

On the Semantics of Snapshot Isolation

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Tuesday 15 January
VMCAI 2019
Lisbon, Portugal

What is STM?

Concurrency Control via *transactions*

- ▶ **atomic** unit of work (set of operations) on shared data
- ▶ **all-or-nothing**

```
// x = y = 0  
  
T: [ x := 1;  
    y := 1;  
    ]  
  
// x = y = 0    OR    x = y = 1
```

Which STM?

Strong consistency - inefficient

- ▶ serialisability
- ▶ strict serialisability
- ▶ ...

Weak consistency

- ▶ parallel snapshot isolation (PSI)
- ▶ snapshot isolation (SI)
- ▶ ...

Which STM?

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Weak consistency

- ▶ parallel snapshot isolation (PSI)
 - ▶ our earlier ESOP paper
- ▶ **snapshot isolation (SI)**
 - ▶ efficient, popular \Rightarrow this talk

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STM Context

- ▶ Shared memory setting
(with **weak** memory consistency)
- ▶ **Mixed** accesses to shared data
(transactional and non-transactional)
- ▶ **Cannot instrument** non-transactional accesses
(weak isolation)

SI STM Desiderata

- ▶ *Declarative* semantics
- ▶ Lock-based reference implementation (*operational* semantics)
 - ➔ **Sound:** Behaviours(imp) \subseteq Behaviours(spec)
 - ➔ **Complete:** Behaviours(spec) \subseteq Behaviours(imp)

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- ▶ *Declarative* semantics with ***mixed*** accesses
- ▶ Reference implementation with ***mixed*** accesses (*operational* semantics)
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SI STM Desiderata: State of Art

✓ *Declarative semantics*

✗ Lock-based reference implementation (*operational semantics*)

→ **Sound:** Behaviours(imp) \subseteq Behaviours(spec)

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SI STM Desiderata: Our Work

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✓ *Declarative semantics*

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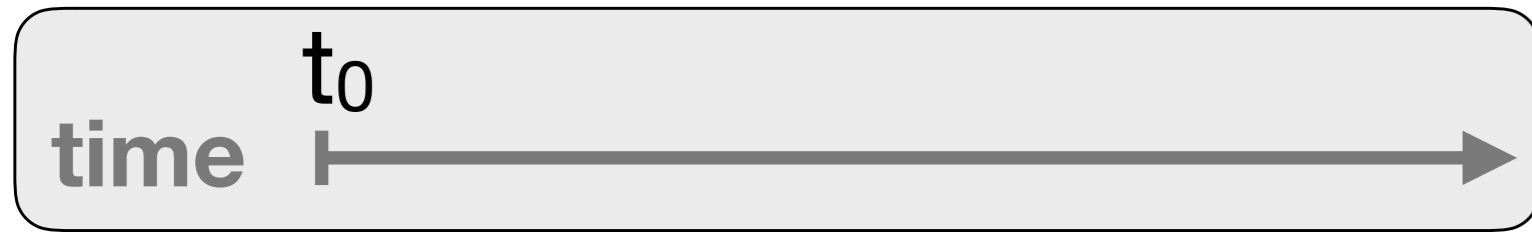
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What is SI?



$\times \mapsto$

$t_0 : 0$

What is SI?

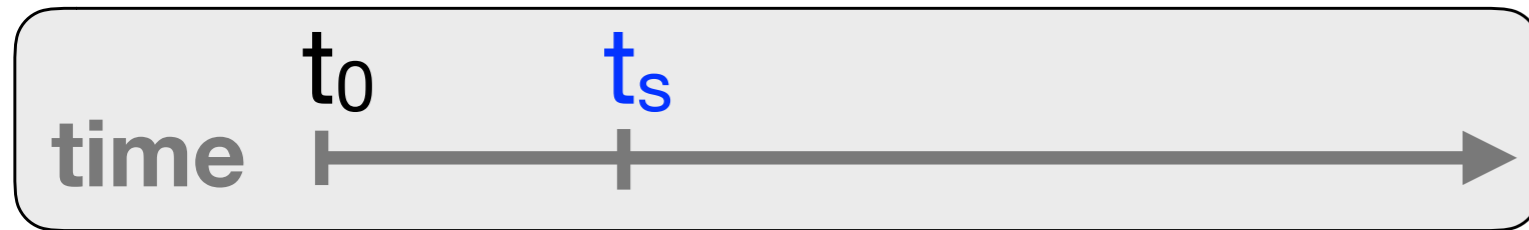


$x \mapsto$

$t_0 : 0$

```
[ x := 1 ;
```

What is SI?



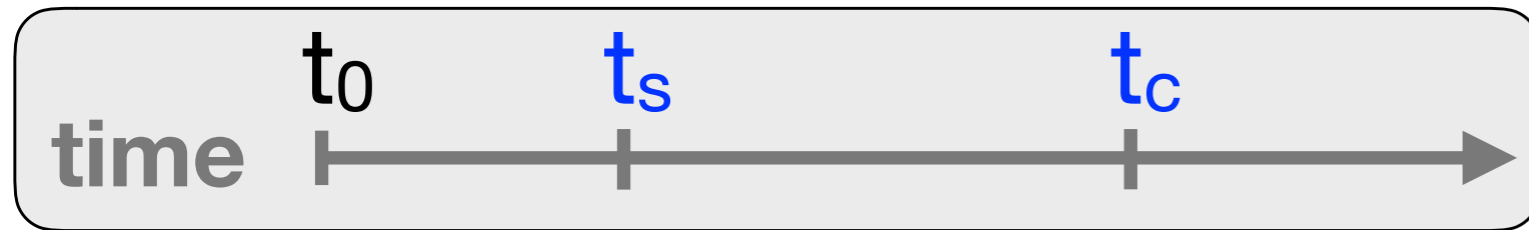
$x \mapsto$

$t_0 : 0$

$t_s : S \triangleq x=0$

$[x := 1 ;$

What is SI?



$x \mapsto$

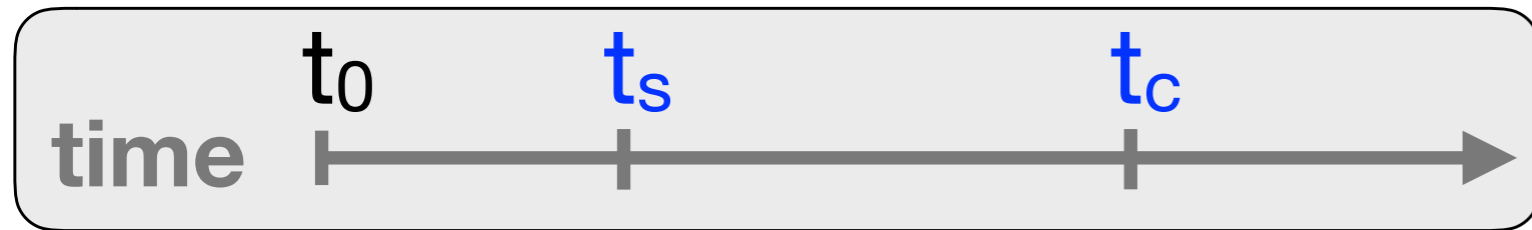
$t_0 : 0$

$t_s : \mathbf{S} \triangleq x=0$

$[x := 1 ;$

$t_c : \mathbf{C} \triangleq x=1$

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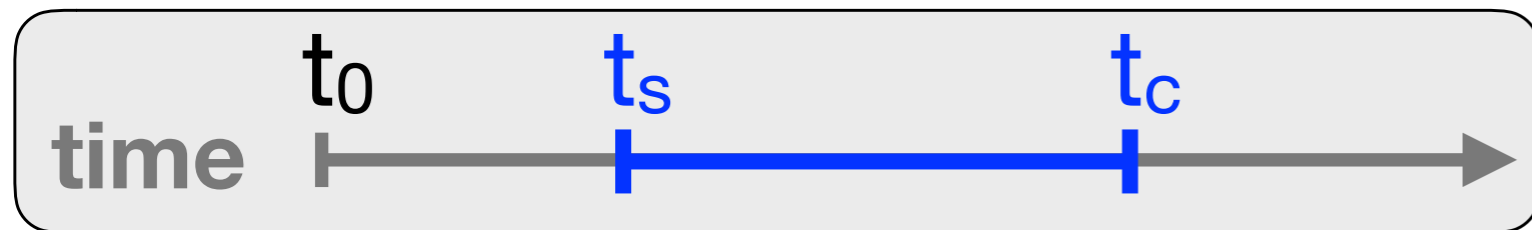
$[x := 1 ;$

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$ww\text{-conflict}(C, t_s, t_c)?$

$ww\text{-conflict}(C, t_s, t_c): \exists x \in C. \text{ a committed trans. updates } x \text{ during } [t_s, t_c]$

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$x \mapsto$

$t_0 : 0 \mid t_c : 1$

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SI: *Allowed* Anomalies

write skew (WS) / store buffering (SB)

$$\mathbf{T1:} \begin{bmatrix} x := 1; \\ a := y; // 0 \end{bmatrix} \parallel \mathbf{T2:} \begin{bmatrix} y := 1; \\ b := x; // 0 \end{bmatrix}$$

// v denotes that v was read

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Not allowed under ***serialisability!***

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T1 executes before T2



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write skew (WS) / store buffering (SB)

T1: $\begin{bmatrix} x := 1; \\ a := y; // 0 \end{bmatrix}$ || **T2**: $\begin{bmatrix} y := 1; \\ b := x; // 0 \end{bmatrix}$ **X**

// v denotes that v was read

Not allowed under ***serialisability!***

T1 executes before T2

X

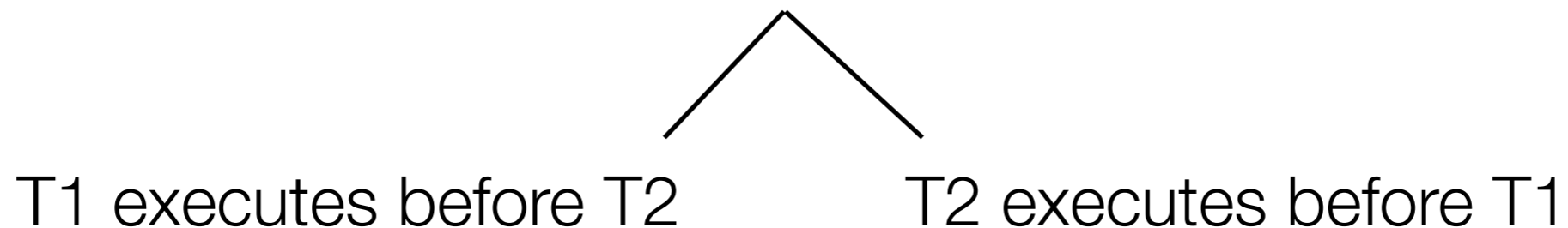
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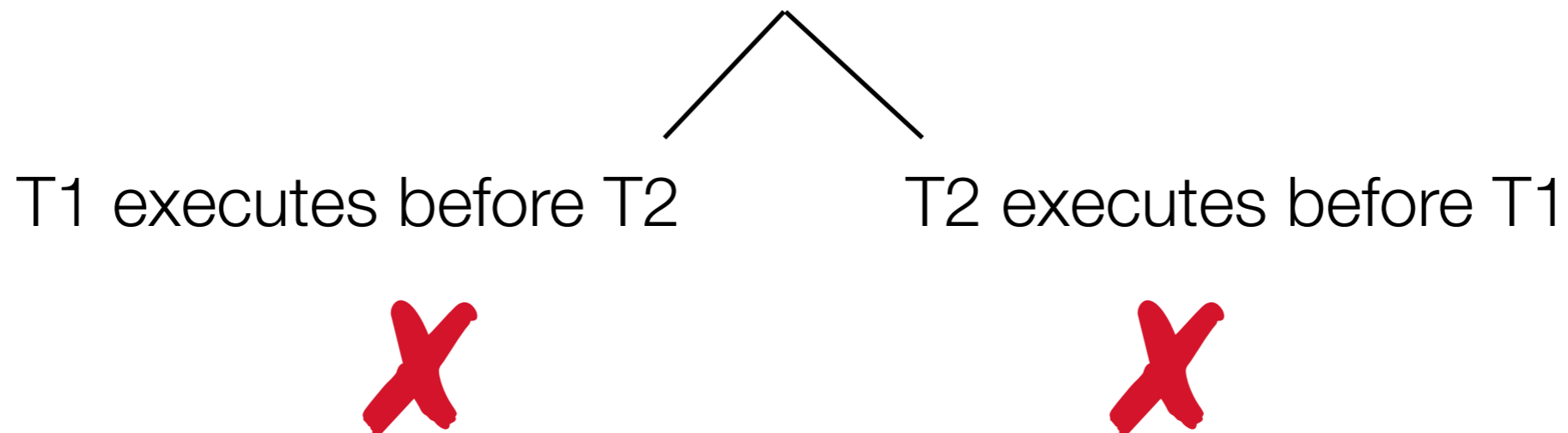
SI: *Allowed* Anomalies

write skew (WS) / store buffering (SB)

T1: $\left[\begin{array}{l} x := 1; \\ a := y; \end{array} // 0 \right. \bigg| \bigg| \left. \begin{array}{l} \mathbf{T2}: \\ y := 1; \\ b := x; \end{array} // 0 \right.$

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T1: $\begin{bmatrix} x := 1; \\ a := y; // 0 \end{bmatrix}$ \parallel **T2:** $\begin{bmatrix} y := 1; \\ b := x; // 0 \end{bmatrix}$

time t_0 \longrightarrow



$x \mapsto$

$t_0 : 0$


$y \mapsto$

$t_0 : 0$

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write skew (WS) / store buffering (SB)

T1: $\begin{bmatrix} x := 1; \\ a := y; // 0 \end{bmatrix} \parallel \begin{bmatrix} y := 1; \\ b := x; // 0 \end{bmatrix}$ **T2**

time t_0 



$x \mapsto$

$t_0 : 0$

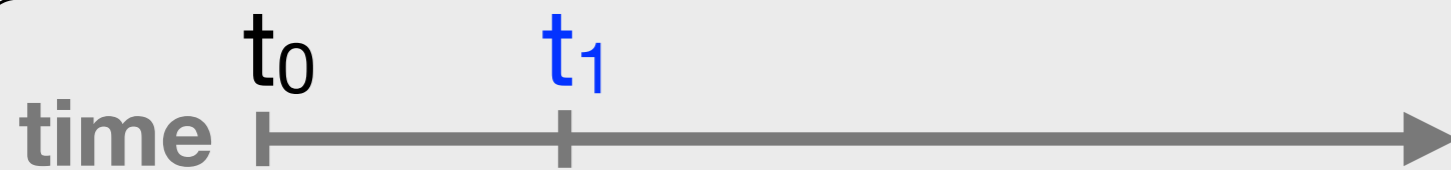
$y \mapsto$

$t_0 : 0$

$\begin{bmatrix} x := 1; \\ a := y; \end{bmatrix}$

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$x \mapsto$

$t_0 : 0$

$y \mapsto$

$t_0 : 0$

$t_1 : \mathbf{S1:} x=y=0$

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$t_0 : 0$

$y \mapsto$

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$t_1 : S1: x=y=0$

$$\begin{bmatrix} x := 1; \\ a := y; // 0 \end{bmatrix}$$

$t_2 : S2: x=y=0$

$$\begin{bmatrix} y := 1; \\ b := x; // 0 \end{bmatrix}$$

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$t_1 : \mathbf{S1:} x=y=0$

$$\begin{bmatrix} x := 1; \\ a := y; // 0 \end{bmatrix}$$

$t_3 : \mathbf{C1:} x=1$

$t_2 : \mathbf{S2:} x=y=0$

$$\begin{bmatrix} y := 1; \\ b := x; // 0 \end{bmatrix}$$

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ww-conflict? no

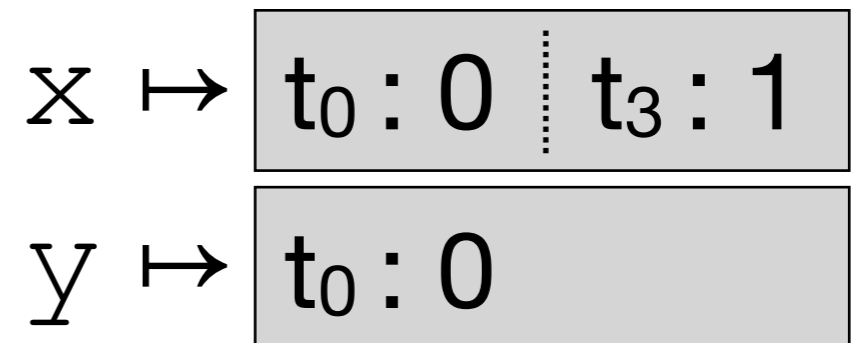
$t_2 : \mathbf{S2:} x=y=0$

$$\begin{bmatrix} y := 1; \\ b := x; // 0 \end{bmatrix}$$

$\text{ww-conflict}(\mathbf{C}, t_s, t_c): \exists x \in \mathbf{C}. \text{ a committed trans. updates } x \text{ during } [t_s, t_c]$

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$x \mapsto \begin{matrix} t_0 : 0 \\ \vdots \\ t_3 : 1 \end{matrix}$
 $y \mapsto \begin{matrix} t_0 : 0 \end{matrix}$

$t_1 : S1: x=y=0$

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$t_3 : C1: x=1$

ww-conflict? no

$t_2 : S2: x=y=0$

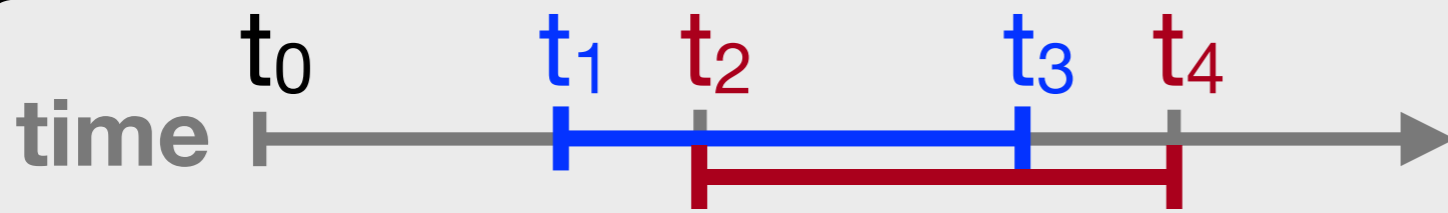
$$\begin{bmatrix} y := 1; \\ b := x; // 0 \end{bmatrix}$$

$t_4 : C2: y=1$

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$x \mapsto t_0 : 0 \mid t_3 : 1$
 $y \mapsto t_0 : 0$

$t_1 : S1: x=y=0$

$$\begin{bmatrix} x := 1; \\ a := y; // 0 \end{bmatrix}$$

$t_3 : C1: x=1$

ww-conflict? no

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$t_2 : S2: x=y=0$

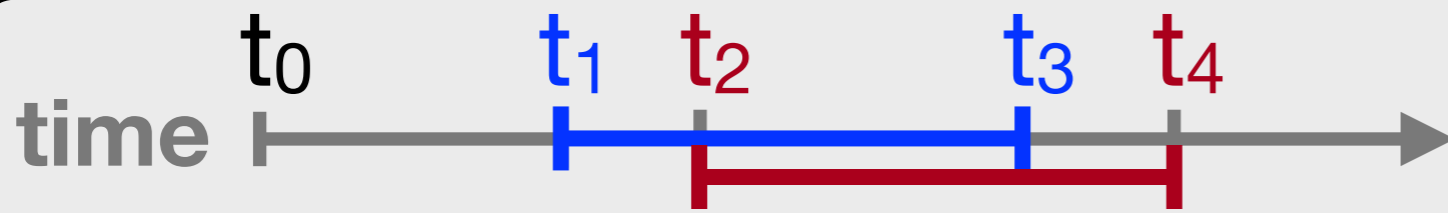
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 $y \mapsto t_0 : 0 \mid t_4 : 1$

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$t_3 : C1: x=1$

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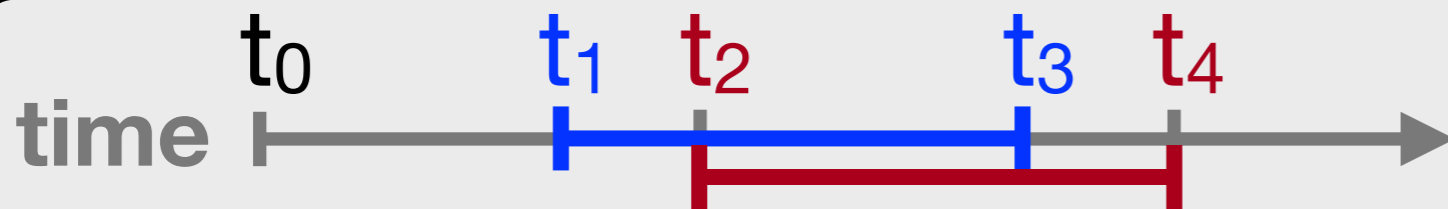
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Interval Order used to detect conflicts:

if: two intervals are **unordered** (they overlap)
then: they have **no write** on the **same location**

t_3 : C1: $x=1$

ww-conflict? no

ww-conflict? no

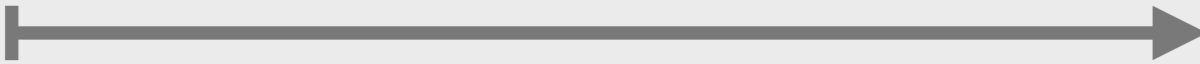
t_4 : C2: $y=1$

$\text{ww-conflict}(\mathbf{C}, t_s, t_c)$: $\exists x \in \mathbf{C}$. a committed trans. updates x during $[t_s, t_c]$

SI: *Disallowed* Anomalies

lost update (LU)

$$\mathbf{T1} : \begin{cases} a := x; & // 0 \\ x := a+1; \end{cases} \quad || \quad \mathbf{T2} : \begin{cases} b := x; & // 0 \\ x := b+1; \end{cases}$$

time t_0 



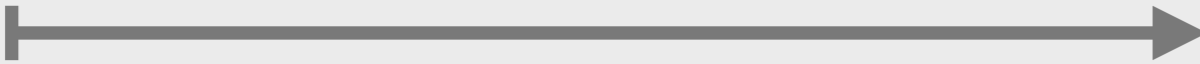
$x \mapsto$

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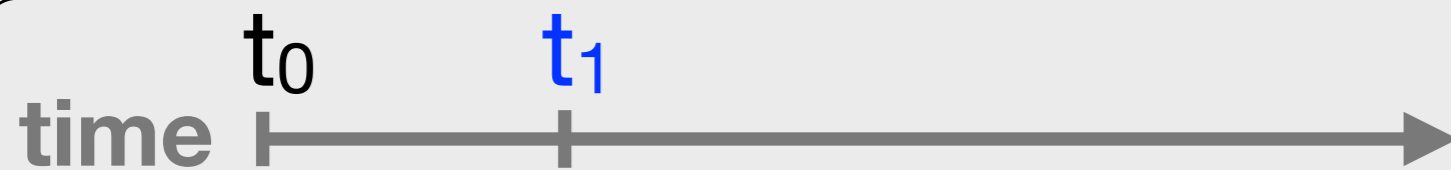
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S1: *Disallowed* Anomalies

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$t_0 : 0$

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$t_2 : S2: x=0$

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$x \mapsto$

$t_0 : 0 \quad \vdots \quad t_3 : 1$

$t_1 : \mathbf{S1} : x=0$

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$t_3 : \mathbf{C1} : x=1$

ww-conflict? no

$t_2 : \mathbf{S2} : x=0$

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$x \mapsto$

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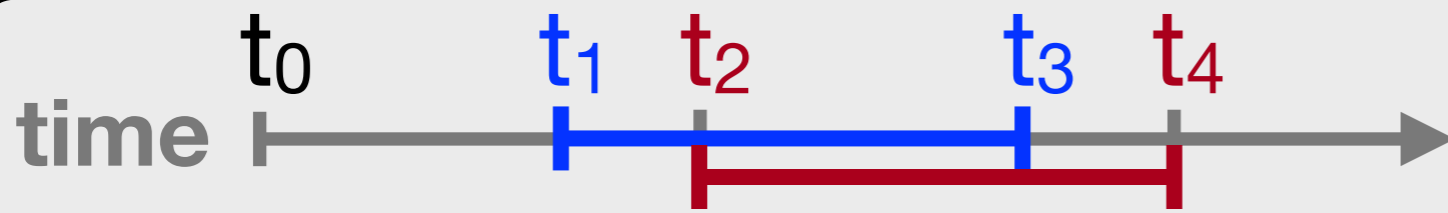
$t_2 : \mathbf{S2} : x=0$

$$\begin{cases} a := x; // 0 \\ x := a+1; \end{cases}$$

$t_4 : \mathbf{C2} : x=1$

S1: *Disallowed* Anomalies

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$x \mapsto$

$t_0 : 0 \quad \vdots \quad t_3 : 1$

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$t_3 : \mathbf{C1} : x=1$

ww-conflict? **no**

ww-conflict? **yes**

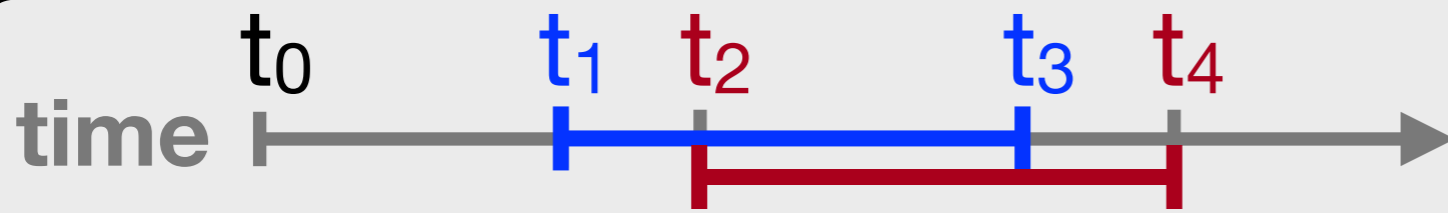
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$t_1 : \mathbf{S1} : x=0$

$$\begin{cases} a := x; & // 0 \\ x := a+1; \end{cases}$$

$t_3 : \mathbf{C1} : x=1$

ww-conflict? **no**

ww-conflict? **yes**

$t_2 : \mathbf{S2} : x=0$

$$\begin{cases} a := x; & // 0 \\ x := a+1; \end{cases}$$

$t_4 : \mathbf{C2} : x=1$

Reasoning about
timestamps and **interval order**
is **difficult**

Reasoning about
timestamps and **interval order**
is **difficult**



Lock-based

SI reference implementation

Which Locks for SI?

X *Global* lock

→ *disjoint* accesses allowed

X *Per-location* locks

→ concurrent reads allowed

✓ *Per-location MRSW* (multiple-readers-single-writer) locks

SI Reference Implementation Pattern

SI (T) \triangleq

```
// must hold read locks on RS  
s := snapshot(RS)  
// must hold write locks on WS  
[ T ] ;
```

snapshot(RS) \triangleq

```
for (x ∈ RS) s[x] := x
```

$[a := x] \triangleq a := s[x]$

$[x := a] \triangleq x := a; s[x] := a$

$[s1; s2] \triangleq [s1] ; [s2]$

...

SI Reference Implementation Pattern

SI (T) \triangleq

```
// must hold read locks on RS  
s := snapshot(RS)  
// must hold write locks on WS  
[[ T ]];
```

snapshot(RS) \triangleq

for (x ∈ RS) s[x] := x

[[a := x] \triangleq a := s[x]

[[x := a] \triangleq x := a; s[x] := a

[[s1; s2] \triangleq [[s1] ; [[s2]

...

➔ **sound:** behaviours(imp) \subseteq behaviours(SI_spec)

e.g. [disallow lost update \(LU\)](#)

➔ **complete:** behaviours(SI_spec) \subseteq behaviours(imp)

e.g. [allow write skew \(WS\)](#)

SI Reference Implementation: Attempt 1

SI (T) \triangleq

```
// must hold read locks on RS  
s := snapshot(RS)  
// must hold write locks on WS  
[ T ] ;
```

sound: disallow lost update (LU)

complete: allow write skew (WS)

SI Reference Implementation: Attempt 1

```
SI (T)  $\triangleq$   
r_lock (RS) ;  
s := snapshot (RS) ;  
r_unlock (RS) ;  
// must hold write locks on WS  
[ T ]
```

sound: disallow lost update (LU)

complete: allow write skew (WS)

SI Reference Implementation: Attempt 1

```
SI (T)  $\triangleq$   
r_lock (RS) ;  
s := snapshot (RS) ;  
r_unlock (RS) ;  
w_lock (WS) ;  
[ T ] ;  
w_unlock (WS) ;
```

sound: [disallow](#) lost update (LU)

complete: [allow](#) write skew (WS)

SI Reference Implementation: Attempt 1

```
SI (T)  $\triangleq$   
r_lock (RS) ;  
s := snapshot (RS) ;  
r_unlock (RS) ;  
w_lock (WS) ;  
[ T ] ;  
w_unlock (WS) ;
```

sound: disallow lost update (LU) 

complete: allow write skew (WS)

SI Reference Implementation: Attempt 1

```
SI (T)  $\triangleq$   
r_lock (RS) ;  
s := snapshot (RS) ;  
r_unlock (RS) ;  
w_lock (WS) ;  
[ T ] ;  
w_unlock (WS) ;
```

sound: disallow lost update (LU) **X**

```
T1: [ a := x ; // 0  
      x := a + 1 ;  
      ] ||  
      (LU)  
T2: [ b := x ; // 0  
      x := b + 1 ;  
      ]
```

SI Reference Implementation: Attempt 1



```
SI (T)  $\triangleq$   
r_lock (RS) ;  
s := snapshot (RS) ;  
r_unlock (RS) ;  
w_lock (WS) ;  
[ T ] ;  
w_unlock (WS) ;
```

sound: disallow lost update (LU) **X**

```
T1: [ a := x; // 0  
      x := a + 1; ]  
      ||  
      (LU)  
T2: [ b := x; // 0  
      x := b + 1; ]
```


SI Reference Implementation: Attempt 1



SI (T) \triangleq

```
r_lock (RS) ;  
s := snapshot (RS) ;  
r_unlock (RS) ;  
w_lock (WS) ;  
[ T ] ;  
w_unlock (WS) ;
```



sound: disallow lost update (LU) **X**

T1:	$\left[\begin{array}{l} a := x; // 0 \\ x := a + 1; \end{array} \right.$	\parallel	T2:	$\left[\begin{array}{l} b := x; // 0 \\ x := b + 1; \end{array} \right.$
		(LU)		

SI Reference Implementation: Attempt 1

SI (T) \triangleq

```
r_lock (RS);  
s := snapshot (RS);  
r_unlock (RS);  
w_lock (WS);  
[ T ];  
w_unlock (WS);
```



sound: disallow lost update (LU) **X**

T1: $\left[\begin{array}{l} a := x; // 0 \\ x := a + 1; \end{array} \right. \parallel \left. \begin{array}{l} \mathbf{T2:} \left[\begin{array}{l} b := x; // 0 \\ x := b + 1; \end{array} \right. \right.$

(LU)

SI Reference Implementation: Attempt 1

SI (T) \triangleq

```
r_lock (RS) ;  
s := snapshot (RS) ;  
r_unlock (RS) ;  
w_lock (WS) ;  
[ T ] ;  
w_unlock (WS) ;
```



sound: disallow lost update (LU) **X**

T1: $\left[\begin{array}{l} a := x; // 0 \\ x := a + 1; \end{array} \right. \parallel \left. \begin{array}{l} \mathbf{T2:} \left[\begin{array}{l} b := x; // 0 \\ x := b + 1; \end{array} \right. \right.$

(LU)

SI Reference Implementation: Attempt 1

```
SI (T)  $\triangleq$   
r_lock (RS) ;  
s := snapshot (RS) ;  
r_unlock (RS) ;  
w_lock (WS) ;  
[ T ] ;  
w_unlock (WS) ;
```

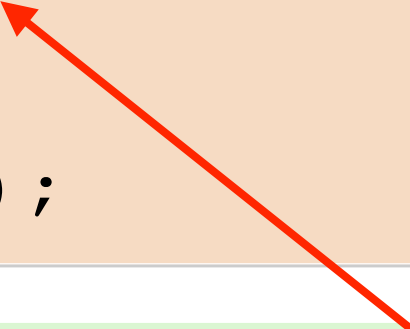


sound: disallow lost update (LU) **X**

```
T1: [ a := x; //0  
       x := a+1; ]  
      ||  
      (LU)  
      ||  
T2: [ b := x; //0  
       x := b+1; ]
```

SI Reference Implementation: Attempt 1

```
SI (T)  $\triangleq$   
r_lock (RS) ;  
s := snapshot (RS) ;  
r_unlock (RS) ;  
w_lock (WS) ;  
[ T ] ;  
w_unlock (WS) ;
```



w_locks are acquired **too late!**
acquire them **before releasing** r_locks!

(LU)

SI Reference Implementation: Attempt 2

```
SI (T)  $\triangleq$   
w_lock (WS) ;  
r_lock (RS \ WS) ;  
s := snapshot (RS) ;  
r_unlock (RS) ;  
[ T ] ;  
w_unlock (WS) ;
```

SI Reference Implementation: Attempt 2

SI (T) \triangleq

`w_lock(WS);`

`r_lock(RS\WS);`

`s := snapshot(RS);`

`r_unlock(RS);`

`[T] ;`

`w_unlock(WS);`

sound: disallow lost update (LU)

complete: allow write skew (WS)

SI Reference Implementation: Attempt 2

SI (T) \triangleq

```
w_lock(WS);
```

```
r_lock(RS\WS);
```

```
s := snapshot(RS);
```

```
r_unlock(RS);
```

```
[ T ] ;
```

```
w_unlock(WS);
```

sound: disallow lost update (LU) 

complete: allow write skew (WS)

SI Reference Implementation: Attempt 2

SI (T) \triangleq

```
w_lock(WS);  
r_lock(RS\WS);  
s := snapshot(RS);  
r_unlock(RS);  
[ T ] ;  
w_unlock(WS);
```

sound: disallow lost update (LU) 

complete: allow write skew (WS) 

T1: $\left[\begin{array}{l} x := 1; \\ a := y; // 0 \end{array} \right. \parallel \left. \begin{array}{l} \mathbf{T2:} \\ y := 1; \\ b := x; // 0 \end{array} \right]$
(WS)

SI Reference Implementation: Attempt 2

SI (T) \triangleq

```
w_lock(WS);  
r_lock(RS\WS);  
s := snapshot(RS);  
r_unlock(RS);  
[ T ] ;  
w_unlock(WS);
```

sound: disallow lost update (LU) 

complete: allow write skew (WS) 

T1: $\left[\begin{array}{l} x := 1; \\ a := y; // 0 \end{array} \right] \parallel \left[\begin{array}{l} y := 1; \\ b := x; // 0 \end{array} \right]$
(WS)

```
1. w_lock({x}); T1  
2. r_lock({y});  
3. s[y] := y;  
4. r_unlock({y});  
5. x := 1; s[x] := 1;  
6. a := s[y];  
7. w_unlock({x});
```

```
8. w_lock({y}); T2  
9. r_lock({x});  
10. s[x] := x;  
11. r_unlock({x});  
12. y := 1; s[y] := 1;  
13. b := s[x];  
14. w_unlock({y});
```

SI Reference Implementation: Attempt 2

SI (T) \triangleq

```
w_lock(WS);  
r_lock(RS\WS);  
s := snapshot(RS);  
r_unlock(RS);  
[ T ] ;  
w_unlock(WS);
```

sound: disallow lost update (LU) ✓

complete: allow write skew (WS) ✗

T1: $\left[\begin{array}{l} x := 1; \\ a := y; // 0 \end{array} \right. \parallel \left. \begin{array}{l} \mathbf{T2:} \\ y := 1; \\ b := x; // 0 \end{array} \right.$
(WS)

```
1. w_lock({x}); T1  
2. r_lock({y});  
3. s[y] := y;  
4. r_unlock({y});  
5. x := 1; s[x] := 1;  
6. a := s[y];  
7. w_unlock({x});
```

```
8. w_lock({y}); T2  
9. r_lock({x});  
10. s[x] := x;  
11. r_unlock({x});  
12. y := 1; s[y] := 1;  
13. b := s[x];  
14. w_unlock({y});
```

1 $\cdots \rightarrow$ 9 9 $\cdots \rightarrow$ 1

$\cdots \rightarrow$: execution order

SI Reference Implementation: Attempt 2

SI (T) \triangleq

```
w_lock(WS);  
r_lock(RS\WS);  
s := snapshot(RS);  
r_unlock(RS);  
[ T ] ;  
w_unlock(WS);
```

sound: disallow lost update (LU) ✓

complete: allow write skew (WS) ✗

T1: $\left[\begin{array}{l} x := 1; \\ a := y; // 0 \end{array} \right] \parallel \left[\begin{array}{l} y := 1; \\ b := x; // 0 \end{array} \right]$
(WS)

```
1. w_lock({x}); T1  
2. r_lock({y});  
3. s[y] := y;  
4. r_unlock({y});  
5. x := 1; s[x] := 1;  
6. a := s[y];  
7. w_unlock({x});
```

```
8. w_lock({y}); T2  
9. r_lock({x});  
10. s[x] := x;  
11. r_unlock({x});  
12. y := 1; s[y] := 1;  
13. b := s[x];  
14. w_unlock({y});
```

1 \dashrightarrow 9 9 \dashrightarrow 1

\dashrightarrow : execution order

SI Reference Implementation: Attempt 2

SI (T) \triangleq

```
w_lock(WS);  
r_lock(RS\WS);  
s := snapshot(RS);  
r_unlock(RS);  
[ T ] ;  
w_unlock(WS);
```

sound: disallow lost update (LU) ✓

complete: allow write skew (WS) ✗

T1: $\left[\begin{array}{l} x := 1; \\ a := y; // 0 \end{array} \right] \parallel \left[\begin{array}{l} y := 1; \\ b := x; // 0 \end{array} \right]$
(WS)

```
1. w_lock({x}); T1  
2. r_lock({y});  
3. s[y] := y;  
4. r_unlock({y});  
5. x := 1; s[x] := 1;  
6. a := s[y];  
7. w_unlock({x});
```

```
8. w_lock({y}); T2  
9. r_lock({x});  
10. s[x] := x;  
11. r_unlock({x});  
12. y := 1; s[y] := 1;  
13. b := s[x];  
14. w_unlock({y});
```

1 \dashrightarrow 9 9 \dashrightarrow 1

\dashrightarrow : execution order

SI Reference Implementation: Attempt 2

SI (T) \triangleq

```
w_lock(WS);
r_lock(RS\WS);
s := snapshot(RS);
r_unlock(RS);
[ T ];
w_unlock(WS);
```

sound: disallow lost update (LU) ✓

complete: allow write skew (WS) ✗

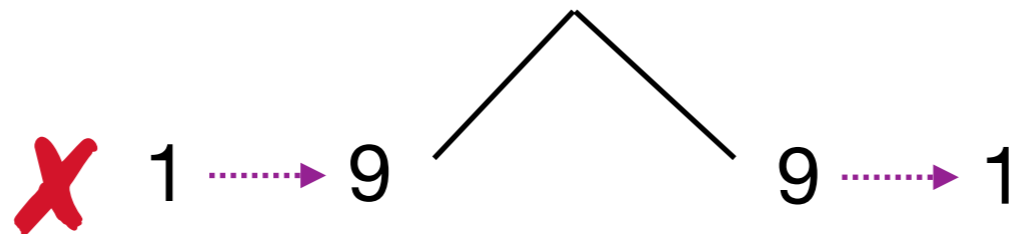
T1: $\left[\begin{array}{l} x := 1; \\ a := y; // 0 \end{array} \right] \parallel \left[\begin{array}{l} y := 1; \\ b := x; // 0 \end{array} \right]$
(WS)

T1

```
1. w_lock({x});
2. r_lock({y});
3. s[y] := y;
4. r_unlock({y});
5. x := 1; s[x] := 1;
6. a := s[y];
7. w_unlock({x});
```

T2

```
8. w_lock({y});
9. r_lock({x});
10. s[x] := x; // 1
11. r_unlock({x});
12. y := 1; s[y] := 1;
13. b := s[x]; // 1
14. w_unlock({y});
```



.....➔ : execution order

SI Reference Implementation: Attempt 2

SI (T) \triangleq

```
w_lock(WS);
r_lock(RS\WS);
s := snapshot(RS);
r_unlock(RS);
[ T ];
w_unlock(WS);
```

sound: disallow lost update (LU) ✓

complete: allow write skew (WS) ✗

T1: $\left[\begin{array}{l} x := 1; \\ a := y; // 0 \end{array} \right] \parallel \left[\begin{array}{l} y := 1; \\ b := x; // 0 \end{array} \right]$
(WS)

T1

```
1. w_lock({x});
2. r_lock({y});
3. s[y] := y;
4. r_unlock({y});
5. x := 1; s[x] := 1;
6. a := s[y];
7. w_unlock({x});
```

T2

```
8. w_lock({y});
9. r_lock({x});
10. s[x] := x; // 1
11. r_unlock({x});
12. y := 1; s[y] := 1;
13. b := s[x]; // 1
14. w_unlock({y});
```

1 \dashrightarrow 9 9 \dashrightarrow 1

\dashrightarrow : execution order

SI Reference Implementation: Attempt 2

SI (T) \triangleq

```
w_lock(WS);  
r_lock(RS\WS);  
s := snapshot(RS);  
r_unlock(RS);  
[ T ] ;  
w_unlock(WS);
```

sound: disallow lost update (LU) ✓

complete: allow write skew (WS) ✗

```
T1: [ x := 1;           || T2: [ y := 1;             
     a := y; // 0      ||      b := x; // 0        
     ]                 ||      ]                   
     (WS)              ||
```

```
1. w_lock({x});           T1  
2. r_lock({y});  
3. s[y] := y;  
4. r_unlock({y});  
5. x := 1; s[x] := 1;  
6. a := s[y];  
7. w_unlock({x});
```

```
8. w_lock({y});           T2  
9. r_lock({x});  
10. s[x] := x; // 1  
11. r_unlock({x});  
12. y := 1; s[y] := 1;  
13. b := s[x]; // 1  
14. w_unlock({y});
```

1 \dashrightarrow 9 9 \dashrightarrow 1

\dashrightarrow : execution order

SI Reference Implementation: Attempt 2

SI (T) \triangleq

```
w_lock(WS);
r_lock(RS\WS);
s := snapshot(RS);
r_unlock(RS);
[ T ];
w_unlock(WS);
```

sound: disallow lost update (LU) ✓

complete: allow write skew (WS) ✗

T1: $\left[\begin{array}{l} x := 1; \\ a := y; // 0 \end{array} \right. \parallel \left. \begin{array}{l} \mathbf{T2:} \\ y := 1; \\ b := x; // 0 \end{array} \right.$
(WS)

```
1. w_lock({x}); T1
2. r_lock({y});
3. s[y] := y;
4. r_unlock({y});
5. x := 1; s[x] := 1;
6. a := s[y];
7. w_unlock({x});
```

```
8. w_lock({y}); T2
9. r_lock({x});
10. s[x] := x; // 1
11. r_unlock({x});
12. y := 1; s[y] := 1;
13. b := s[x]; // 1
14. w_unlock({y});
```

1 \dashrightarrow 9 9 \dashrightarrow 1

\dashrightarrow : execution order

SI Reference Implementation: Attempt 2

SI (T) \triangleq

```
w_lock(WS);  
r_lock(RS\WS);  
s := snapshot(RS);  
r_unlock(RS);  
[ T ] ;  
w_unlock(WS);
```

sound: disallow lost update (LU) ✓

complete: allow write skew (WS) ✗

```
T1: [ x := 1;           || T2: [ y := 1;  
      a := y; // 0     ||      b := x; // 0  
      ] (WS)           ]
```

```
1. w_lock({x});           T1  
2. r_lock({y});  
3. s[y] := y; // 1  
4. r_unlock({y});  
5. x := 1; s[x] := 1;  
6. a := s[y]; // 1  
7. w_unlock({x});
```

```
8. w_lock({y});           T2  
9. r_lock({x});  
10. s[x] := x; // 1  
11. r_unlock({x});  
12. y := 1; s[y] := 1  
13. b := s[x]; // 1  
14. w_unlock({y});
```

1 \dashrightarrow 9 9 \dashrightarrow 1

\dashrightarrow : execution order

SI Reference Implementation: Attempt 2

SI (T) \triangleq

```
w_lock(WS);
r_lock(RS\WS);
s := snapshot(RS);
r_unlock(RS);
[ T ];
w_unlock(WS);
```

sound: disallow lost update (LU) ✓

complete: allow write skew (WS) ✗

T1: $\left[\begin{array}{l} x := 1; \\ a := y; // 0 \end{array} \right] \parallel \left[\begin{array}{l} y := 1; \\ b := x; // 0 \end{array} \right]$
(WS)

T1

```
1. w_lock({x});
2. r_lock({y});
3. s[y] := y; // 1
4. r_unlock({y});
5. x := 1; s[x] := 1;
6. a := s[y]; // 1
7. w_unlock({x});
```

T2

```
8. w_lock({y});
9. r_lock({x});
10. s[x] := x; // 1
11. r_unlock({x});
12. y := 1; s[y] := 1;
13. b := s[x]; // 1
14. w_unlock({y});
```

1 $\dots \rightarrow$ 9 9 $\dots \rightarrow$ 1 ✗

$\dots \rightarrow$: execution order

SI Reference Implementation: Attempt 2

SI (T) \triangleq

```
w_lock(WS);  
r_lock(RS\WS);  
s := snapshot(RS);  
r_unlock(RS);  
[ T ] ;  
w_unlock(WS);
```

sound: disallow lost update (LU) ✓

complete: allow write skew (WS) ✗

T1: $\left[\begin{array}{l} x := 1; \\ a := y; // 0 \end{array} \right. \parallel \left. \begin{array}{l} \mathbf{T2:} \\ y := 1; \\ b := x; // 0 \end{array} \right.$
(WS)

w_locks are acquired **too early!**

1. acquire them **as r_locks** first;
2. **promote them** to writers later

.....➔ : execution order

SI Reference Implementation: Attempt 3

SI (T) \triangleq

r_lock (RS \cup WS) ;

s := snapshot (RS) ;

r_unlock (RS \ WS) ;

promote (WS) ;

[T] ;

w_unlock (WS) ;

sound: disallow lost update (LU)

complete: allow write skew (WS)

SI Reference Implementation: Attempt 3

SI (T) \triangleq

r_lock (RS \cup WS) ;

s := snapshot (RS) ;

r_unlock (RS \ WS) ;

promote (WS) ;

[T] ;

w_unlock (WS) ;

sound: disallow lost update (LU) 

complete: allow write skew (WS) 

SI Reference Implementation: Attempt 3

SI (T) \triangleq

r_lock (RS \cup WS) ;

s := snapshot (RS) ;

r_unlock (RS \ WS) ;

promote (WS) ;

[T] ;

w_unlock (WS) ;

sound: disallow lost update (LU) ✓

complete: allow write skew (WS) ✓

1. **Prescient** implementation:

requires prior knowledge of RS and WS

\Rightarrow see paper for **non-prescient** variant

2. May **deadlock**

\Rightarrow see paper for **non-deadlocking** implementations

(prescient and non-prescient)

What about **mixed** accesses?

RSI = SI + **mixed** accesses

What about **mixed** accesses?

$$\text{acyclic}(\text{rsi-hb}_{loc} \cup \text{mo} \cup \text{rb})$$

where

$$\text{rsi-hb} \triangleq (\text{rsi-po} \cup \text{rsi-rf} \cup \text{mo}_T \cup \text{si-rb})^+$$

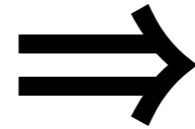
$$\text{rsi-po} \triangleq (\text{po} \setminus \text{po}_I) \cup [\mathcal{W}]; \text{po}_I; [\mathcal{W}]$$

$$\text{rsi-rf} \triangleq (\text{rf}; [\mathcal{NT}]) \cup ([\mathcal{NT}]; \text{rf}; \text{st}) \cup \text{rf}_T \cup (\text{mo}; \text{rf})_T$$

$$\text{si-rb} \triangleq [\mathcal{R}_E]; \text{rb}_T; [\mathcal{W}] \quad \text{and} \quad \mathcal{R}_E \triangleq \{r \mid \exists w. (w, r) \in \text{rf}_E\}$$

RSI = SI + **mixed** accesses

What about **mixed** accesses?



Lock-based

RSI reference implementation

$$\mathbf{RSI} = SI + \mathbf{mixed} \text{ accesses}$$

RSI: SI + Mixed Accesses

```
x := 1;
```

```
y := 1;
```

```
||
```

```
T:
```

```
[ a := x; // 0
```

```
  b := y; // 1
```

RSI: SI + Mixed Accesses

```
x := 1;
y := 1;
||
T:
[ a := x; // 0
  b := y; // 1
```

snapshot (RS) \triangleq

for (x ∈ RS) s[x] := x

RSI: SI + Mixed Accesses

```
x := 1;
y := 1;
||
T:
[ a := x; // 0
  b := y; // 1
```

Solution: read every location *twice!*

`snapshot (RS) \triangleq`

```
for (x $\in$ RS) s[x] := x
```

`snapshot_RSI (RS) \triangleq`

```
// tentative snapshot
```

```
for (x $\in$ RS) s[x] := x
```

```
// validate snapshot
```

```
for (x $\in$ RS)
```

```
    if (s[x]  $\neq$  x) snapshot (RS)
```


RSI: SI + Mixed Accesses

Caveat: non-transactional writes
with same value
cannot race
with the same transaction

```
// 0  
// 1
```

Solution: read every location *twice!*

`snapshot (RS) \triangleq`

```
for (x $\in$ RS) s[x] := x
```

`snapshot_RSI (RS) \triangleq`

```
// tentative snapshot
```

```
for (x $\in$ RS) s[x] := x
```

```
// validate snapshot
```

```
for (x $\in$ RS)
```

```
    if (s[x]  $\neq$  x) snapshot (RS)
```

RSI Reference Implementation

SI (T) \triangleq

```
r_lock (RS  $\cup$  WS) ;
```

```
s := snapshot (RS) ;
```

```
r_unlock (RS  $\setminus$  WS) ;
```

```
promote (WS) ;
```

```
[ T ] ;
```

```
w_unlock (WS) ;
```

RSI Reference Implementation

```
RSI (T)  $\triangleq$   
r_lock (RS  $\cup$  WS) ;  
s := snapshot_RSI (RS) ;  
r_unlock (RS  $\setminus$  WS) ;  
promote (WS) ;  
[ T ] ;  
w_unlock (WS) ;
```

- **Sound:** behaviours(imp) \subseteq behaviours(**RSI_spec**)
- **Complete:** behaviours(**RSI_spec**) \subseteq behaviours(imp)

RSI Reference Implementation

```
RSI (T)  $\triangleq$   
r_lock (RS  $\cup$  WS) ;  
s := snapshot_RSI (RS) ;  
r_unlock (RS  $\setminus$  WS) ;  
promote (WS) ;  
[ T ] ;  
w_unlock (WS) ;
```

- ➔ **Sound:** behaviours(imp) \subseteq behaviours(**RSI_spec**)
- ➔ **Complete:** behaviours(**RSI_spec**) \subseteq behaviours(imp)

1. **Prescient** implementation

⇒ see paper for **non-prescient** variant

2. May **deadlock**

⇒ see our paper for **non-deadlocking** implementations
(prescient and non-prescient)

Conclusions

- ✓ **Sound & complete** lock-based reference implementation for **SI**
 - ▶ **prescient** and **non-prescient** variants
- ✓ *Declarative* **RSI** semantics with **mixed** accesses
- ✓ **Sound & complete** reference implementation for **RSI**
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Conclusions

- ✓ **Sound & complete** lock-based reference implementation for **SI**
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Thank you for listening!