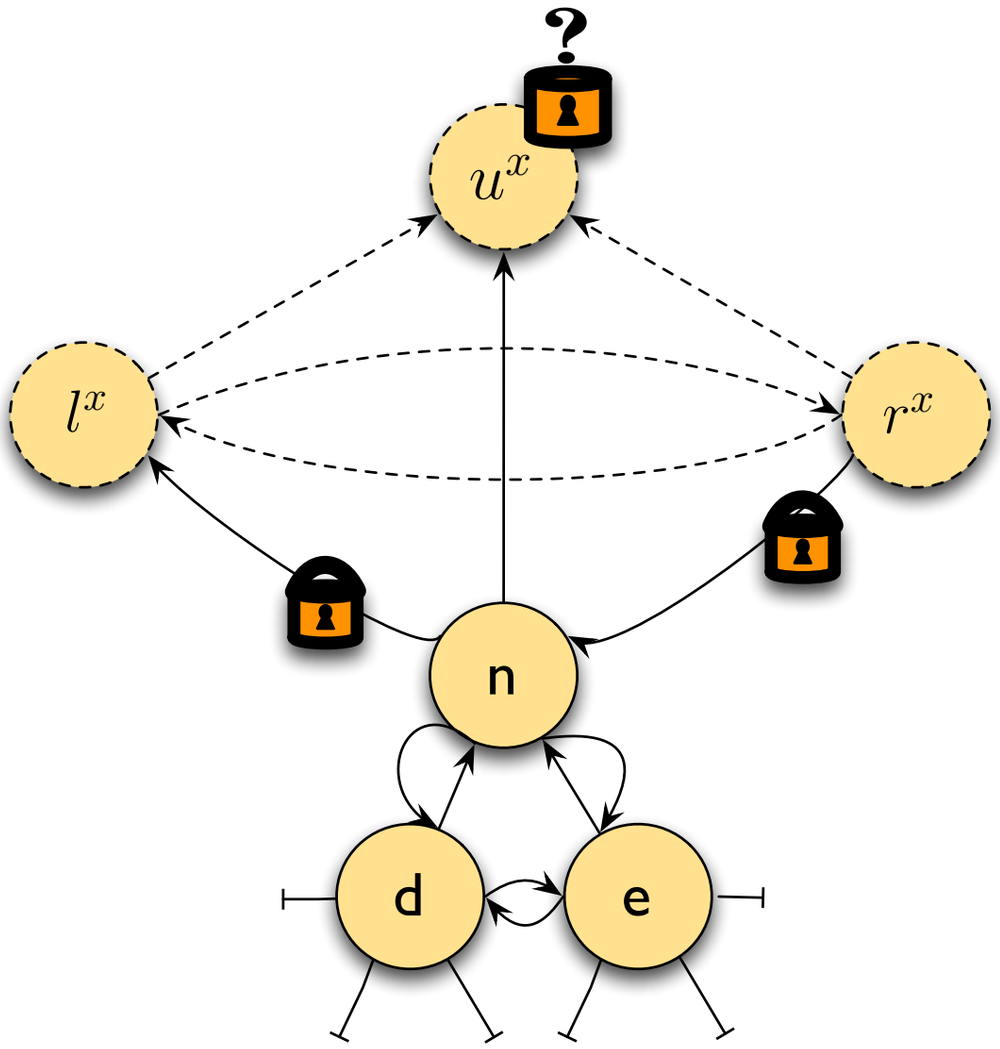


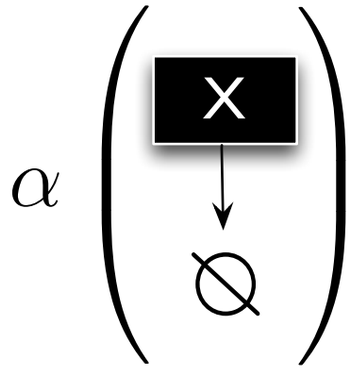


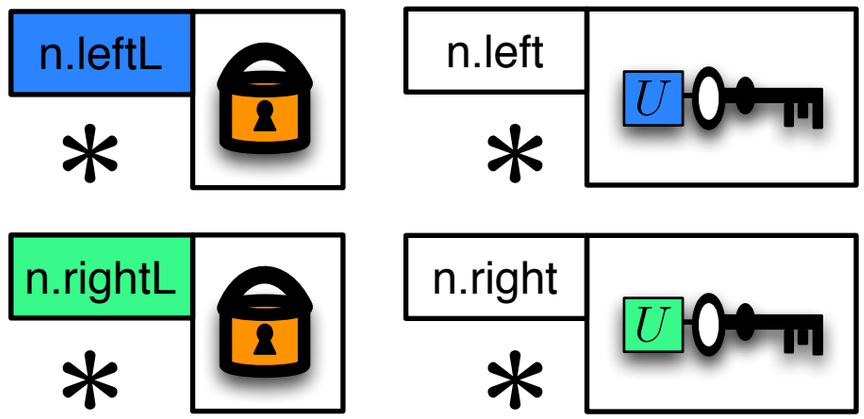


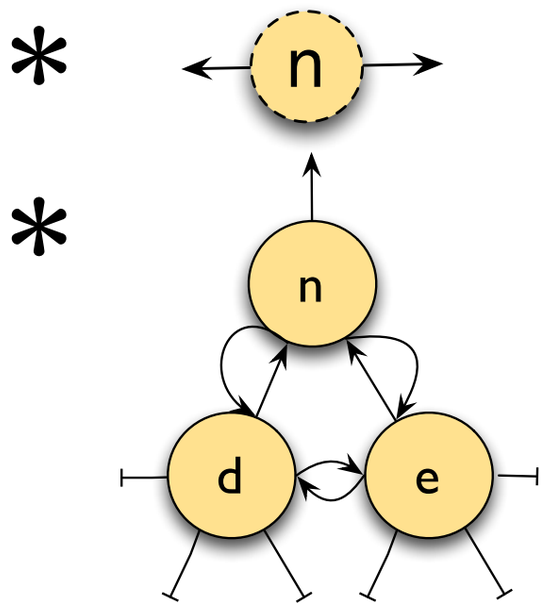
Refinement (Axiomatic Correctness)

```
proc deleteTree(n){
  local l,u,d,r,ul in
    u := [n.up] ; d := [n.first] ; ul:= [n.upL] ;
    //Acquiring the necessary locks.
    lock(ul) ;
    lock(n.leftL) ; l:= [n.left] ;
    if l ≠ null then lock(l.rightL)
    else if u ≠ null then lock(u.firstL) ;
    lock(n.rightL) ; r:= [n.right] ;
    if r ≠ null then lock(r.leftL) ;
    else if u ≠ null then lock(u.lastL) ;
    unlock(ul) ;
    //Pointer Swinging.
    if l ≠ null then      [l.right] := r ;
    else if u ≠ null then [u.first] := r ;
    if r ≠ null then [r.left] := l ;
    else if u ≠ null then [u.last] := l ;
    //Unlocking the acquired locks.
    if l ≠ null then unlock(l.rightL) ;
    else if u ≠ null then unlock(u.firstL) ;
    if r ≠ null then unlock(r.leftL) ;
    else if u ≠ null then unlock(u.lastL) ;
    call disposeForest(d) ;
    disposeNode(n) ;
}
```



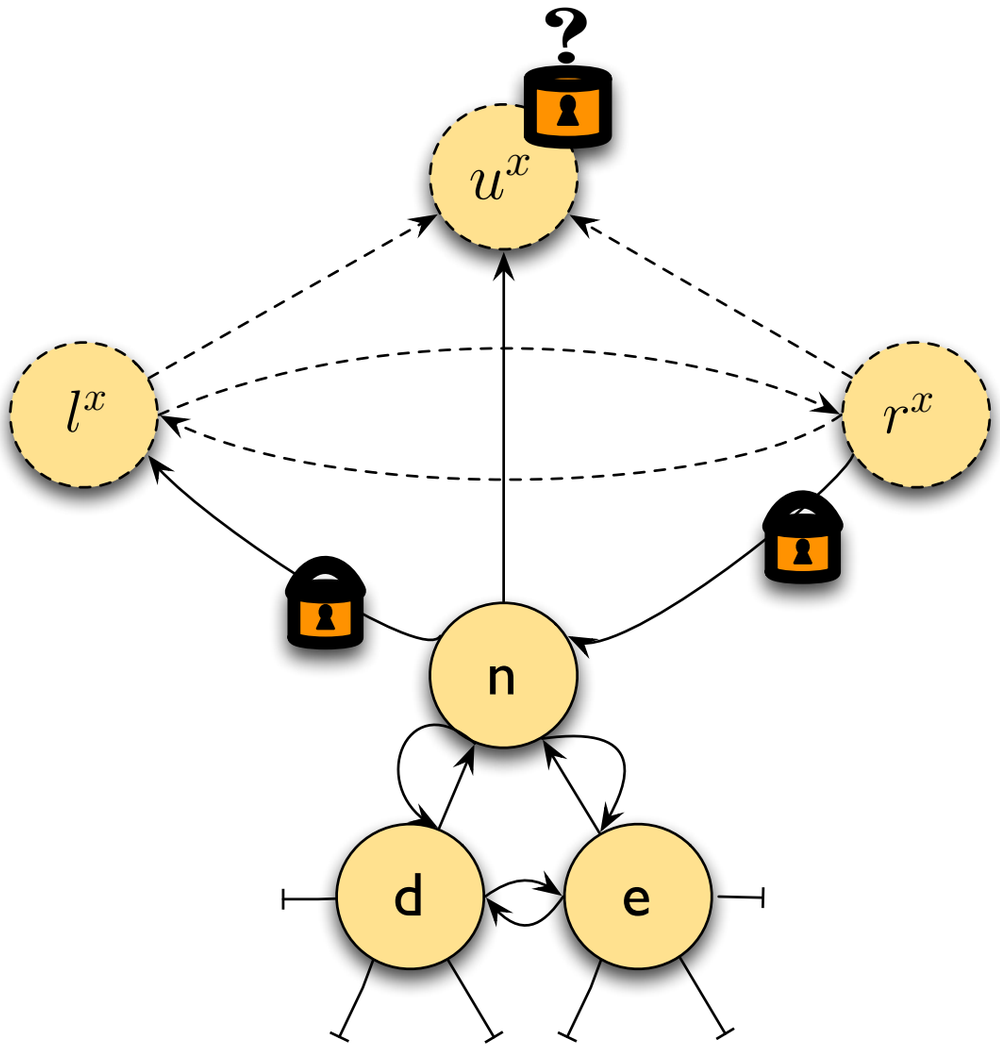


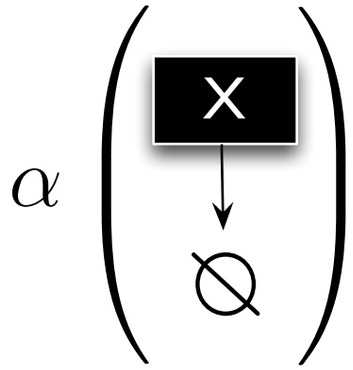


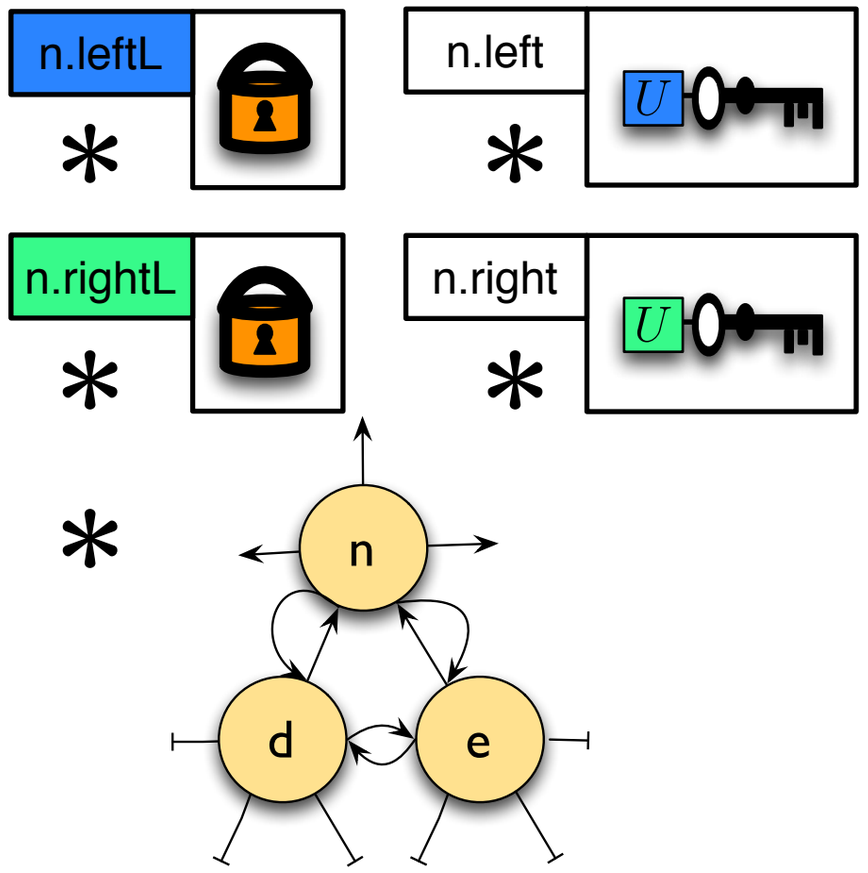


Refinement (Axiomatic Correctness)

```
proc deleteTree(n){
  local l,u,d,r,ul in
    u := [n.up] ; d := [n.first] ; ul:= [n.upL] ;
    //Acquiring the necessary locks.
    lock(ul) ;
    lock(n.leftL) ; l:= [n.left] ;
    if l ≠ null then lock(l.rightL)
    else if u ≠ null then lock(u.firstL) ;
    lock(n.rightL) ; r:= [n.right] ;
    if r ≠ null then lock(r.leftL) ;
    else if u ≠ null then lock(u.lastL) ;
    unlock(ul) ;
    //Pointer Swinging.
    if l ≠ null then      [l.right] := r ;
    else if u ≠ null then [u.first] := r ;
    if r ≠ null then [r.left] := l ;
    else if u ≠ null then [u.last] := l ;
    //Unlocking the acquired locks.
    if l ≠ null then unlock(l.rightL) ;
    else if u ≠ null then unlock(u.firstL) ;
    if r ≠ null then unlock(r.leftL) ;
    else if u ≠ null then unlock(u.lastL) ;
    call disposeForest(d) ;
    disposeNode(n) ;
}
```







Refinement (Axiomatic Correctness)

```
proc deleteTree(n){
  local l,u,d,r,ul in
    u := [n.up] ; d := [n.first] ; ul:= [n.upL] ;
    //Acquiring the necessary locks.
    lock(ul) ;
    lock(n.leftL) ; l:= [n.left] ;
    if l ≠ null then lock(l.rightL)
    else if u ≠ null then lock(u.firstL) ;
    lock(n.rightL) ; r:= [n.right] ;
    if r ≠ null then lock(r.leftL) ;
    //Pointer Swinging.
    if l ≠ null then      [l.right] := r ;
    else if u ≠ null then [u.first] := r ;
    if r ≠ null then [r.left] := l ;
    else if u ≠ null then [u.last] := l ;
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    if l ≠ null then unlock(l.rightL) ;
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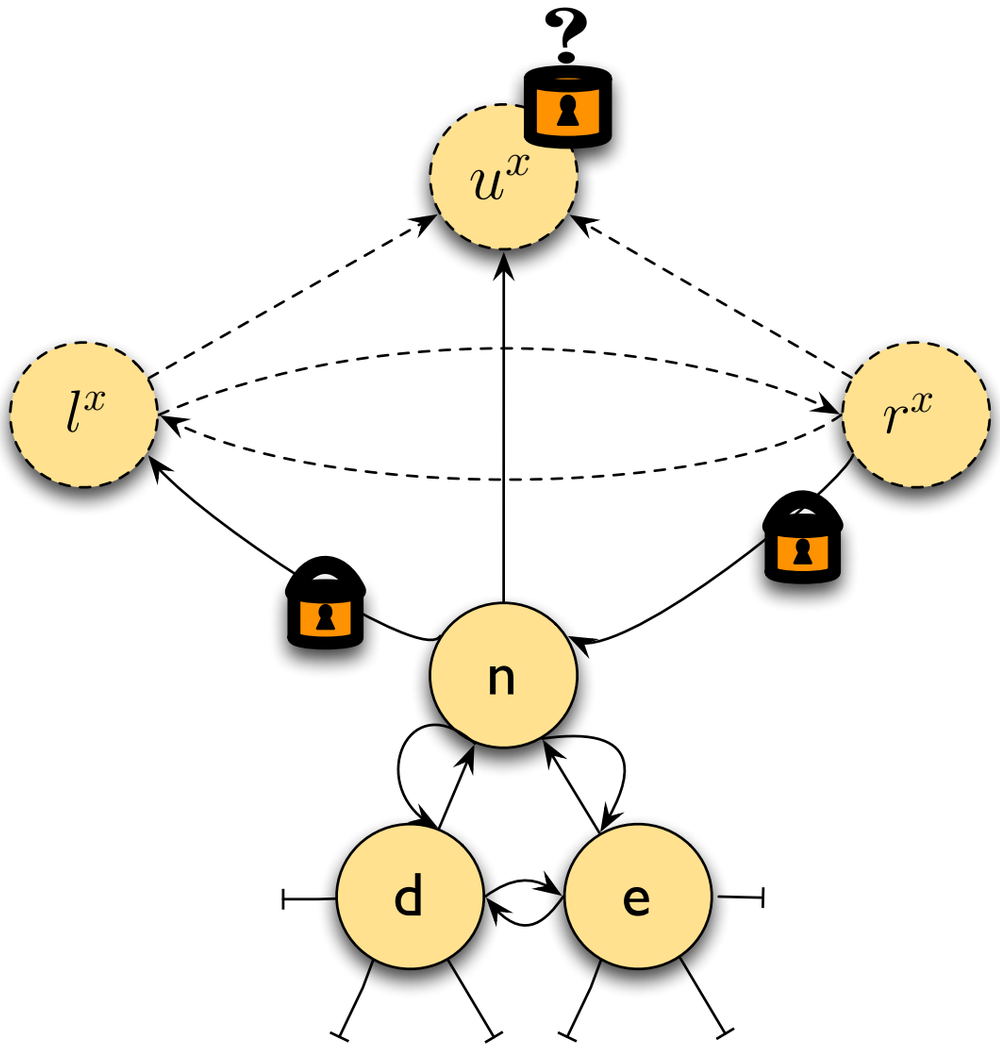
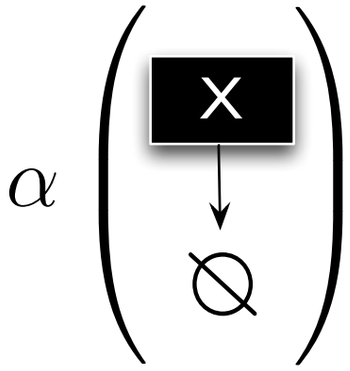
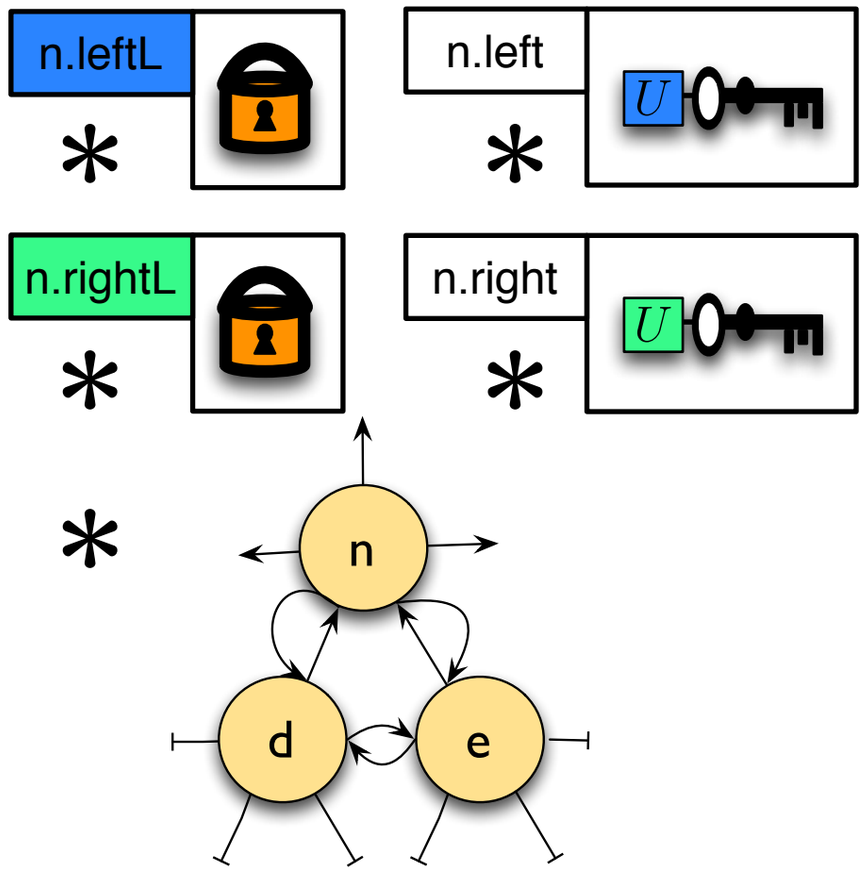
    if r ≠ null then unlock(r.leftL) ;
    else if u ≠ null then unlock(u.lastL) ;
    call disposeForest(d) ;
    disposeNode(n) ;
}
```

disposeTree(n)







Refinement (Axiomatic Correctness)

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    else if u ≠ null then unlock(u.lastL) ;
    call disposeForest(d) ;
    disposeNode(n) ;
}
```
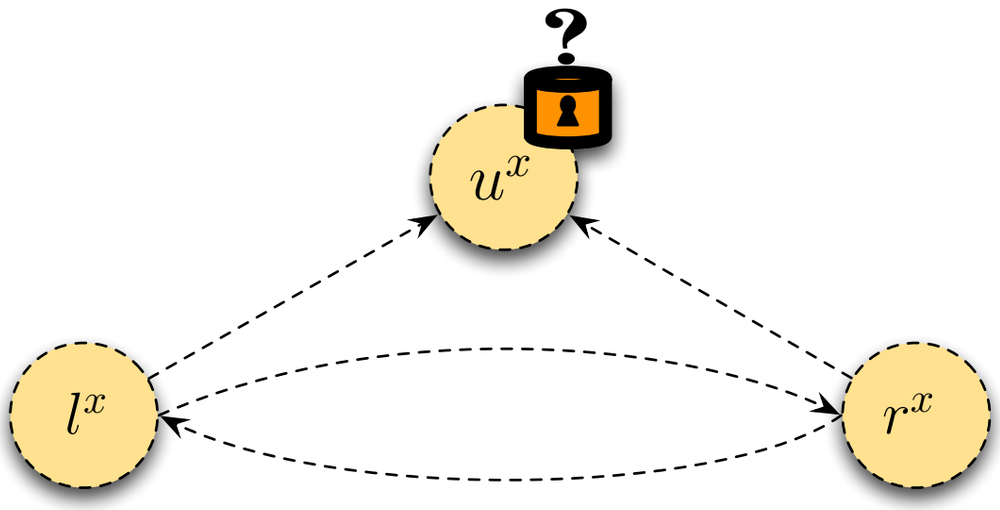
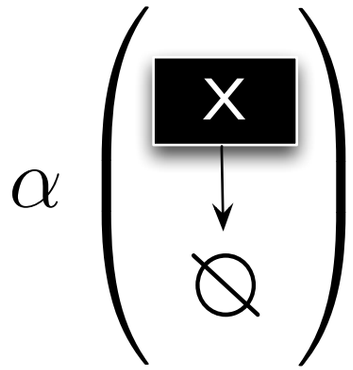
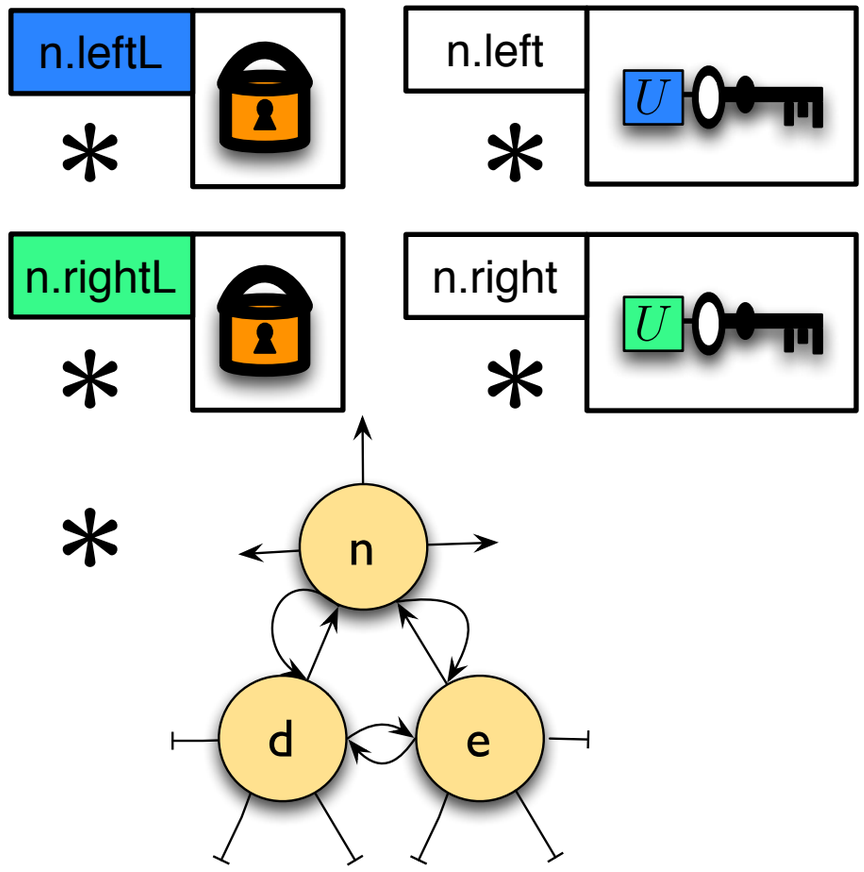

disposeTree(n)







Refinement (Axiomatic Correctness)

```
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    lock(ul) ;
    lock(n.leftL) ; l:= [n.left] ;
    if l ≠ null then lock(l.rightL)
    else if u ≠ null then lock(u.firstL) ;
    lock(n.rightL) ; r:= [n.right] ;
    if r ≠ null then lock(r.leftL) ;
    //Pointer Swinging.
    if l ≠ null then      [l.right] := r ;
    else if u ≠ null then [u.first] := r ;
    if r ≠ null then [r.left] := l ;
    else if u ≠ null then [u.last] := l ;
    //Unlocking the acquired locks.
    if l ≠ null then unlock(l.rightL) ;
    else if u ≠ null then unlock(u.firstL) ;
    if r ≠ null then unlock(r.leftL) ;
    else if u ≠ null then unlock(u.lastL) ;
    call disposeForest(d) ;
    disposeNode(n) ;
}
```
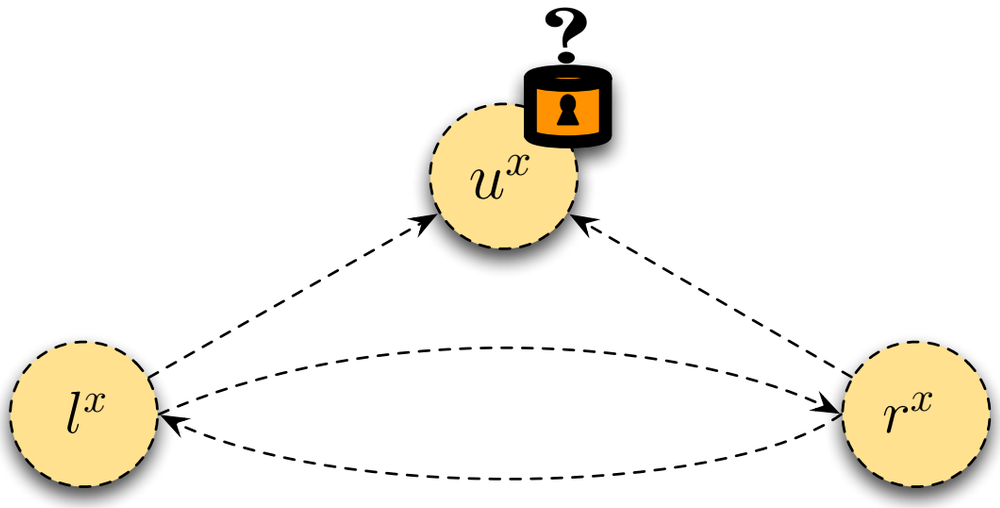
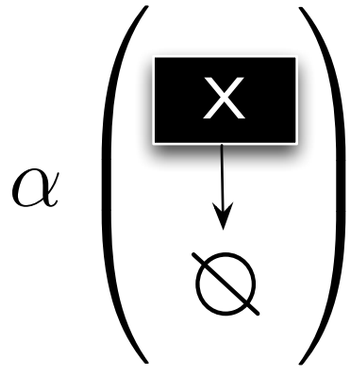

disposeTree(n)





Thank you for listening.

Questions?